Moment of Inertia and The Parallel Axis Theorem

The following questions are concept based. For questions with higher level difficulty, watch videos in the course.

Q1. A solid disk of mass 2 kg and radius 0.5 m is rotating about its central axis at an angular velocity of 10 rad/s. What is its rotational kinetic energy?

A. 5 J
B. 10 J
C. 25 J
D. 50 J

Answer: B. 10 J

Explanation: The rotational kinetic energy (K) is given by $K = 1/2\ I\omega^2$. For a solid disk, the moment of inertia (I) is $1/2\ MR^2$. Substituting the given values, $I = 1/2\ (2\ kg)\ (0.5\ m)^2 = 0.25\ kg \cdot m^2$. Therefore, $K = 1/2\ (0.25\ kg \cdot m^2)\ (10\ rad/s)^2 = 10\ J$.

Q2. Which of the following statements about the moment of inertia is correct?

A. It depends only on the mass of the object.
B. It is the same for all axes of rotation.
C. It depends on both the mass of the object and its distribution relative to the axis of rotation.
D. It is always less when the object is rotated about an axis through its centre of mass.

Answer: C. It depends on both the mass of the object and its distribution relative to the axis of rotation.

Explanation: The moment of inertia (I) depends on the mass of the object and how the mass is distributed with respect to the axis of rotation. It changes if the axis of rotation changes, as different mass distributions relative to different axes affect the value of I.

Q3. A thin rod of length L and mass M is pivoted about one end and rotates in a plane perpendicular to its length. What is its moment of inertia about the pivot?

A. $(1/12)\ ML^2$
B. $(1/3)\ ML^2$
C. $(1/2)\ ML^2$
D. ML^2

Answer: B. $(1/3)\ ML^2$

Explanation: For a thin rod pivoted about one end, the moment of inertia (I) is given by $I = (1/3)\ ML^2$.

Q4. If the same thin rod (length L, mass M) is pivoted about its center, what would be its moment of inertia?

A. $(1/12)\ ML^2$
B. $(1/3)\ ML^2$
C. $(1/2)\ ML^2$
D. ML^2

Answer: A. $(1/12)\ ML^2$

Explanation: For a thin rod pivoted about its center, the moment of inertia (I) is given by $I = (1/12)\ ML^2$.

Q5. A particle of mass 0.5 kg is moving in a circular path of radius 2 m with an angular velocity of 4 rad/s. Calculate its linear velocity.

A. 1 m/s
B. 2 m/s
C. 4 m/s
D. 8 m/s

Answer: C. 4 m/s

Explanation: The linear velocity (v) is related to the angular velocity (ω) by the equation $v = \omega r$. Substituting the given values, $v = (4\ rad/s)(2\ m) = 8\ m/s$.

Q6. What is the total kinetic energy of a system of particles if each particle has a different speed but the same angular velocity around a fixed axis?

A. $1/2\ I\omega^2$
B. $\Sigma\ 1/2\ m_i v_i^2$

C. $\Sigma m_i r_i^2$
D. $1/2\ Mv^2$

Answer: A. $1/2\ I\omega^2$

Explanation: The total kinetic energy of a rotating system of particles is given by the rotational kinetic energy formula $K = 1/2\ I\omega^2$, where I is the moment of inertia and ω is the angular velocity.

Q7. A ring and a disk of the same mass and radius are rolling down an incline without slipping. Which one reaches the bottom first and why?

A. The ring, because it has a higher moment of inertia.
B. The disk, because it has a lower moment of inertia.
C. Both reach at the same time because they have the same mass.
D. The disk, because it has a higher mass distribution near the axis.

Answer: B. The disk, because it has a lower moment of inertia.

Explanation: The disk reaches the bottom first because it has a lower moment of inertia ($I_{disk} = 1/2\ MR^2$) compared to the ring ($I_{ring} = MR^2$). This means the disk has less rotational inertia to overcome and thus accelerates faster.

Q8. Calculate the rotational inertia of a thin rectangular plate of mass M, length L, and width W rotating about an axis through one of its edges and parallel to its width.

A. $(1/12)\ M(L^2 + W^2)$
B. $(1/3)\ M(L^2)$
C. $(1/3)\ M(W^2)$
D. $(1/2)\ M(L^2)$

Answer: B. $(1/3)\ M(L^2)$

Explanation: For a thin rectangular plate rotating about an axis along its length, the moment of inertia is $I = (1/3)\ ML^2$.

Q9. If the angular velocity of a rotating object is doubled, how does its rotational kinetic energy change?

A. It remains the same.
B. It doubles.
C. It quadruples.
D. It halves.

Answer: C. It quadruples.

Explanation: Rotational kinetic energy (K) is given by $K = 1/2\ I\omega^2$. If the angular velocity (ω) is doubled, the kinetic energy becomes $(1/2)\ I(2\omega)^2 = 4\ (1/2\ I\omega^2)$. Therefore, it quadruples.

Q10. A cylindrical hoop of mass 3 kg and radius 0.4 m is rotating about its central axis with an angular velocity of 15 rad/s. What is its rotational kinetic energy?

A. 36 J
B. 45 J
C. 54 J
D. 90 J

Answer: C. 54 J

Explanation: The rotational kinetic energy (K) is given by $K = 1/2\ I\omega^2$. For a cylindrical hoop, the moment of inertia (I) is MR^2. Substituting the given values, $I = (3\ kg)\ (0.4\ m)^2 = 0.48\ kg \cdot m^2$. Therefore, $K = 1/2\ (0.48\ kg \cdot m^2)\ (15\ rad/s)^2 = 54\ J$.