# 

# A-LEVEL MATHEMATICS 7357/2

Paper 2

Mark scheme

June 2019

Version: 1.0 Final



Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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# Mark scheme instructions to examiners

# General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

# Key to mark types

Μ	mark is for method
R	mark is for reasoning
А	mark is dependent on M or m marks and is for accuracy
В	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

# Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

# AS/A-level Maths/Further Maths assessment objectives

A	0	Description					
	AO1.1a	Select routine procedures					
AO1	AO1.1b	Correctly carry out routine procedures					
	AO1.2	Accurately recall facts, terminology and definitions					
	AO2.1	Construct rigorous mathematical arguments (including proofs)					
	AO2.2a	Make deductions					
AO2	AO2.2b	Make inferences					
	AO2.3	Assess the validity of mathematical arguments					
	AO2.4	Explain their reasoning					
	AO2.5	Use mathematical language and notation correctly					
	AO3.1a	Translate problems in mathematical contexts into mathematical processes					
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes					
	AO3.2a	Interpret solutions to problems in their original context					
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems					
AO3	AO3.3	Translate situations in context into mathematical models					
	AO3.4	Use mathematical models					
	AO3.5a	Evaluate the outcomes of modelling in context					
	AO3.5b	Recognise the limitations of models					
	AO3.5c	Where appropriate, explain how to refine models					

Examiners should consistently apply the following general marking principles

#### **No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

#### Otherwise we require evidence of a correct method for any marks to be awarded.

#### Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

## Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

## Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q	Marking instructions	AO	Mark	Typical solution
1	Ticks the correct response	2.2a	R1	
	Total		1	

Q	Marking instructions	AO	Mark	Typical solution
2	Circles the correct response	1.1b	B1	$a^{\frac{8}{15}}$
	Total		1	

Q	Marking instructions	AO	Mark	Typical solution
3	Circles the correct response	1.2	B1	$f(x) = x^2$
	Total		1	

Q	Marking instructions	AO	Mark	Typical solution
4	Explains how the factor theorem	2.4	E1	As $(x+2)$ is a factor, then when
	applies with reference to			x = -2, $f(x) = 0$
	f(-2) = 0 for either function			
	Explains that either quadratic			4 - 2b + c = 0
	expression can be factorised in			4 - 2d + e = 0
	the form $(x+2)(x+p)$ as			4 - 2h + c = 4 - 2d + e
	(x+2) is a factor			2d - 2b - a - a
	or Fundaine that an division by			2a - 2b = e - c
	Explains that on division by $(r + 2)$ the remainder would be			2(d-b) = e-c
	(x + 2) the remainder would be zero			
	Uses the factor theorem	1.1a	M1	
	with $x = -2$ substituted into one			
	of the expressions to obtain a			
	correct expression			
	to zero for this mark			
	or			
	Expands one of their factorised			
	forms and equates coefficients			
	$(x+2)(x+p) = x^{2} + (p+2)x + 2p$			
	p+2=b			
	2p = c			
	or Divides and the second second			
	Divides one of the expressions by $(r + 2)$ to obtain a correct			
	(x + 2) to obtain a conject remainder. Fither one of			
	4-2b+c			
	4 - 2d + e			
	1 20 10			
	Deduces both correct equations	2.2a	A1	
	using factor theorem or division			
	4 - 2b + c = 0			
	4 - 2d + e = 0			
	<b>PI</b> by $4-2b+c = 4-2d+e$			
	OF Expands both of their factorised			
	forms and equates coefficients to			
	deduce the correct equations –			
	must not use $p$ in both			
	Forms a single equation for	2.1	R1	
	b, c, d and $e$ and completes			
	rigorous argument to show the			
	<b>NB</b> R1 can be awarded even if			
	E1 was <b>not</b> awarded			
	Total		4	

Q	Marking instructions	AO	Mark	Typical solution
5	Separates the variables – one	3.1a	M1	
	side correct			
	Condone missing integral signs			$\frac{1}{r^2} \ln x  \mathrm{d}x = t  \mathrm{d}t$
	PI by correct integration			
	Integrates their ftdt correctly	1.1b	A1F	$t dt = \frac{t^2}{c} + c$
				<b>J</b> 2
		1.1b	B1	$u = \ln x$
	Obtains $u' = -and v =r$ OE			. 1
		1.1a	M1	$u' = \frac{1}{r}$
	Integrates $\frac{1}{x^2} \ln x  dx$			
	JX			$v' = x^2$
	Substitutes their $u = u' = v$ and $v'$			$v = -x^{-1}$
	into the correct formula for			
	integration by parts			$\left  -\frac{1}{2} \ln x - \right  \frac{1}{2} (-x^{-1}) dx$
	Condone sign errors in formula			$\int -\frac{1}{2} \ln x + \int \frac{1}{2} dx$
	Obtains	1.1b	A1	$x$ J $x^2$
	1 1			1, 1
	$\ln x x$			$\int \frac{\ln x}{x} \frac{\pi}{x}$
	Substitutes $t = 2$ and $x = 1$ into	1.1a	M1	
	their integrated equation to find			
	their + $c$			$\int -\frac{1}{2} \ln x - \frac{1}{2} = \frac{t^2}{2} + c$
				x $x$ $x$ $2$
	Obtains correct solution must	2.5	A1	$t = 2, x = 1 \Longrightarrow -1 = 2 + c$
	have $t^2 =$			c = -3
	ACF			
				$t^{2} = 6 - 2 \left( \frac{1 + \ln x}{1 + \ln x} \right)$
	Total		7	

Q	Marking instructions	AO	Mark	Typical solution
6	Compares with $R\cos(x\pm \alpha)$ or	3.1a	M1	$R\sin(x+\alpha) = a\sin x + b\cos x$
	$R\sin(x+\alpha)$			、 <i>,</i> ,
	by forming an identity e.g.			R - A
	$R\sin(r+\alpha) = a\sin r + b\cos r$			$\Lambda - 4$
	$A\sin(x+\alpha) = a\sin x + b\cos x$			
	or			$4\sin\left(\frac{\pi}{2}+\alpha\right)=2\sqrt{3}$
	Differentiates correctly and			$\left(3^{+\alpha}\right)^{-2\sqrt{3}}$
	equates to zero CAO PI by			$\pi$
	$a\cos x = b\sin x$			$\alpha = \frac{1}{3}$
	<b>PI</b> by			
	$R = 4 \text{ or } a^2 + b^2 = 16$			$\pi$ -
	Deduces $R = 4$	2.2a	A1	$a=4\cos\frac{\pi}{3}=2$
	or			$\pi$ , $\pi$ , $\pi$
	$a^2 + b^2 = 16$			$b = 4\sin\frac{\pi}{3} = 2\sqrt{3}$
	Forms a correct equation for $\alpha$	1.1b	B1	5
	PI by			
	or $\alpha$			
	Forms the equation shown			
	below			
	$2\sqrt{3} = \frac{a\sqrt{3}}{2} + \frac{b}{2}$ <b>OE</b>			
	Must substitute correct exact			
	Values for the trig functions	1 1 2	N/1	
	any correct value of $\alpha$	1.1a		
	Correct values are shown below			
	$\alpha = \frac{\pi}{3}$ or 0 for $R\sin(x \pm \alpha)$			
	$\alpha = \pm \frac{\pi}{6} \text{ for } R \cos(x \pm \alpha)$			
	Or Fliminatos o variable correctly			
	Finales a variable correctly from their two equations – <b>must</b>			
	obtain a correct simplified			
	equation			
	Deduces $a = 2$	2.2a	R1	
	Deduces $b = 2\sqrt{3}$	2.2a	K1	
	Total		6	

Q	Marking instructions	AO	Mark	Typical solution
7(a)	Sketches any cubic graph,	1.2	B1	
	crossing the <i>x</i> -axis in three			
	Sketches any cubic graph with a	12	B1	
	positive coefficient of $x^3$	1.2		
				$\land$
7(b)(i)		1 1 2	M1	
<i>'</i> (S)(I)	Differentiates to obtain $\Gamma(x)$	1.14		For a turning point $\Gamma(x) = 0$
	I wo terms with at least one correct without $2x^2$ or 6 mm			$f(x) = x^3 + 3px^2 + q$
	$\frac{1}{2} \frac{1}{2} \frac{1}$	1 1b	Δ1	$f'(x) = 3x^2 + 6px$
	Solves $3x^2 + 6px = 0$ to obtain	1.10		
	x = 0 or $x = -2p$			$3x^2 + 6px = 0$
	Substitutes $x = 0$ in			3x(x+2p) = 0
	$f'(x) = 3x^2 + 6px$ and obtains 0			x = 0
	Obtains the correct two roots	2.4	R1	x = -2p
	x = 0 and $x = -2p$ OE			
	and states why there must be a turning point referring to root			Since one of the roots is $x = 0$ there
	x = 0			must be a turning point on the y axis
7(b)(ii)	Deduces that turning point at	2.2a	B1	Since $p > 0$
	x = -2p is a maximum or			x = -2p is the maximum
	deduces that turning point $x = 0$			x = 0 is the minimum
	May have been seen in part			
	(b)(i)			f(0) = q
	Accept a sketch showing correct			$f(-2p) = (-2p)^3 + 3p(-2p)^2 + q$
	relative positions of turning			$-4n^3+a$
	Substitutes their $x = -2p$ into	1.1a	M1	-+p+q
	f(x)			
	Obtains correct $f(0) = q$ and	1.1b	A1	1 3 0
	$f\left(-2p\right) = 4p^3 + q$			-+p < q < 0
	Deduces	2.2a	R1	
	either $q < 0$ or $-4p^3 < q$			
	Condone ≤			
	Deduces $-4p^3 < q < 0$ CAO	2.2a	R1	
	Total		10	

Q	Marking instructions	AO	Mark	Typical solution
8(a)	Takes logs of both sides of the	1.1a	M1	$\log_{10} V = \log_{10} p q^t$
	equation and applies addition			
	rule			$\log_{10} V = \log_{10} p + \log_{10} q^t$
	Completes rigorous argument to	2.1	R1	
	show required result			$\log_{10} V = \log_{10} p + t \log_{10} q$
	Condono missing bass			
8(b)	Equators log to 3.90	34	M1	$\log n = 3.90$
0(D)	Equales $\log_{10} p$ to 3.90	5.4		$\log_{10} p = 3.90$
	Forms two simultaneous			n = 7940
	equations using points from the			p = 7340
	line of best fit only			528 - 390
	Calculates gradient and	3.4	M1	$\log q = \frac{3.26 + 3.96}{40 - 0} = 0.0345$
	equates to $\log_{10} q$			40-0
	or			a = 1.08
	Solves their pair of simultaneous			<i>q</i> = 1.00
	equations to obtain $p$ and $q$			
	Obtains correct	1.1b	A1	
	AWRT 8000			
	CSO			-
	Obtains correct q	1.1b	A1	
	AWRT 1.1			
9(a)		2.4	N/4	
0(0)	Substitutes $V = 500000$ into their $V = 7040 + 4.00^{4}$	3.4		$500000 = 7940 \times 1.08^{\circ}$
	$V = 7940 \times 1.00$			t = 53.82
	$\log V = \log 7040 + t \log 1.08$			
	$\log_{10} v = \log_{10} 7940 + i \log_{10} 1.08$			
	PI by correct t value			
	Solves their equation for t	1.1a	M1	
	Must have $t > 40$			
	States their correct year using	3.2a	A1F	The house will first be worth half a
	1970+ their integer part of $t$			million pounds during 2023
	Must be later than 2010			
8(d)	Explains that their 2023	3.5b	E1F	The model is only based on data
	(FT later than 2010) is outside			between 1970 and 2010
	the range of data collected			
	Explains that house prices may	3.2b	E1	House prices may not continue to
	not continue to grow in the same			grow in the same way indefinitely
	way			
	Must refer to context not just to			
	extrapolation/pattern			
	Can be implied by comments			
	such as:			
	Theresa may have made			
	improvements by adding a new			
	room			
	Prices could fall in a market			
	crash		11	
	I otal		1.1	

Q	Marking instructions	AO	Mark	Typical solution
9(a)	Write in a form to which the	3.1a	M1	$(-r^2)^{\frac{1}{2}}$
	applied			$\sqrt{4-2x^2} = 2\left(1-\frac{x}{2}\right)$
	Must be of form $a \left(1 - \frac{x^2}{2}\right)^{\frac{1}{2}}$			$\approx 2\left(1 + \frac{1}{2}\left(-\frac{x^2}{2}\right)\right)$
	Completes rigorous argument to obtain correct expansion <b>AG</b>	2.1	R1	$\approx 2 - \frac{x^2}{2}$
9(b)	Compares their $\frac{x^2}{2}$ to 1	1.1a	M1	$\left  -\frac{x^2}{2} \right  < 1$
	Condone incorrect inequality			
	<b>PI</b> by $ -2x^2  < 4$			$\Rightarrow  x  < \sqrt{2}$
	Obtains correct range of values ACF	1.1b	A1	
9(c)	Explains that as 0.4 radians is	2.4	E1	As 0.4 is small
	small therefore $\cos x \approx 1 - \frac{x^2}{2}$			$\cos x \approx 1 - \frac{x^2}{2}$
	Must refer to 0.4 and small			$\int 0.4$ $\int 0.4$ $\int x^2$
	angle approximation for cos x	1 1a	M1	$\int_{0} \sqrt{\cos x}  \mathrm{d}x \approx \int_{0} \sqrt{1 - \frac{x}{2}}  \mathrm{d}x$
	from <b>9(a)</b> as the integrand	1.10		$1 \int_{0.4}^{0.4} x^2 dx$
	Integrates their expression with	1.1a	M1	$\approx \frac{1}{2} \int_{0}^{1} 2 - \frac{1}{2} dx$
	Obtains correct value must be at least five decimal places	1.1b	A1	$\approx \int_{0}^{0.4} 1 - \frac{x^2}{4} dx$
	140			$[ r^3]^{0.4}$
	Condone $\frac{148}{375}$			$\approx \left\lfloor x - \frac{x}{12} \right\rfloor_{0}$
	CAO			$\approx 0.4 - \frac{0.4^3}{2}$
				12
9(d)	States that 1.4 radians is not a	2.4	E1	$\approx 0.39467$ Since 1.4 is not a small angle the
	small angle so the			approximation is not suitable
	approximation is not valid Must refer to small angle			
	approximation <b>and</b> 1.4			
	or State invalid as 1.4 is bigger			
	than 0.664			
	<b>NB</b> 0.664 is the limiting value for approximation to be valid			
	Total		9	

Q	Marking Instructions	AO	Marks	Typical Solution
10	Ticks correct box	2.2a	B1	The particle was decelerating for $12 \le t \le 20$
	Total		1	

Q	Marking Instructions	AO	Marks	Typical Solution
11	Circles correct answer	1.1b	B1	1000 N
	Total		1	

Q	Marking Instructions	AO	Marks	Typical Solution
12	Circles correct answer	1.1b	B1	-400
	Total		1	

Q	Marking Instructions	AO	Marks	Typical Solution
	States appropriate <i>suvat</i> equation	1.1a	M1	$v^2 = u^2 + 2as$
13(a)	and clearly identifies			u = 0 $a = g$ s=h
	s = h, a = g and $u = 0$			
	<b>PI</b> by $v^2 = 0^2 + 2gh$			$v^2 = 0^2 + 2gh$
	OE			$v = \sqrt{2gh}$
	Completes rigorous argument by	2.1	R1	V C
	rearranging correctly for $v$			
	Must have used consistent signs			
	for s and a			
(0(1))	AG	0.41		
13(b)	Substitutes two values in $v = \sqrt{2gh}$	3.1b	M1	When $g = 9.8$ and $h = 18$
	to find the third value <b>OE</b>			
	Obtains correct third value	1.1b	A1	$v = \sqrt{2 \times 9.8 \times 18} = 18.8$
	If finding v then accept <b>AWRT</b> 19			18.8 < 20
	If finding <i>q</i> then accept <b>AWRT</b> 11			
	If finding $h$ then accept <b>AWRT</b> 20			Machine is faulty
	Makes an appropriate comparison	2.2b	R1	
	for correct $v, g$ or $h$ and infers that			
	the teacher's claim is correct. The			
	comparison can be implied in their			
	comment, eg the value of v is less			
	than 20			
	or Makes an appropriate comparison			
	using $a = 10$ and infers that the			
	teacher's claim is incorrect. Their			
	answer <b>must</b> be rounded to 20.			
	The comparison can be implied in			
	their comment, eg the value			
	matches the given value of v			
	Total		5	

Q	Marking Instructions	AO	Marks	Typical Solution
14(a)	Finds a moment of a force about	1.1b	B1	Take moments about A
	any point. Must have the form			
	force x distance			$mg \times 0.04 = 0.28g \times 0.03$
	Can be awarded for 6R			
	PI by fully correct equation	0.0		
	Forms a fully correct moments	3.3	M1	m = 0.21
	equation using the correct model			
	Must have included $g$ on both sides			
	Moments about <i>B</i> gives (in metres)			
	0.28g(0.03) + 0.1mg =			
	0.06(0.28g + mg)			
	Solves equation to show $m = 0.21$ <b>AG</b>	1.1b	A1	
14(b)	Forms a moments equation for	3.1b	M1	Take moments about A
	equilibrium of rod with correct			
	number of terms – can use $m$ , 0.21			$0.21g \times 0.04 = 0.048g \times 0.05 \times n$
	or their value for <i>m</i> from part <b>14(a)</b>			
	Condone omission of $g$			n-35
	Enroughout part 14(b)	2.4		
	Forms a moments equation for	3.4	AIF	
	involving <i>n</i> correct – can use <i>m</i>			Maximum $n = 3$
	0.21 or their <i>m</i> value from <b>part</b>			
	14(a)			
	FT their incorrect <i>m</i>			
	Moments about <i>B</i> gives			
	0.06R = 0.00048ng + 0.1mg			
	Obtains a fully correct moments	1.1b	A1	
	equation with $m = 0.21$ substituted			
	Manager (a share) David			
	Noments about <i>B</i> gives $0.06(0.21 \pm 0.048 \pm 1)$			
	0.00(0.21g + 0.048ng) = 0.00048ng + 0.1(0.21)g			
	0.00046ng + 0.1(0.21)g Must have substituted correct			
	expression for $R$			
	States $n - 3$	1 1h	A1	-
	CSO	1.10		
14(c)	States an assumption about the	3.5b	E1	The rod is uniform
	rod	2.00		
	Accept			
	The mass/weight of the rod acts in			
	the middle			
	The rod is in limiting equilibrium <b>OE</b>			
	The rod is rigid			
	Total		8	

Q	Marking Instructions	AO	Marks	Typical Solution
15(a)	Finds $\overrightarrow{AB}$ or $\overrightarrow{CD}$ or $\overrightarrow{BC}$ or $\overrightarrow{DA}$	3.1a	M1	$\overrightarrow{AB} = \begin{bmatrix} -620\\ 100 \end{bmatrix}  \overrightarrow{CD} = \begin{bmatrix} 930\\ 270 \end{bmatrix}$
	correctly			L-1803 L2703
	label			
	$\overrightarrow{BC} = \begin{bmatrix} -130 \end{bmatrix}$ $\overrightarrow{DA} = \begin{bmatrix} -180 \end{bmatrix}$ OE			$CD = -1.5 \times AB$
	or			
	Finds gradient of <i>AB</i> or <i>CD</i> or <i>BC</i> or <i>DA</i> correctly			Thus <i>AB</i> and <i>CD</i> are parallel but not equal in length
	Gradient $AB = CD = \frac{9}{31}$ <b>OE</b>			ABCD is a trapezium but not a
	Gradient $BC = \frac{84}{13}$ <b>OE</b>			
	Gradient $DA = -\frac{25}{6}$ <b>OE</b>			
	Accept ratios $\frac{31}{9}, \frac{13}{84}, -\frac{6}{25}$ <b>OE</b>			
	Ignore any incorrect labelling of ratios here			-
	Finds <i>AB</i> and <i>CD</i> correctly <b>OE</b>	1.1b	A1	
	Or Finds gradients of $AB$ and $CD$			
	correctly			
	or			
	Finds a corresponding pair of ratios			
	reciprocals of gradients are			
	labelled as gradients or vectors			
	Shows/states $\overrightarrow{CD} = \pm 1.5 \times \overrightarrow{AB}$ <b>OE</b>	_		
	or	1.1b	A1	
	Shows/states that $BC \neq k \times DA$			
	Finds $\overrightarrow{BC}$ and $\overrightarrow{D4}$ correctly			
	or			
	Finds gradients of BC and DA			
	correctly			
	or Finds a second corresponding pair			
	of ratios correctly– <b>Do not</b> award if			
	reciprocals of gradients are			
	labelled as gradients or vectors			
	If incorrect labelling used for ratios			
	then maximum mark is			
	M1 A0 A0 E1 R0			

	Deduces that $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel - implied by reference to equal gradients or Deduces correctly that $\overrightarrow{BC}$ and $\overrightarrow{DA}$ are not parallel <b>NB</b> E1 is Independent of any other marks	3.2a	E1	
	Completes rigorous proof by deducing correctly that the scalar multiple of $\pm 1.5$ <b>OE</b> means the parallel sides are not equal in length or Completes rigorous proof by deducing correctly that $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel giving justification and that $\overrightarrow{BC}$ and $\overrightarrow{DA}$ are not parallel giving justification Must include a statement that <i>ABCD</i> is not a parallelogram at some point <b>NB</b> R1 can be awarded even if <b>E1</b> was <b>not</b> awarded <b>CSO</b>	2.1	R1	
15(b)	Uses velocity/displacement/time relationship Evidenced by dividing any vector /distance from part <b>15(a)</b> by 50	3.1b	M1	$\boldsymbol{\nu} = \frac{1}{50} \times \begin{bmatrix} -130\\ -840 \end{bmatrix}$ $\boldsymbol{\nu} = \begin{bmatrix} -2.6\\ -16.8 \end{bmatrix}$
	Finds the magnitude of their $\overrightarrow{BC}$ or $v$	1.1a	M1	<i>Speed</i> = $ v  = \sqrt{2.6^2 + 16.8^2}$
	Obtains 17	1.1a	A1	$Speed = 17 \text{ m s}^{-1}$
	States correct speed with correct units	3.2a	A1	•
	Total		9	

Q	Marking Instructions	AO	Marks	Typical Solution
16(a)	Differentiates to obtain $\frac{dv}{dv}$ with at least	3.4	M1	
	one exponent term correct			$\frac{dv}{dt} = 10.512e^{-0.9t} - 0.009e^{0.3t}$
	Obtains fully correct expression for $\frac{dv}{dt}$	1.1b	A1	Maximum $v$ occurs when $\frac{dv}{dt} = 0$
	Explains that maximum $v$ occurs when $\frac{dv}{dt} = 0$ Accept reference to stationary point	2.4	E1	$10.512e^{-0.9t} - 0.009e^{0.3t} = 0$
	Forms equation $\frac{dv}{dt} = 0$ and solves to find a value for <i>t</i>	1.1a	M1	
	PI by correct t Obtains correct value of t AWRT 5.9	1.1b	A1	t = 5.886
	Substitutes their $t$ into the given model <b>PI</b> by correct $v$	1.1b	M1	$v = 11.71 - 11.68e^{-0.9 \times 5.886} -0.03e^{0.3 \times 5.886}$
	Finds value for maximum v AWRT 11.5	1.1b	A1	v = 11.5
	Justifies final answer as being a maximum value eg: • This is the maximum value as it is the <b>only</b> value which relates to $\frac{dv}{dt} = 0$ • Evaluates second derivative at t = 5.9 where $\frac{d^2v}{dt^2} = -9.4608e^{-0.9t} - 0.0027e^{0.3t}$ obtaining correct value of -0.063 or explains both terms are negative so it is less than 0 • Tests first derivative considering gradient either side of <i>t</i> =5.9	2.4	R1	This is the maximum value as it is the <b>only</b> value which relates to $\frac{dv}{dt} = 0$
	<ul> <li>Sketches curve with maximum identified at (5.9, 11.5)</li> <li>CSO</li> </ul>			
	<b>NB</b> R1 can be awarded even if E1 was <b>not</b> awarded			

16(b)	Integrates at least one term correct	3.4	M1	$s = \int u dt$
	Integrates at least two terms correct	1.1a	M1	$=$ $3 - \int v u v$
	Obtains a fully correct integrated expression including a constant	1.1b	A1	$s = 11.71t + 12.978e^{-0.9t} - 0.1e^{0.3t} + c$
	Interprets initial conditions - states s = 0 when $t = 0PI by substitution of correct values$	3.4	B1	s = 0 when $t = 0$
	Substitutes $s = 0$ and $t = 0$ to find their constant – must be clear evidence of substitution seen if incorrect <i>c</i> obtained	1.1a	M1	c = -12.878
	Obtains fully correct expression for distance – coefficients can be in any form and do not have to be evaluated as a single decimal <b>ACF</b>	3.2a	A1	$-0.1e^{0.3t} - 12.878$
16(c)	Substitutes $t = 9.8$ into their expression for distance to find s <b>PI</b> by sight of 99.99 m for s or Substitutes $s = 100$ into their expression for distance to find t <b>PI</b> by sight of 9.801 for t	1.1a	M1	s = 99.99  m Model predicts distance to be 99.99 which is very near to 100 Accurate
	Compares s value with 100 metres or t value with 9.8 and concludes that it is a good model	3.5a	A1	
	Total		16	

Q	Marking Instructions	AO	Marks	Typical Solution
17(a)	Resolves vertically to form a three	3.1b	M1	
	term equation			
	Condone sign error or sin/cos error			$R + T\sin\theta = Mg$
	Obtains fully correct equation for	1.1b	A1	
	resolving vertically	0.41		$T\cos\theta - F = Ma$
	Uses Newton's second law	3.10	MT	
	equation			
	Condone sign error or consistent			$F = \mu R$
	cos/sin error			
	Obtains fully correct equation for	1.1b	A1	$T\cos\theta - \mu R = Ma$
	resolving horizontally			$T\cos\theta = \mu(M\sigma - T\sin\theta) = M\sigma$
	Uses $F = \mu R$ to replace F with $\mu R$ in	3.3	B1	$1\cos\theta = \mu(Mg - 1\sin\theta) = Mu$
	their horizontal equation			$T(\cos\theta + \mu\sin\theta) = Ma + \mu Mg$
	Eliminates R to form a single	1.1a	MT	
				$T - \frac{M(a + \mu g)}{m}$
	Completes rigorous argument to find	2.1	R1	$I = \frac{1}{\cos\theta + \mu\sin\theta}$
	required expression.			
	Must see Tas a factor before			
	division e a $T(\cos \theta + \mu \sin \theta)$			
	AG			
	Explains that the relationship may	2.4	B1	The sledge is at rest so the
17(b)	not be valid because the sledge is at			relationship may not be valid as
	rest			friction may not be acting at its
	Identifies that friction may not be at	3.5b	B1	limiting value
	Its limiting value			
	Accept reference to $F \leq \mu R$			
	sledge may not be on the point of			
	Total		9	