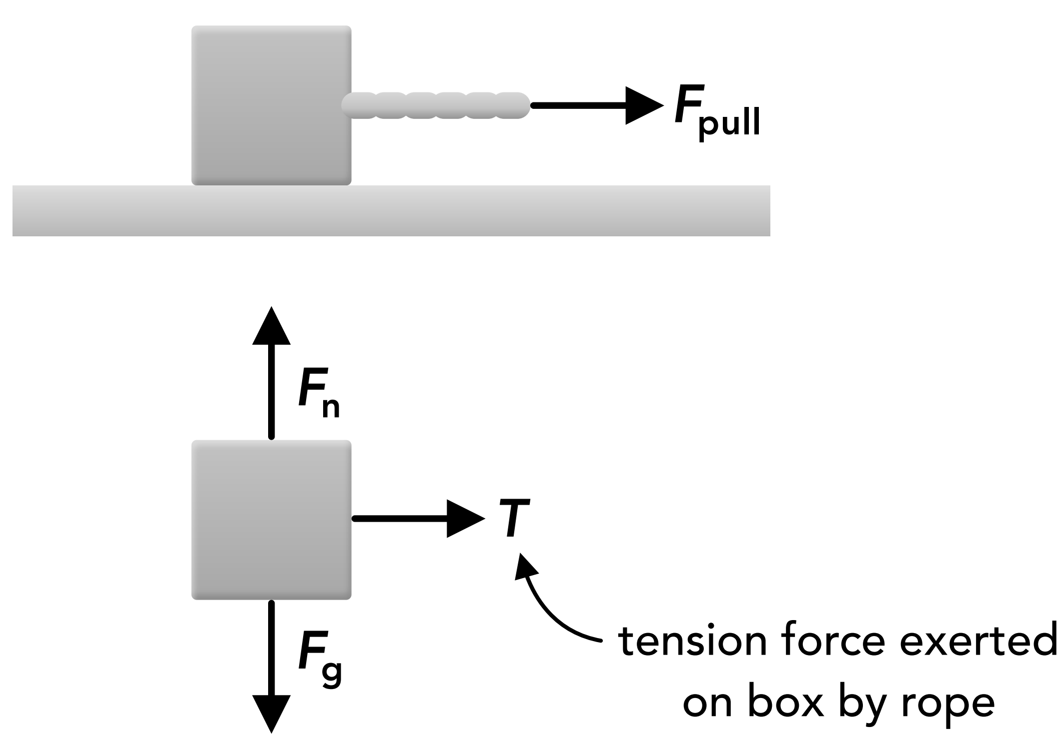
STUDY GUIDE

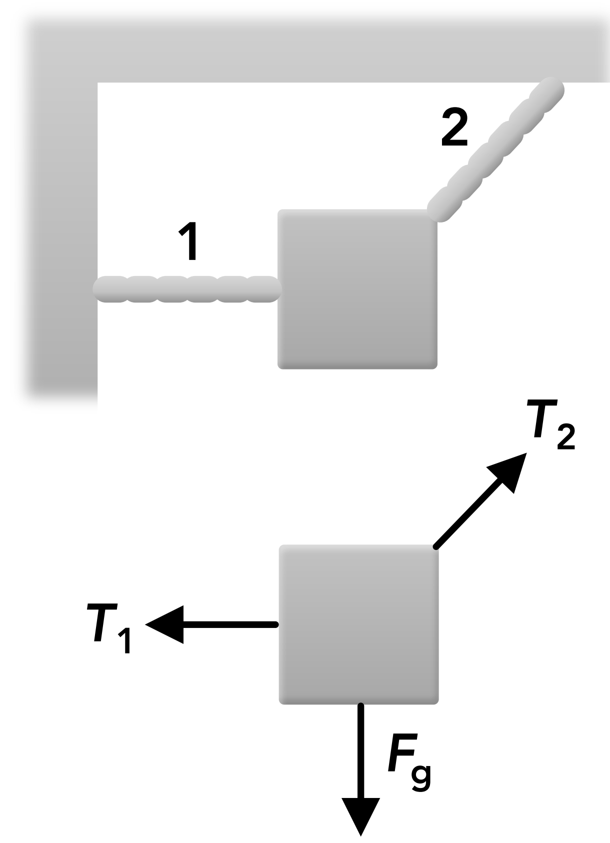
TENSION & PULLEY SYSTEMS

Tension

Variables		SI Unit
T	tension force	N

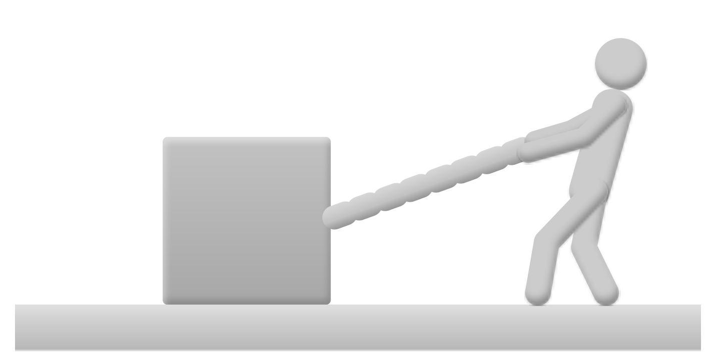
- **Tension** is a **pulling force** that we usually associate with ropes, strings, cables, wires, or other long and thin objects.
- If a rope is attached to an object or surface and the rope is pulled, a tension force arises in the rope and that same tension force is exerted on the objects at both ends of the rope.
- A tension force always acts in the same direction as the rope, and is always a pulling force (not a pushing force).



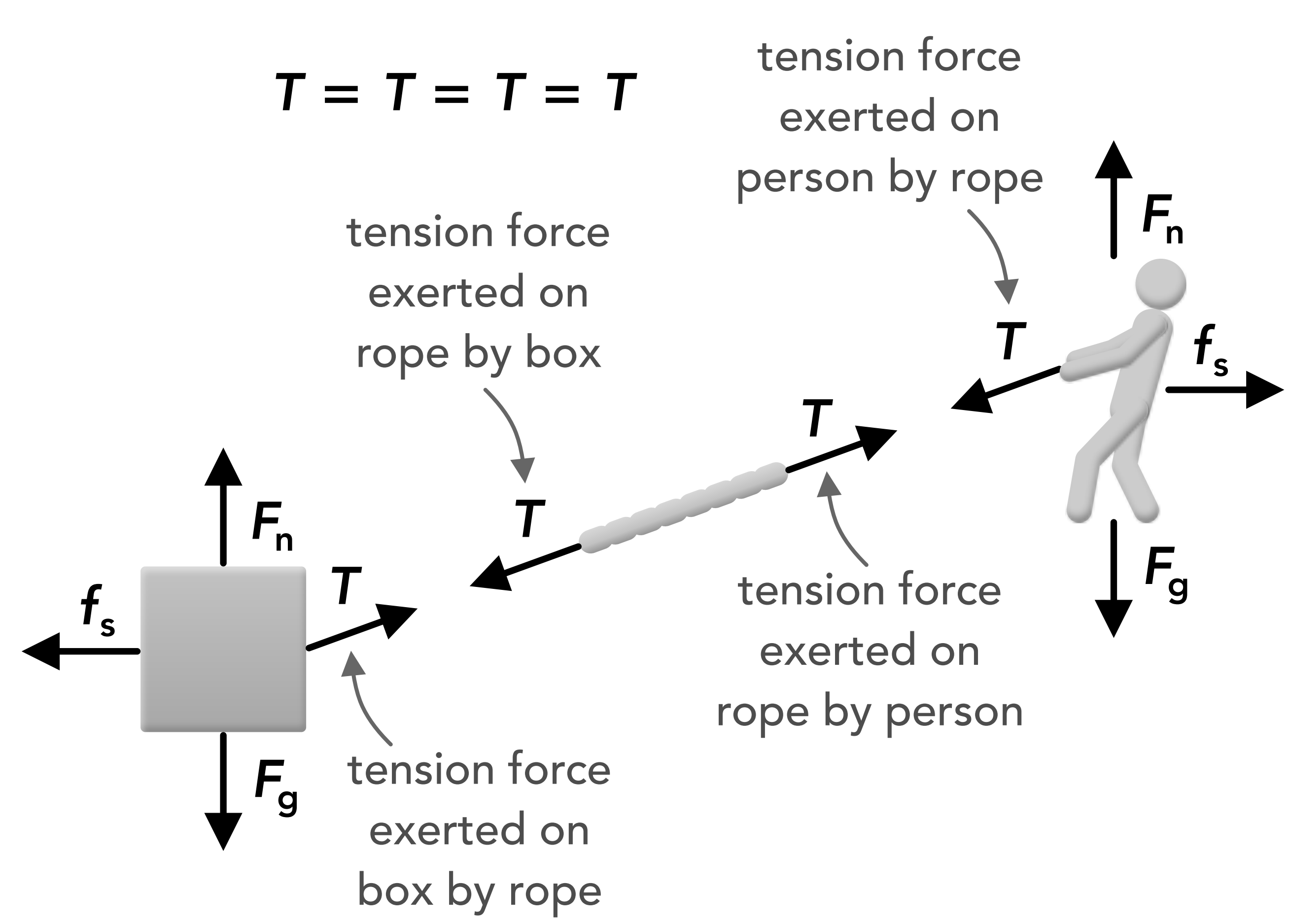


- An **ideal** rope (string, cable, etc.)…
  - is massless: the rope itself has no mass, no inertia and no weight
  - does not change length, regardless of the tension
- We assume that the **tension force is the same at both ends of an ideal rope but acts in opposite directions**. any change in the force at one end is instantly transmitted to the other end. This also applies to ropes passing around ideal pulleys.
- We could think of a rope as a spring with an infinite spring constant (stiffness) which never changes length no matter how much force is applied. Tension force in a rope is like the spring force in a spring.

A person pulling a rope attached to a box



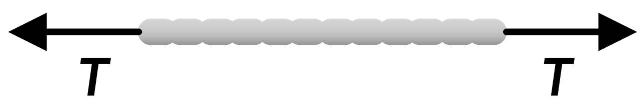
Free body diagrams of the box, rope and person



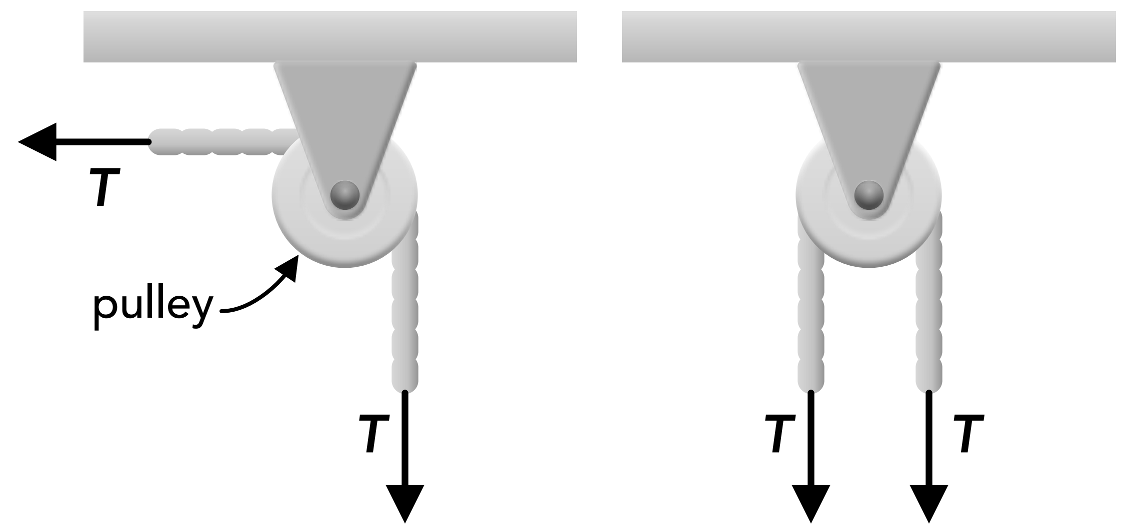
Pulley Systems

- When a rope passes around a pulley, **the rope and the tension force change direction**. The pulley rotates as the rope moves in either direction.

The tension force at the end of a rope acts in the direction of the end of the rope

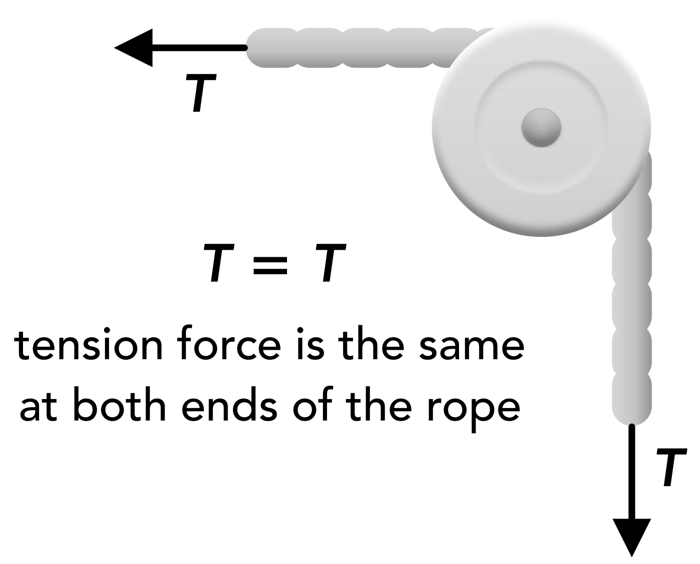


The rope and the tension force change direction around a pulley

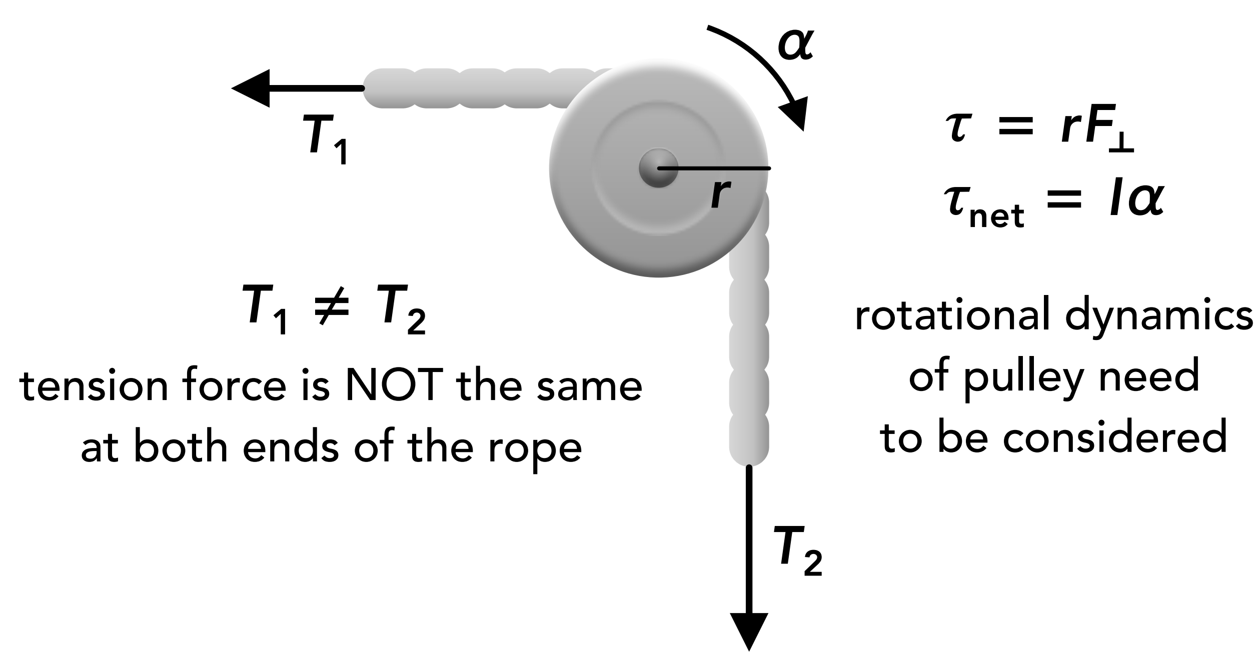


- An **ideal** pulley is...
  - massless: the pulley has no mass, no inertia and no weight
  - frictionless: there are no friction forces acting within the pulley and it would rotate freely forever
- When a rope passes around an ideal pulley, **the tension force is still the same at both ends of the rope**.
- When a rope passes around a real or non-ideal pulley that has mass, the pulley has rotational inertia and the torque and rotational dynamics of the pulley need to be considered in addition to the other objects involved.
- If a pulley is not frictionless, the mechanical energy of the system (the ropes and other objects) may be lost as thermal energy due to friction.

Ideal pulley (massless, frictionless)

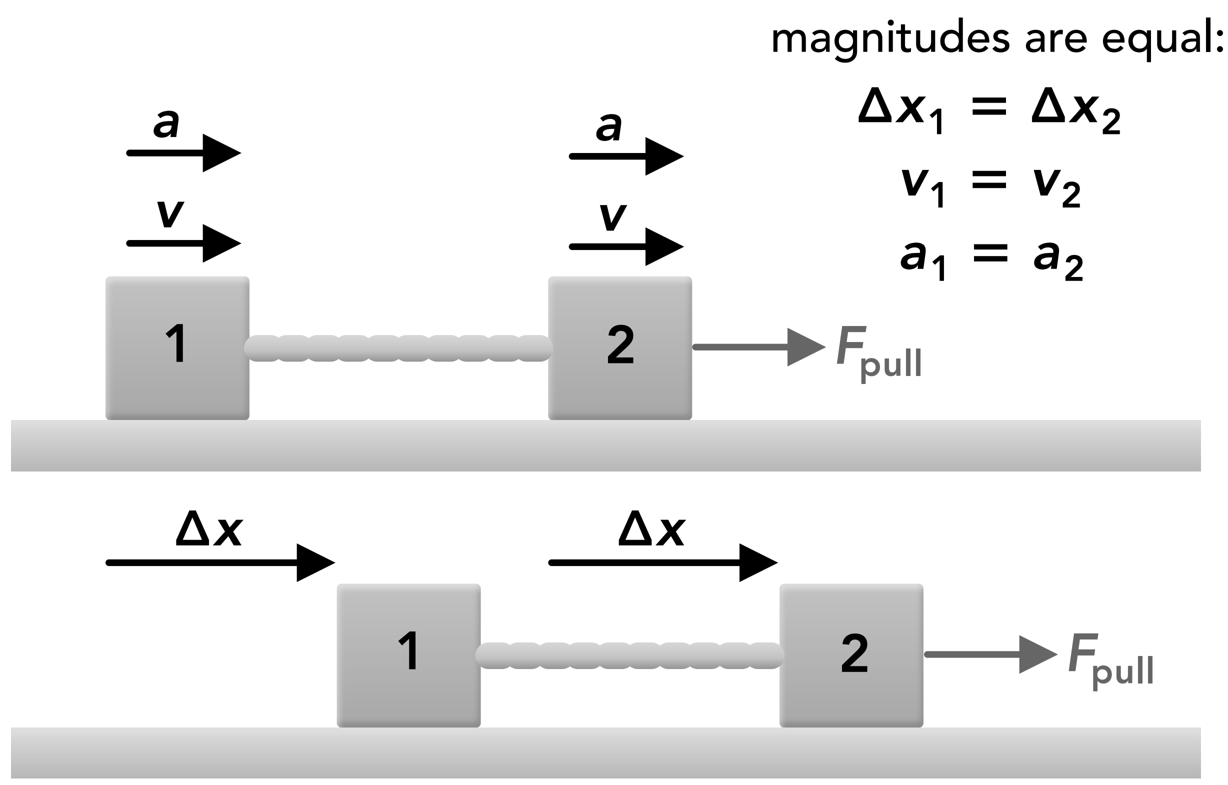


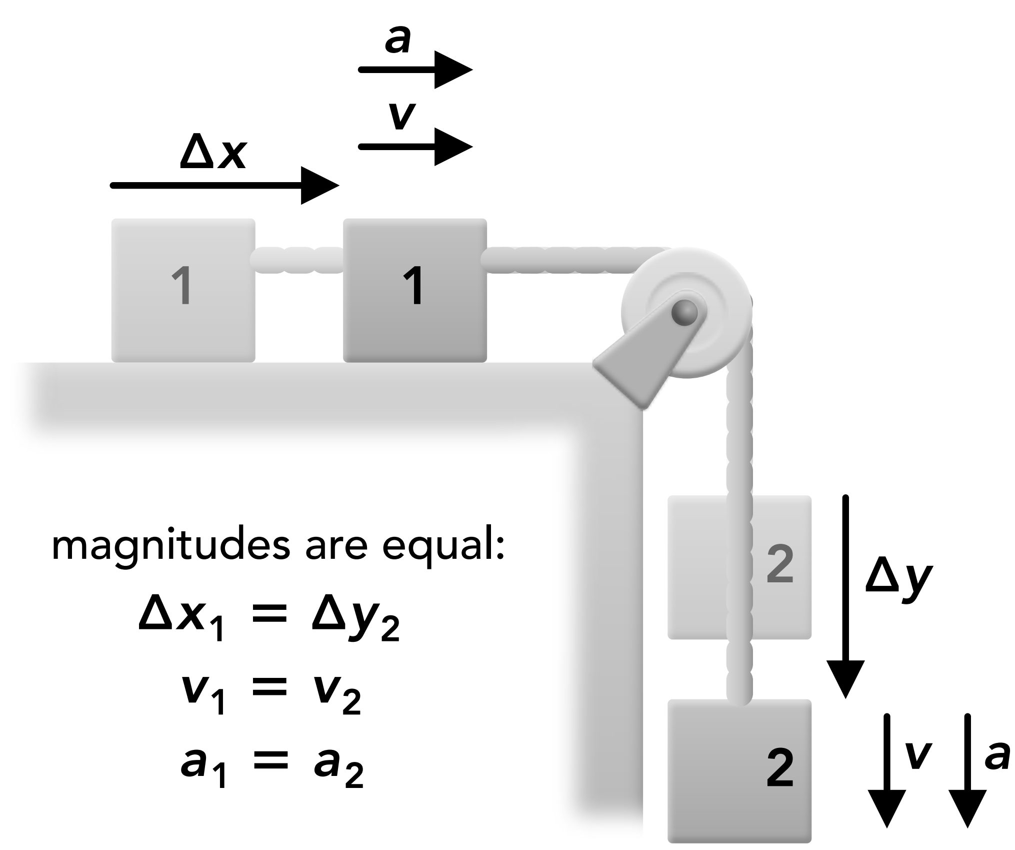
Real pulley (with mass)



- If the rope is ideal and doesn't change length, **both objects at each end of the rope move together**. During any period of time, both objects must move the same displacement (even if the directions are different) because they're attached to the same rope. If the two objects did not move the same displacement, the rope would have to change length or break.
- Since one object can't move faster than the other, **their displacements, velocities and accelerations have the same magnitude** (but the directions may be different).
- This means we can set the magnitudes of the accelerations equal to each other in order to solve a system of equations that we get from Newton's 2nd law for both obects.

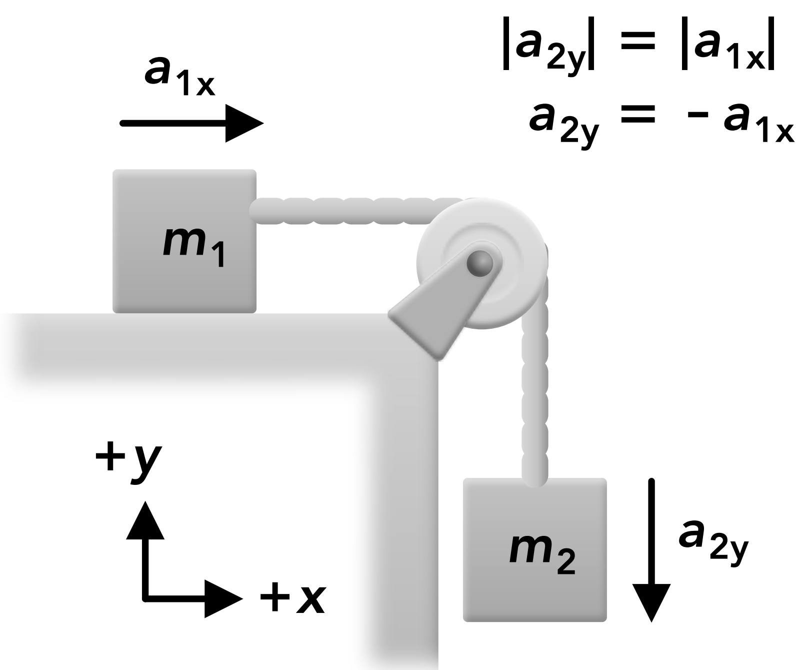
Objects connected by a rope have the same displacement, velocity and acceleration (magnitudes)



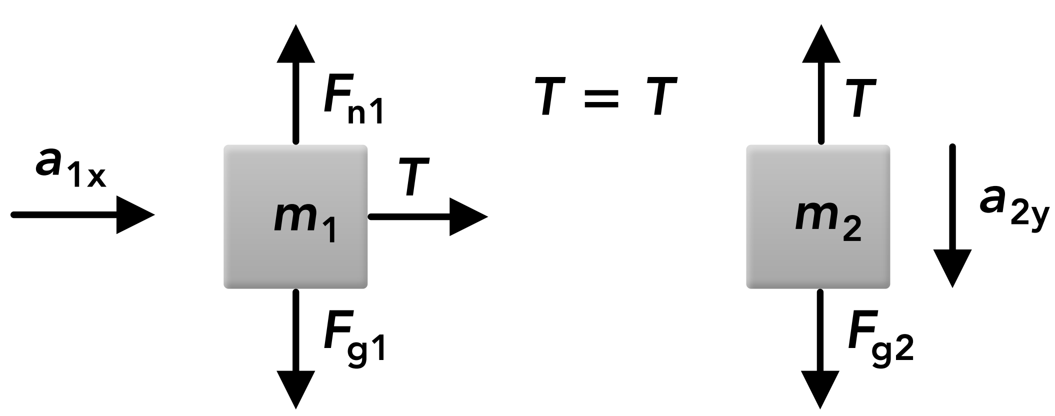


- This means that we can **treat both objects and the rope as a single system (or object)**. If we're trying to find the acceleration, we can draw free body diagrams and apply Newton's 2nd law to each object. Or we can draw a free body diagram and apply Newton's 2nd law to the **system using the total mass of each object**.
- If we do that, **the tension force becomes an internal force and is not included in the free body diagram**. So we can use this method to find the acceleration, but not the tension.
- Instead of using the x and y directions for the forces and acceleration, we can use a **new direction that is always parallel to the rope** as it bends around the pulley, and just focus on the positive and negative directions. Think of this as "straightening out" the rope into a "rope axis" while keeping the direction of the forces relative to the rope.

Separate free body diagrams and Newton's 2nd law for each block



Block 1: Block 2:



$$\sum F_y = ma_y$$
$$F_{n1} - F_{g1} = m_1(0)$$

$$\sum F_x = ma_x$$
$$T = m_1 a_{1x}$$

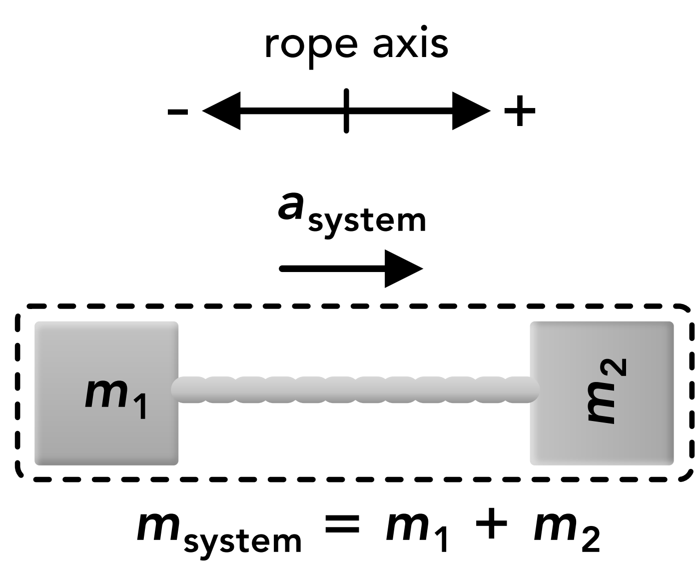
$$\sum F_y = ma_y$$
$$T - F_{g2} = m_2 a_{2y}$$
$$T = m_2 a_{2y} + F_{g2}$$
$$T = m_2 a_{2y} + m_2 g$$

$$T = T$$
$$m_1 a_{1x} = m_2 a_{2y} + m_2 g$$
$$a_{2y} = -a_{1x} \rightarrow m_1 a_{1x} = m_2(-a_{1x}) + m_2 g$$

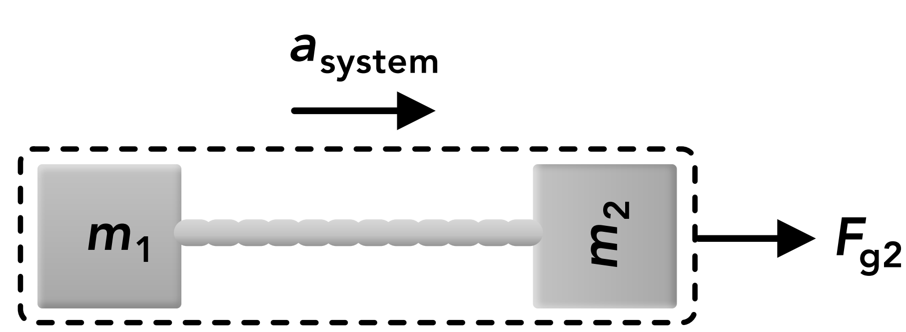
$$a_{1x} = \frac{m_2 g}{m_1 + m_2}$$

$$a_{2y} = -\frac{m_2 g}{m_1 + m_2}$$

The same blocks and rope are treated as a single system with a total mass, using a shared "rope axis"



System:



tension is an internal force, not included

net force in the "rope axis" direction:

$$\sum F = m_{system} a_{system}$$
$$F_{g2} = (m_1 + m_2) a_{system}$$
$$m_2 g = (m_1 + m_2) a_{system}$$

$$a_{system} = \frac{m_2 g}{m_1 + m_2}$$