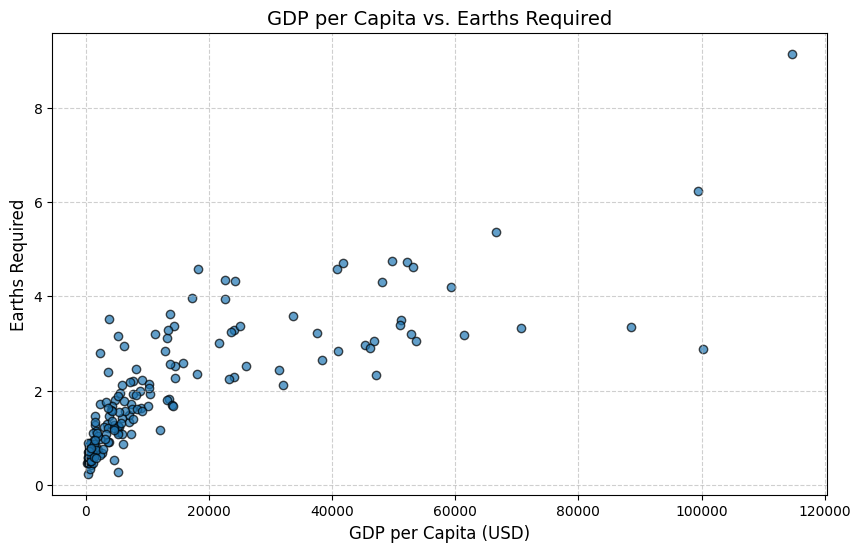
# **Guide to Interpreting Regression Models: Linear, Logarithmic, and Binary Transformations**

This document outlines how to interpret regression models under various transformations of the dependent and independent variables. We'll cover five scenarios:

1. **Linear - Linear**
2. **Linear - Log**
3. **Log - Linear**
4. **Log - Binary**
5. **Log - Log**

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Each section includes the mathematical expression, interpretation of coefficients, and practical implications.

## **1) Linear - Linear Regression**

### **Mathematical Form:**

Y=β0+β1X+ϵ

Where:

* Y = Dependent variable (Earths Required)
* X = Independent variable (GDP per Capita)
* β0 = Intercept
* β1 = Slope coefficient
* ϵ = Error term

### **Interpretation:**

In a linear-linear regression model, the slope coefficient β1 tells us the change in the dependent variable for a one-unit change in the independent variable.

**Example:** If β1=0.0000514, it means that for every **$1 increase in GDP per Capita**, the **Earths Required increases by 0.0000514 units**.

### **Practical Implication:**

This type of regression assumes a **constant linear relationship** between the variables. It works well when both variables are linearly related without significant skewness.

## **2) Linear - Log Regression**

### **Mathematical Form:**

Y=β0+β1log⁡(X)+ϵ

Where:

* Y = Dependent variable (Earths Required)
* log⁡(X) = Log-transformed independent variable (Log of GDP per Capita)

### **Interpretation:**

The slope coefficient β1 represents the change in the dependent variable for a **1% change in the independent variable**.

**Example:** If β1=0.73, it means that a **1% (0.01) increase in GDP per Capita** results in a **0.0073-unit increase in Earths Required**.

### **Practical Implication:**

This model is useful when the independent variable is highly skewed, and we want to interpret the effect of **percentage changes** rather than absolute changes.

## **3) Log - Linear Regression**

### **Mathematical Form:**

log⁡(Y)=β0+β1X+ϵ

Where:

* log⁡(Y) = Log-transformed dependent variable (Log of Earths Required)
* X = Independent variable (GDP per Capita)

### **Interpretation:**

The slope coefficient β1 represents the **percentage change in the dependent variable** for a **one-unit change in the independent variable**.

**Example:** If β1=0.00002357, it means that for every **$1 increase in GDP per Capita**, the **Earths Required increases by 0.002357%**.

### **Practical Implication:**

This model is useful when the dependent variable has exponential growth patterns, such as population growth or resource consumption.

## **4) Log - Binary Regression**

### **Mathematical Form:**

log⁡(Y)=β0+β1Xbinary+ϵ

Where:

* log⁡(Y) = Log-transformed dependent variable (Log of Earths Required)
* Xbinary = Binary independent variable (e.g., GDP Binary: 1 = Above Median, 0 = Below Median)

### **Interpretation:**

The slope coefficient β1 represents the **difference in the log of the dependent variable** between the two binary groups.

**Example:** If β1=1.06, it means that countries with a GDP per Capita above the median require **about exp(1.06) = 2.88 = 288% or 188% more Earths** compared to countries below the median.

### **Practical Implication:**

This model is useful when the independent variable is categorical (binary) and we want to compare the differences between two groups.

## **5) Log - Log Regression**

### **Mathematical Form:**

log⁡(Y)=β0+β1log⁡(X)+ϵ

Where:

* log⁡(Y) = Log-transformed dependent variable (Log of Earths Required)
* log⁡(X) = Log-transformed independent variable (Log of GDP per Capita)

### **Interpretation:**

The slope coefficient β1 represents the **elasticity** of the dependent variable with respect to the independent variable. It tells us the **percentage change in the dependent variable for a 1% change in the independent variable**.

**Example:** If β1=0.41, it means that a **1% increase in GDP per Capita** results in a **0.41% increase in Earths Required**.

### **Practical Implication:**

Log-log models are particularly useful for analyzing relationships where both variables exhibit exponential growth patterns. This model provides insights into **elasticity** and is commonly used in economics.

## **Summary Table of Interpretations**

| **Model Type** | **Mathematical Form** | **Interpretation** |
| --- | --- | --- |
| Linear - Linear | Y=β0+β1X | Absolute change in Y for a 1-unit change in X |
| Linear - Log | Y=β0+β1log⁡(X) | Change in Y for a 1% change in X |
| Log - Linear | log⁡(Y)=β0+β1X | Percentage change in Y for a 1-unit change in X |
| Log - Binary | log⁡(Y)=β0+β1Xbinary | Difference in log(Y) between binary groups |
| Log - Log | log⁡(Y)=β0+β1log⁡(X) | Percentage change in Y for a 1% change in X |

This guide provides a structured way to interpret various types of regressions, making it easier to explain relationships between variables under different transformations.