STATICS

Module 4

CIVIL ENGINEERING ACADEMY

- ✓ Resultants of force systems
- ✓ Equivalent force systems
- ✓ Equilibrium of rigid bodies
- ✓ Frames and trusses
- ✓ Centroid of area
- ✓ Area moments of inertia
- ✓ Static friction

(FE Reference)

OUTLINE

2-D EQUILIBRIUM STATE

There are 3 conditions that must be satisfied for static equilibrium:

$$\sum F_x = 0$$

Horizontal forces

$$\sum F_y = 0$$

Vertical forces

$$\sum M = 0$$

Moment

CASES

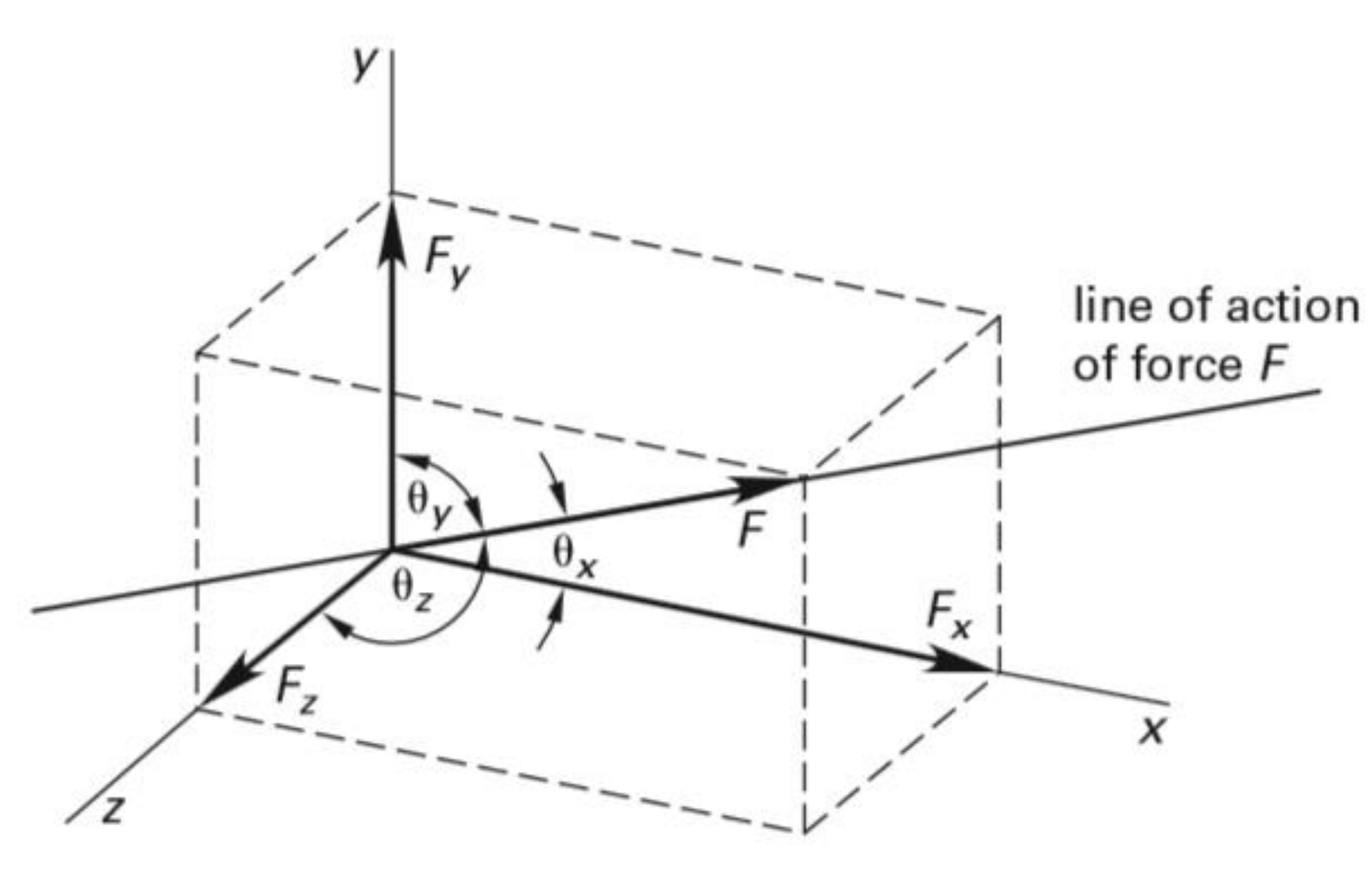
1. Structures at rest
2. Object with a constant velocity
3. Stable structures

FORCE SYSTEMS

Resultant force (F) is vector representation of three-dimensional force.

$$F = F_x i + F_y j + F_z k$$

The component of forces can be found by using the cosines of angle with respect to the corresponding axis:



$$F_x = F\cos\theta_x$$

$$F_y = F\cos\theta_y$$

$$F_z = F\cos\theta_z$$

$$F = \sqrt{F_x{}^2 + F_y{}^2 + F_z{}^2}$$

FORCE SYSTEMS

MOMENT → a force that rotates, turns, or twists an object. Also the cross product of the radius vector **r** and for vector **F**:

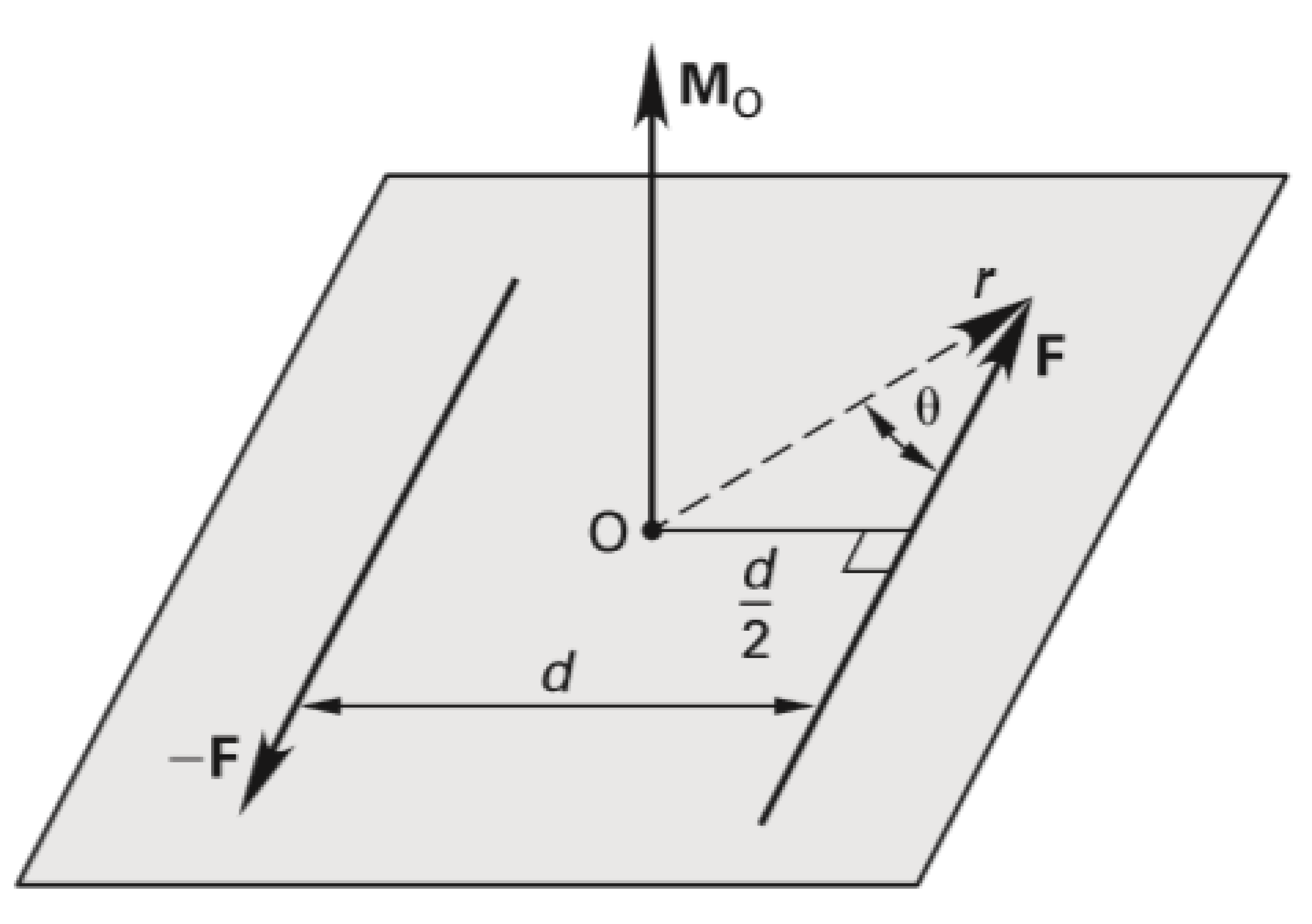
$$M = r \times F$$

A point (O) on an object experiences a moment whenever there is a force applied to it at a distance, but can be zero if the force passes through the point. Two forces that are equal in magnitude, opposite in direction, and parallel to each other is called a couple.

SYSTEMS OF FORCES

$$F = \sum F_n$$

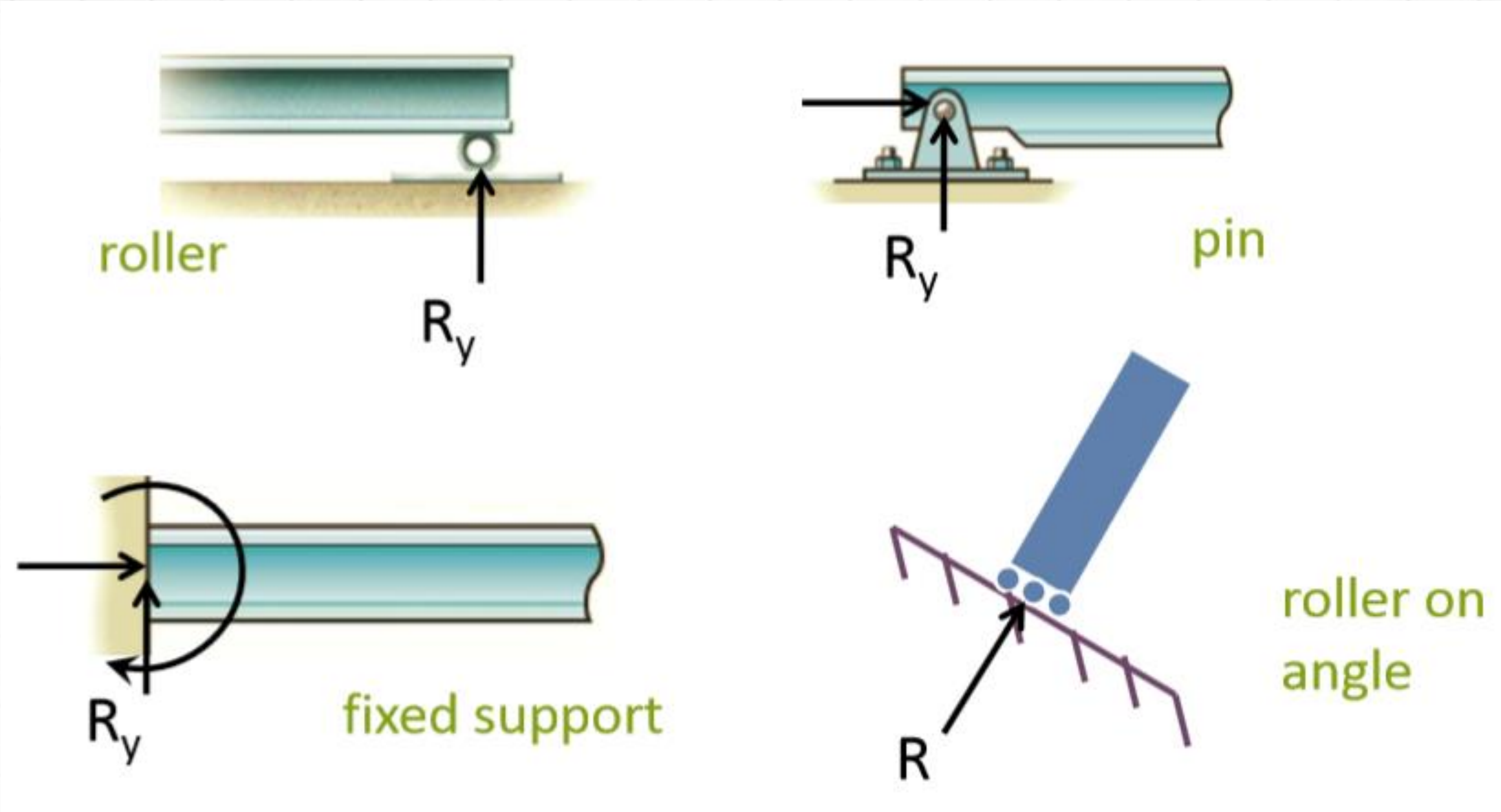
$$M = \sum (r_n \times F_n)$$



FORCE SYSTEMS

For static Equilibrium: $\sum F = 0 \qquad \sum M = 0$

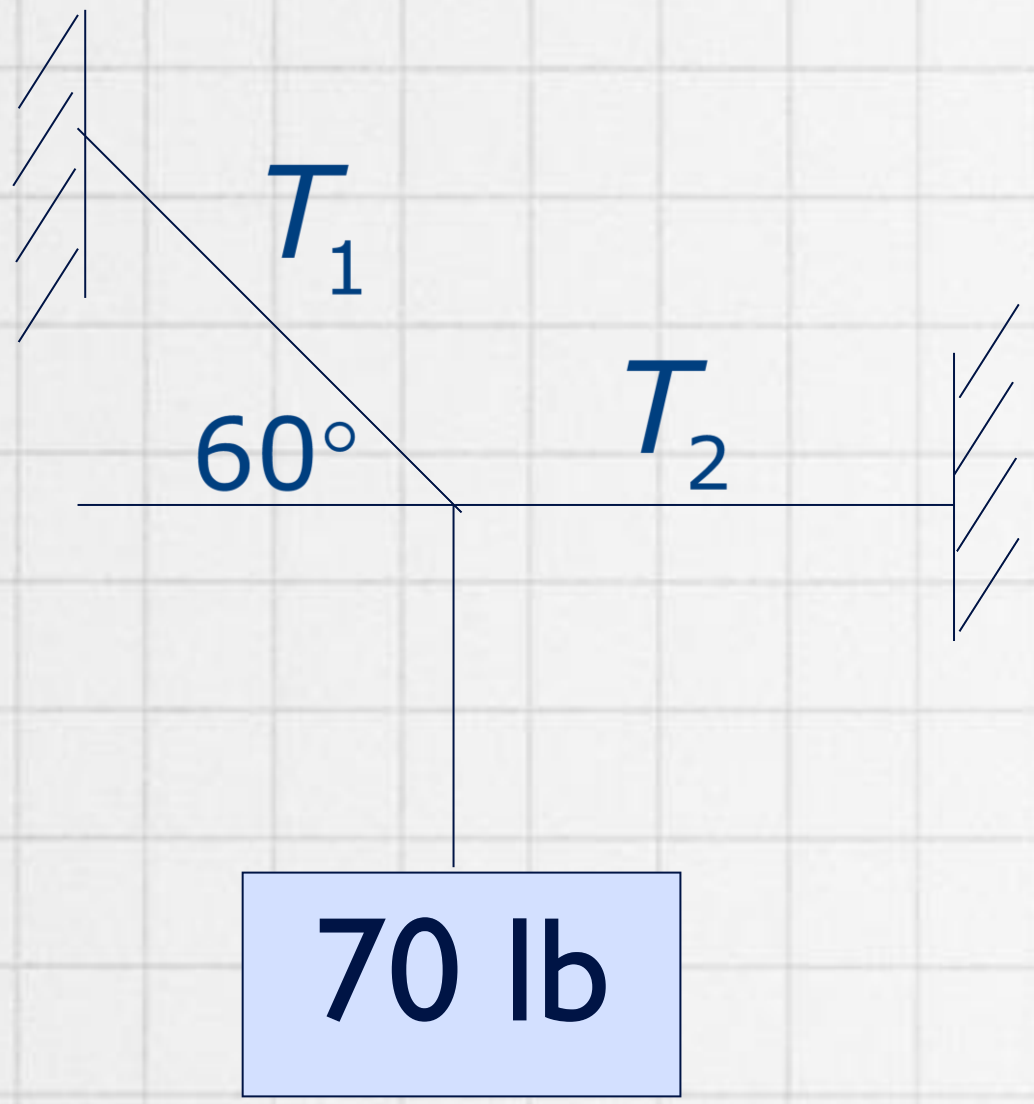
Support reactions (force and moment) resist translation or rotation.

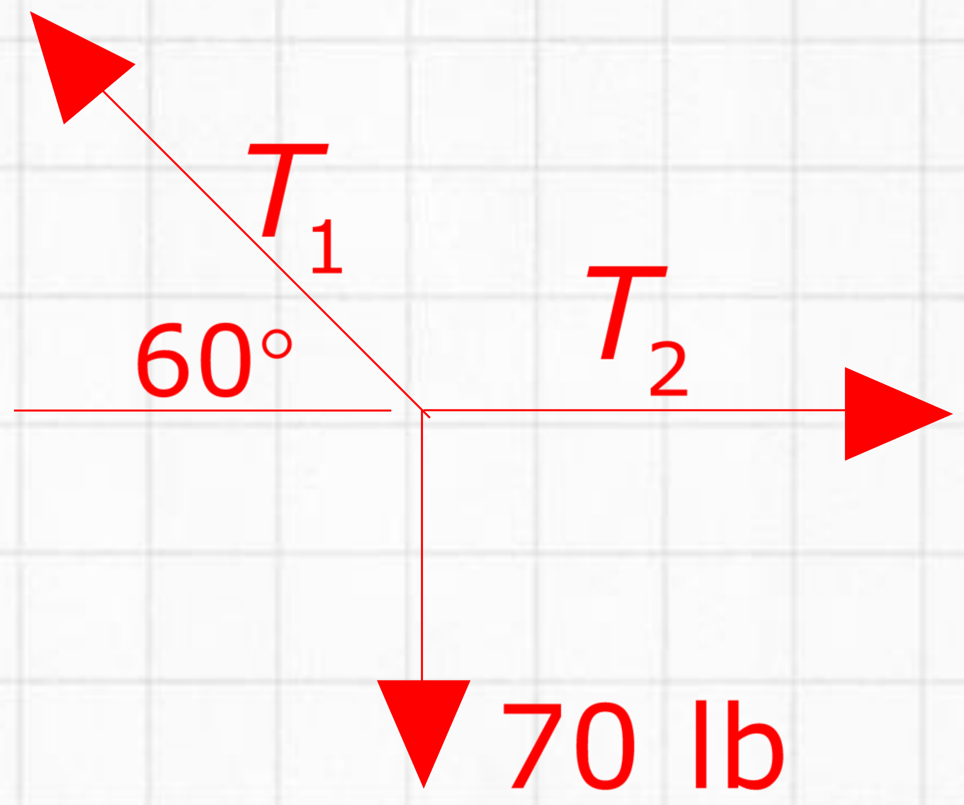


EQUILIBRIUM OF FORCE SYSTEMS

EXAMPLE #1

Three ropes are connected each other to hold a box in the equilibrium state. If the weight of the box is 70 lb, find the internal forces T_1 and T_2





Solution:

$$\sum F_y = 0$$

$$T_1 \sin 60° - 70 = 0$$

$$T_1 = 80.83 \text{ lb}$$

$$\sum F_x = 0$$

$$T_1 \cos 60° - T_2 = 0$$

$$T_2 = 40.41 \text{ lb}$$

FORCE SYSTEMS

TRUSS : set of pin-connected **axial** members.

A stable truss consists of a system of **triangular** structural cells.

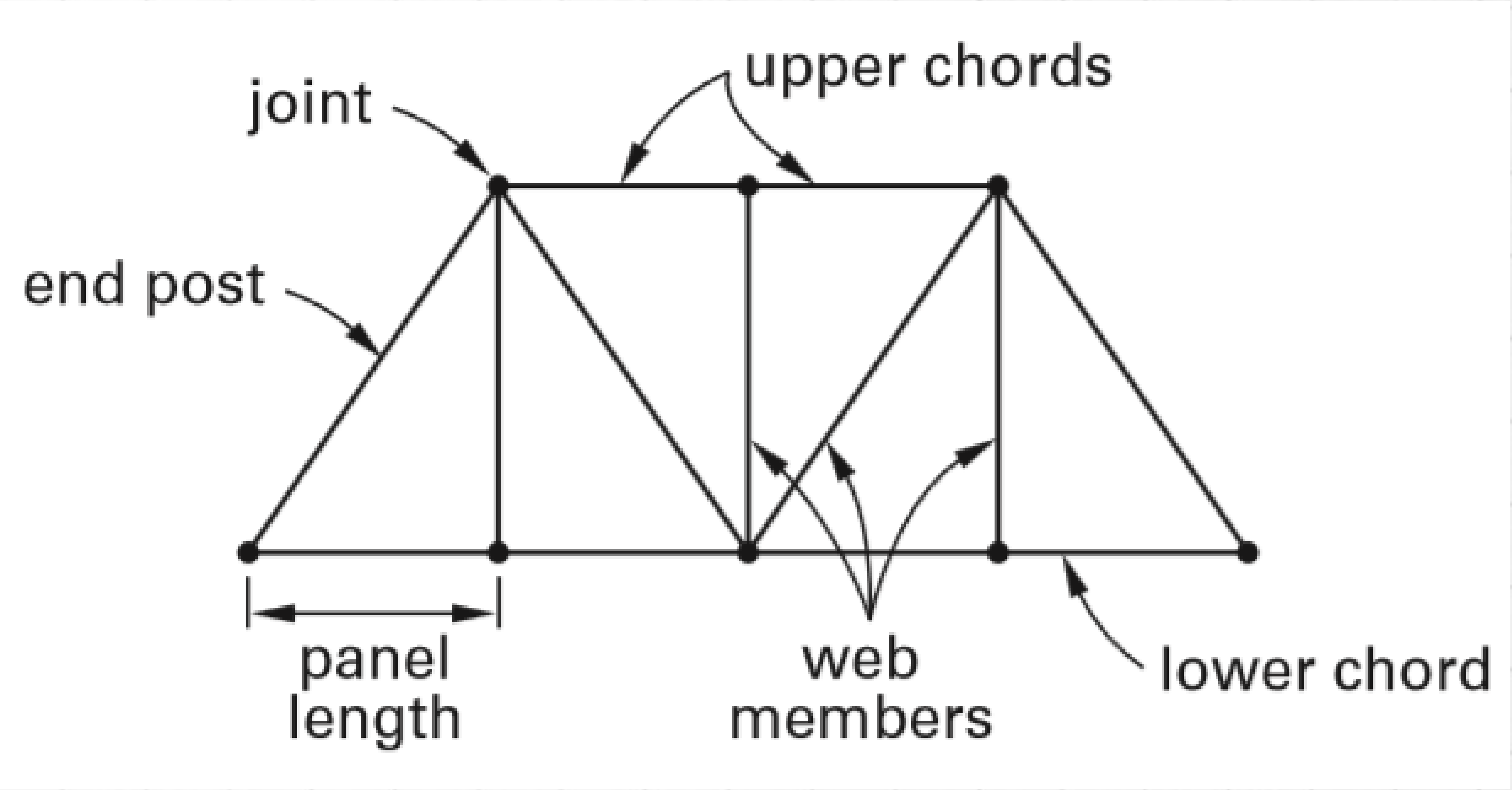
Truss Assumptions:

- The connection between the members are pinned
- All forces and reactions are applied at connection points
- All truss members only have axial forces (compression or tension)

Methods to solve member forces:

1. Method of joints
2. Method of sections

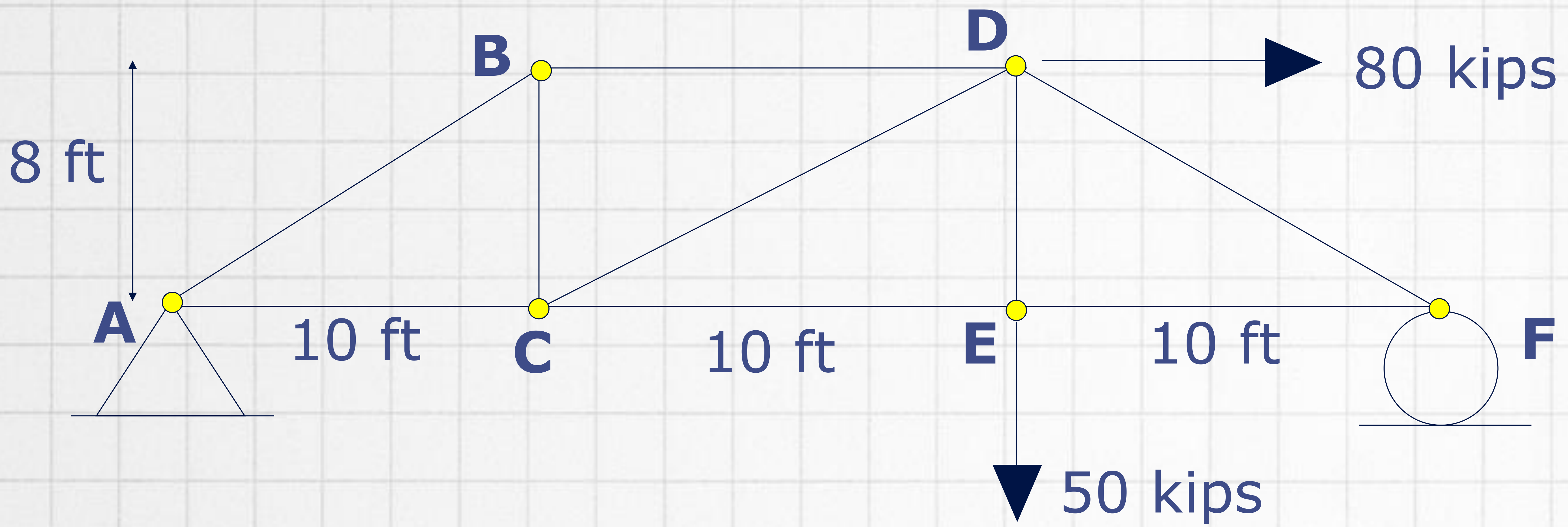
$$\sum F = 0 \qquad \sum M = 0$$



TRUSSES

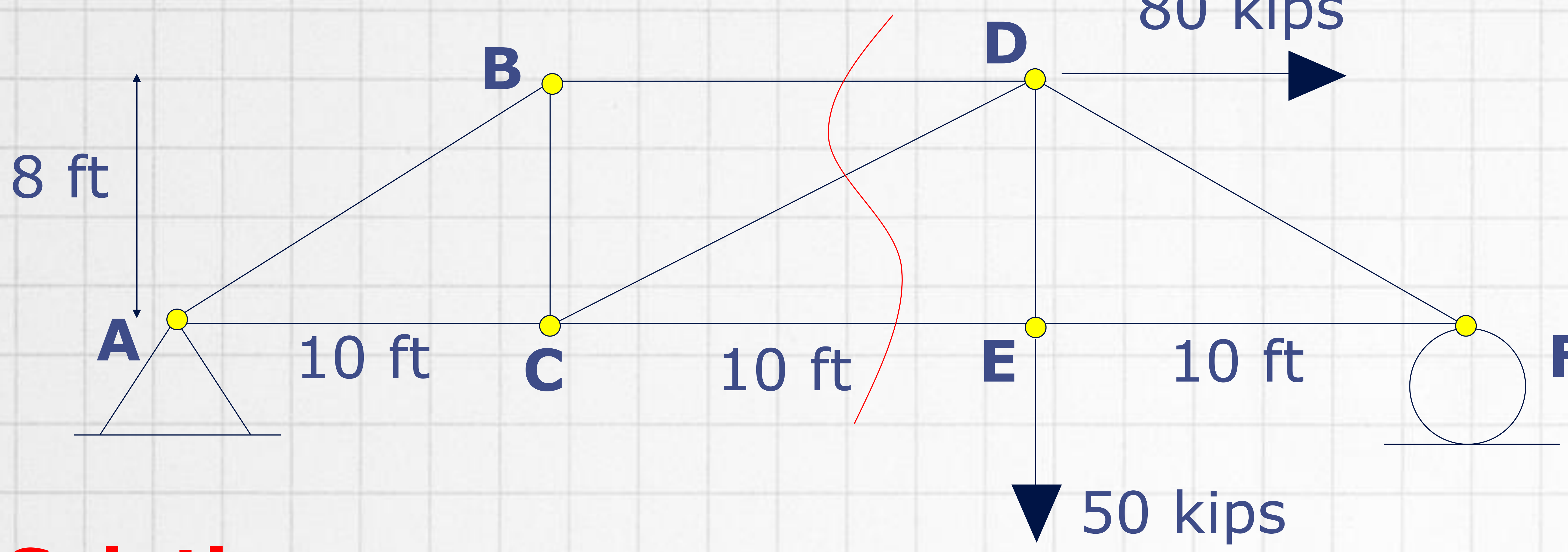
EXAMPLE #2

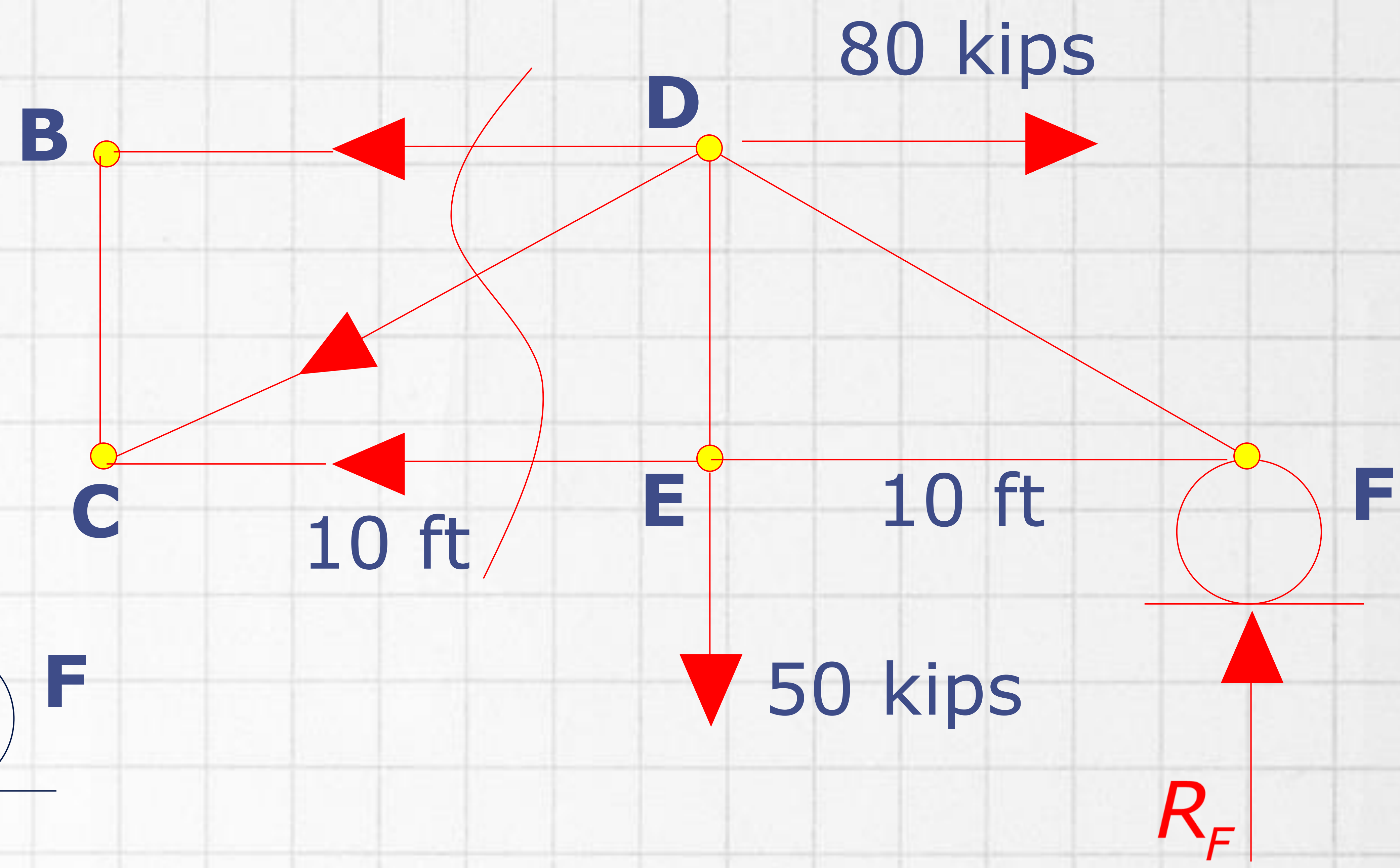
Determine the force in member BD shown below.



TRUSSES

EXAMPLE #2





Solution:

$$\sum M_A = 0$$

$$R_F(30\text{ ft}) - 50(20\text{ ft}) - 80(8\text{ ft}) = 0$$

$$R_F = 54.667\text{ kips}(\uparrow)$$

Using Method of Sections
(cut vertically as shown above)

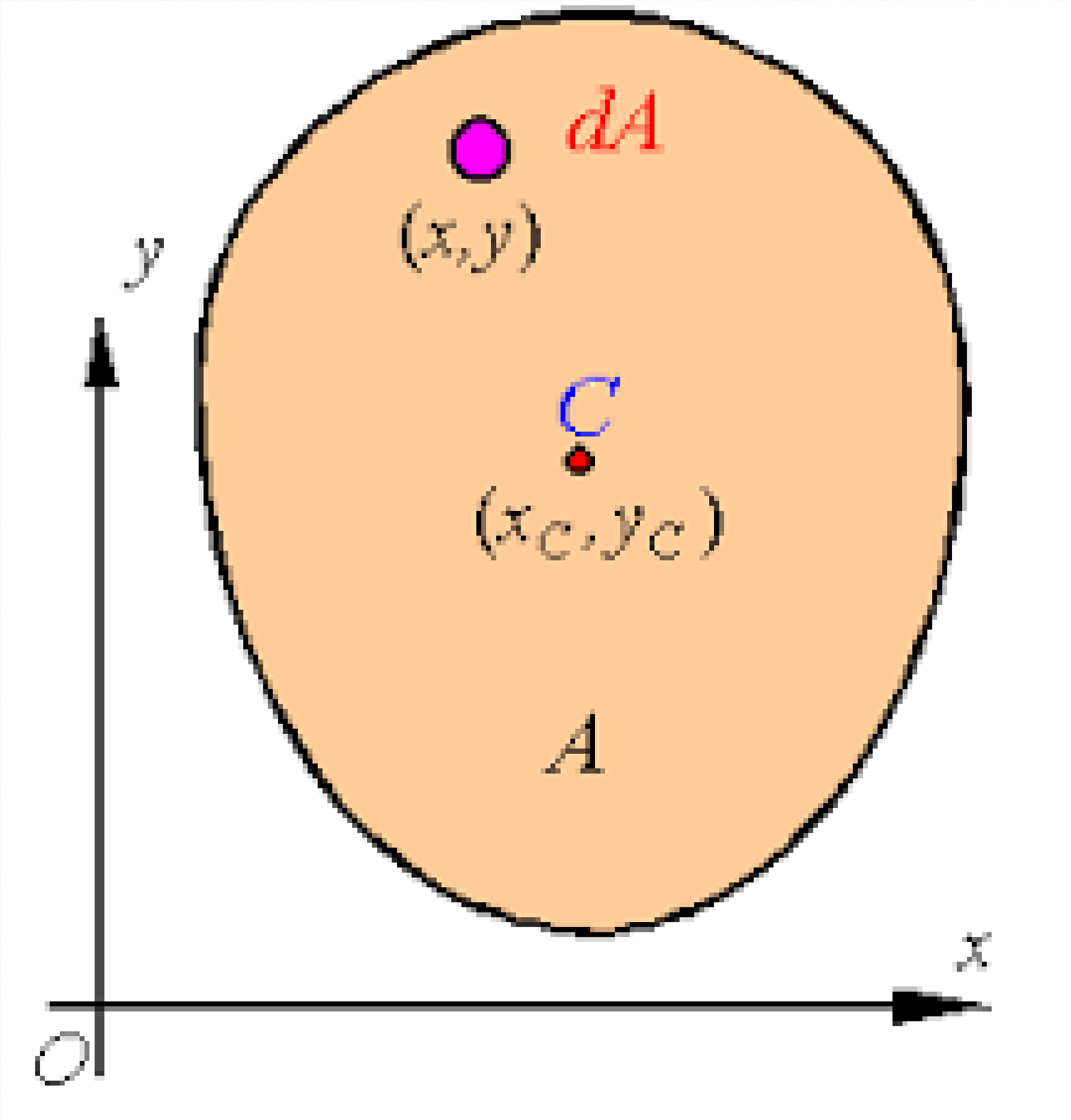
$$\sum M_C = 0\text{ (right)}$$

$$T_{BD}(8\text{ ft}) - 80(8\text{ ft}) - 50(10\text{ ft}) + R_F(20\text{ ft}) = 0$$

$$T_{BD} = 5.8325\text{ kips (tension)}$$

TRUSSES

Centroid of an area is the arithmetic mean location or geometric center of the area of a body. For a homogeneous body, it is the same as the center of gravity:



(First Moment of Area)

$$x_c = \frac{\sum (x_n a_n)}{A}$$ → centroid coordinate in X-component

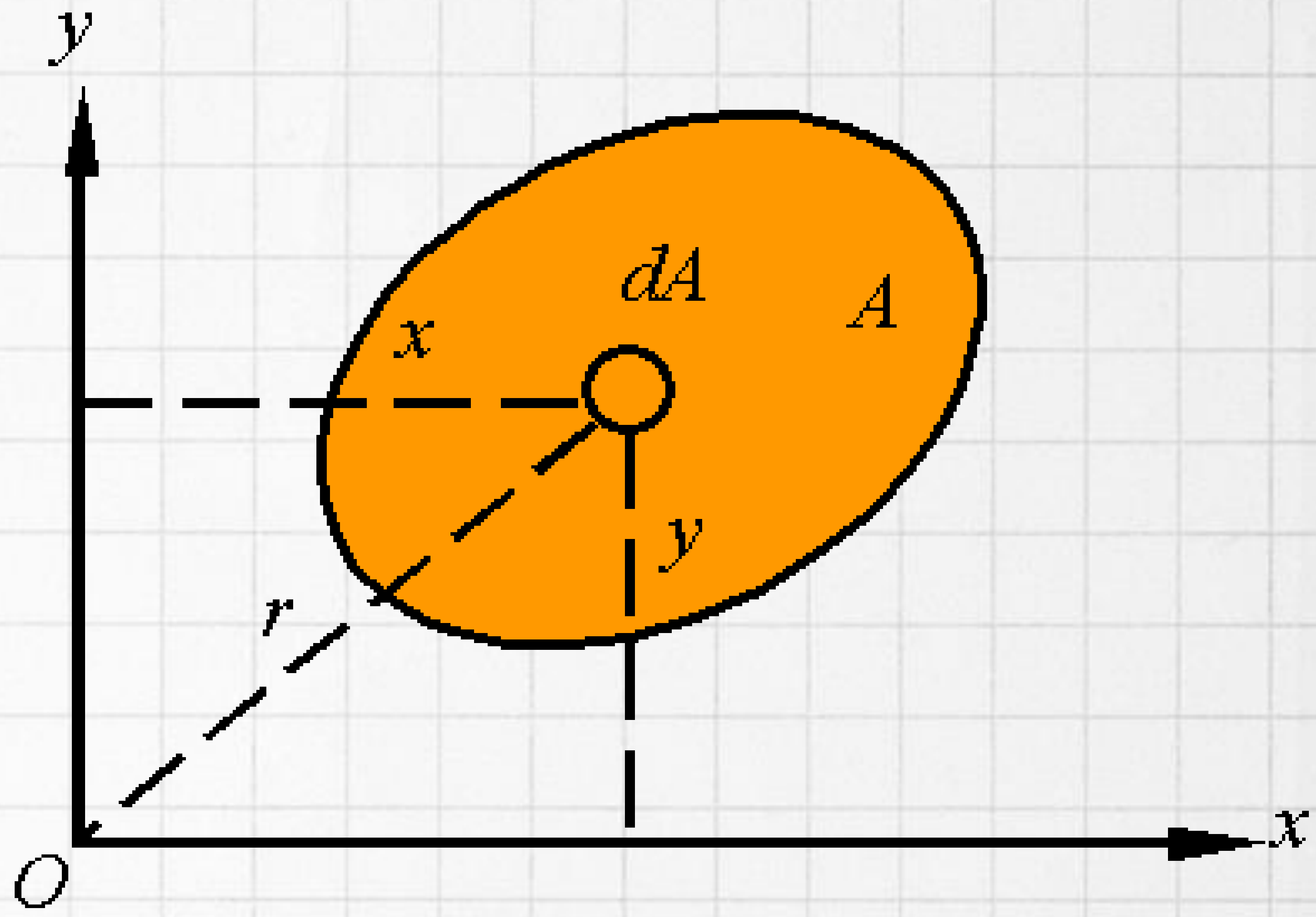
$$y_c = \frac{\sum (y_n a_n)}{A}$$ → centroid coordinate in Y-component

where $A = \sum a_n$ → total area

CENTROID OF AREA

Moment of inertia (I) or second moment of area is defined as:

$$I_x = \int y^2 dA \qquad I_y = \int x^2 dA$$



Polar moment of inertia (J) is defined as the sum of themoments of inertia about any 2 perpendicular axes in the area and passing through the same point.

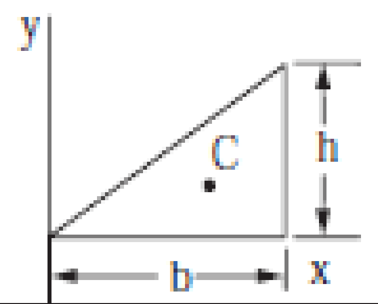
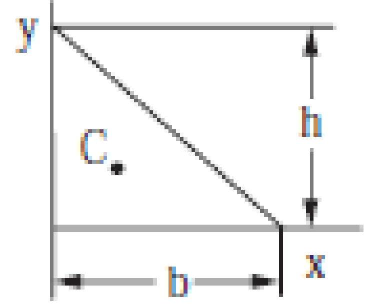
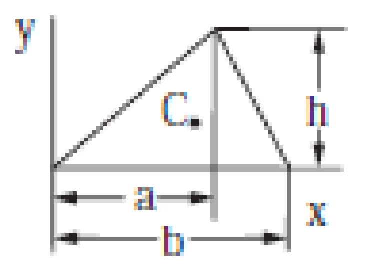
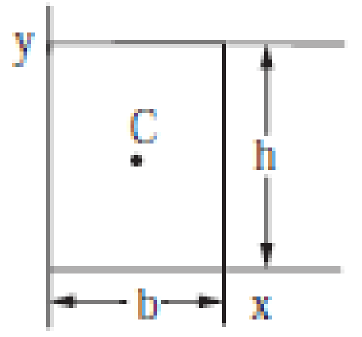
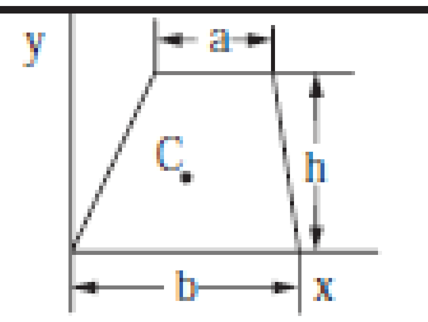
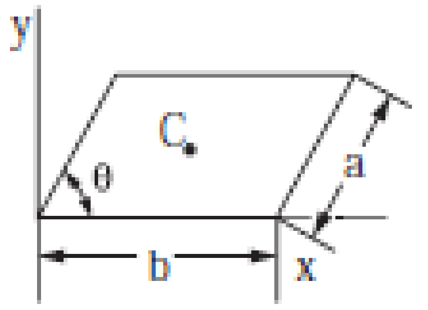
$$I_z = J = I_x + I_y = \int \left(x^2 + y^2\right) dA$$

Radius of gyration (r) is the distance from a reference axis at which all of the area can be considered to be concentrated to produce the moment of inertia.

$$r_x = \sqrt{I_x/A} \qquad r_y = \sqrt{I_y/A} \qquad r_p = \sqrt{J/A}$$

MOMENT OF INERTIA

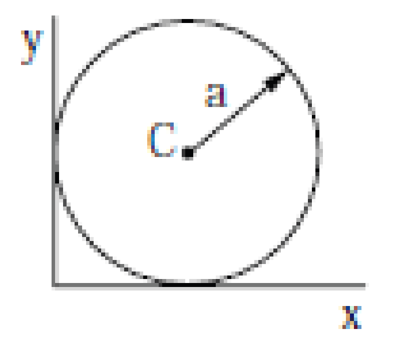
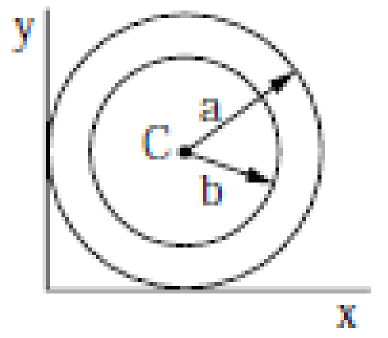
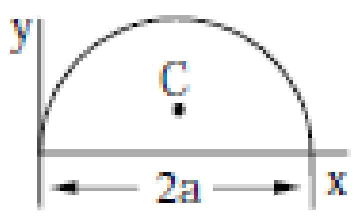
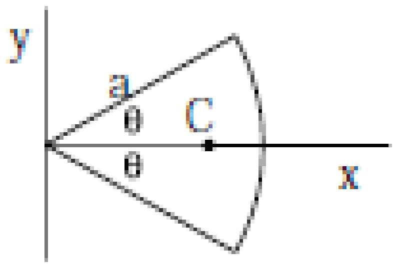
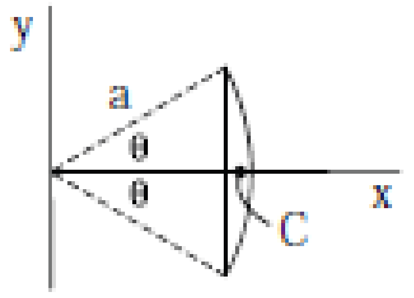
List of Basic Shapes

Figure	Area & Centroid	Area Moment of Inertia	(Radius of Gyration)^2	Product of Inertia
	$A = bh/2$ $x_c = 2b/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = b^3h/36$ $I_x = bh^3/12$ $I_y = b^3h/4$	$r^2_{x_c} = h^2/18$ $r^2_{y_c} = b^2/18$ $r^2_x = h^2/6$ $r^2_y = b^2/2$	$I_{x_c y_c} = Abh/36 = b^2h^2/72$ $I_{xy} = Abh/4 = b^2h^2/8$
	$A = bh/2$ $x_c = b/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = b^3h/36$ $I_x = bh^3/12$ $I_y = b^3h/12$	$r^2_{x_c} = h^2/18$ $r^2_{y_c} = b^2/18$ $r^2_x = h^2/6$ $r^2_y = b^2/6$	$I_{x_c y_c} = -Abh/36 = -b^2h^2/72$ $I_{xy} = Abh/12 = b^2h^2/24$
	$A = bh/2$ $x_c = (a+b)/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = [bh(b^2 - ab + a^2)]/36$ $I_x = bh^3/12$ $I_y = [bh(b^2 + ab + a^2)]/12$	$r^2_{x_c} = h^2/18$ $r^2_{y_c} = (b^2 - ab + a^2)/18$ $r^2_x = h^2/6$ $r^2_y = (b^2 + ab + a^2)/6$	$I_{x_c y_c} = [Ah(2a-b)]/36$ $= [bh^2(2a-b)]/72$ $I_{xy} = [Ah(2a+b)]/12$ $= [bh^2(2a+b)]/24$
	$A = bh$ $x_c = b/2$ $y_c = h/2$	$I_{x_c} = bh^3/12$ $I_{y_c} = b^3h/12$ $I_x = bh^3/3$ $I_y = b^3h/3$ $J = [bh(b^2 + h^2)]/12$	$r^2_{x_c} = h^2/12$ $r^2_{y_c} = b^2/12$ $r^2_x = h^2/3$ $r^2_y = b^2/3$ $r^2_p = (b^2 + h^2)/12$	$I_{x_c y_c} = 0$ $I_{xy} = Abh/4 = b^2h^2/4$
	$A = h(a+b)/2$ $y_c = \dfrac{h(2a+b)}{3(a+b)}$	$I_{x_c} = \dfrac{h^3(a^2 + 4ab + b^2)}{36(a+b)}$ $I_x = \dfrac{h^3(3a+b)}{12}$	$r^2_{x_c} = \dfrac{h^2(a^2 + 4ab + b^2)}{18(a+b)}$ $r^2_x = \dfrac{h^2(3a+b)}{6(a+b)}$	
	$A = ab \sin\theta$ $x_c = (b + a\cos\theta)/2$ $y_c = (a \sin\theta)/2$	$I_{x_c} = (a^3 b \sin^3\theta)/12$ $I_{y_c} = [ab \sin\theta(b^2 + a^2\cos^2\theta)]/12$ $I_x = (a^3 b \sin^3\theta)/3$ $I_y = [ab \sin\theta(b + a\cos\theta)^2]/3$ $-(a^2b^2 \sin\theta\cos\theta)/6$	$r^2_{x_c} = (a \sin\theta)^2/12$ $r^2_{y_c} = (b^2 + a^2\cos^2\theta)/12$ $r^2_x = (a \sin\theta)^2/3$ $r^2_y = (b + a\cos\theta)^2/3$ $-(ab\cos\theta)/6$	$I_{x_c y_c} = (a^3 b \sin^2\theta \cos\theta)/12$

Housner, George W., and Donald E. Hudson, Applied Mechanics Dynamics, D. Van Nostrand Company, Inc., Princeton, NJ, 1959. Table reprinted by permission of G.W. Housner & D.E. Hudson.

SUMMARY (1)

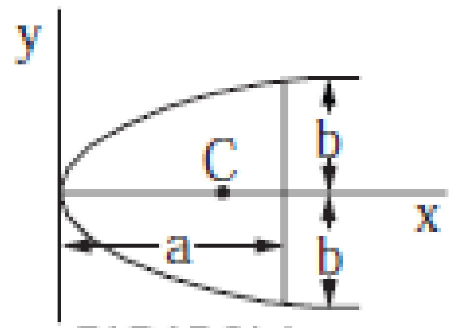
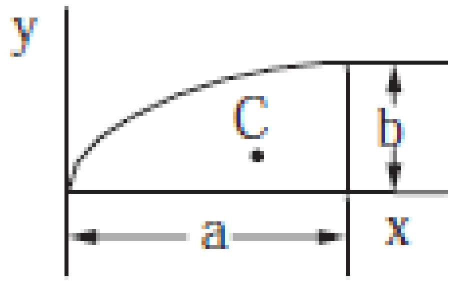
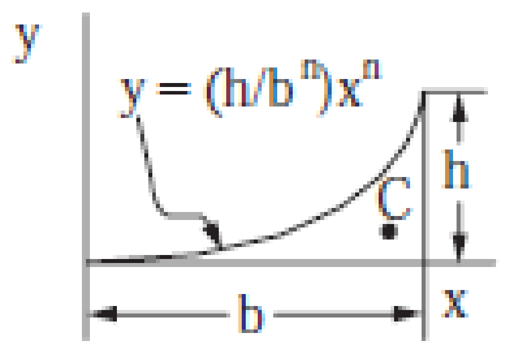
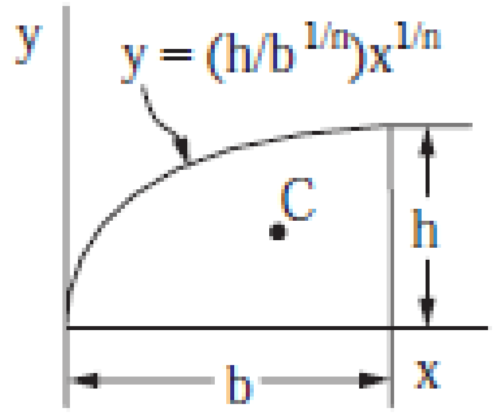
List of Basic Shapes

Figure	Area & Centroid	Area Moment of Inertia	(Radius of Gyration)^2	Product of Inertia
	$A = \pi a^2$ $x_c = a$ $y_c = a$	$I_{x_c} = I_{y_c} = \pi a^4/4$ $I_x = I_y = 5\pi a^4/4$ $J = \pi a^4/2$	$r_{x_c}^2 = r_{y_c}^2 = a^2/4$ $r_x^2 = r_y^2 = 5a^2/4$ $r_p^2 = a^2/2$	$I_{x_c y_c} = 0$ $I_{xy} = Aa^2$
	$A = \pi(a^2 - b^2)$ $x_c = a$ $y_c = a$	$I_{x_c} = I_{y_c} = \pi(a^4 - b^4)/4$ $I_x = I_y = \frac{5\pi a^4}{4} - \pi a^2 b^2 - \frac{\pi b^4}{4}$ $J = \pi(a^4 - b^4)/2$	$r_{x_c}^2 = r_{y_c}^2 = (a^2 + b^2)/4$ $r_x^2 = r_y^2 = (5a^2 + b^2)/4$ $r_p^2 = (a^2 + b^2)/2$	$I_{x_c y_c} = 0$ $I_{xy} = Aa^2$ $= \pi a^2 (a^2 - b^2)$
	$A = \pi a^2/2$ $x_c = a$ $y_c = 4a/(3\pi)$	$I_{x_c} = \frac{a^4(9\pi^2 - 64)}{72\pi}$ $I_{y_c} = \pi a^4/8$ $I_x = \pi a^4/8$ $I_y = 5\pi a^4/8$	$r_{x_c}^2 = \frac{a^2(9\pi^2 - 64)}{36\pi^2}$ $r_{y_c}^2 = a^2/4$ $r_x^2 = a^2/4$ $r_y^2 = 5a^2/4$	$I_{x_c y_c} = 0$ $I_{xy} = 2a^4/3$
 CIRCULAR SECTOR	$A = a^2\theta$ $x_c = \frac{2a}{3}\frac{\sin\theta}{\theta}$ $y_c = 0$	$I_x = a^4(\theta - \sin\theta\cos\theta)/4$ $I_y = a^4(\theta + \sin\theta\cos\theta)/4$	$r_x^2 = \frac{a^2}{4}\frac{(\theta - \sin\theta\cos\theta)}{\theta}$ $r_y^2 = \frac{a^2}{4}\frac{(\theta + \sin\theta\cos\theta)}{\theta}$	$I_{x_c y_c} = 0$ $I_{xy} = 0$
 CIRCULAR SEGMENT	$A = a^2\left[\theta - \frac{\sin 2\theta}{2}\right]$ $x_c = \frac{2a}{3}\frac{\sin^3\theta}{\theta - \sin\theta\cos\theta}$ $y_c = 0$	$I_x = \frac{Aa^2}{4}\left[1 - \frac{2\sin^3\theta\cos\theta}{3\theta - 3\sin\theta\cos\theta}\right]$ $I_y = \frac{Aa^2}{4}\left[1 + \frac{2\sin^3\theta\cos\theta}{\theta - \sin\theta\cos\theta}\right]$	$r_x^2 = \frac{a^2}{4}\left[1 - \frac{2\sin^3\theta\cos\theta}{3\theta - 3\sin\theta\cos\theta}\right]$ $r_y^2 = \frac{a^2}{4}\left[1 + \frac{2\sin^3\theta\cos\theta}{\theta - \sin\theta\cos\theta}\right]$	$I_{x_c y_c} = 0$ $I_{xy} = 0$

Housner, George W., and Donald E. Hudson, Applied Mechanics Dynamics, D. Van Nostrand Company, Inc., Princeton, NJ, 1959. Table reprinted by permission of G.W. Housner & D.E. Hudson.

SUMMARY (2)

List of Basic Shapes

Figure	Area & Centroid	Area Moment of Inertia	(Radius of Gyration)^2	Product of Inertia
 PARABOLA	$A = 4ab/3$ $x_c = 3a/5$ $y_c = 0$	$I_{x_c} = I_x = 4ab^3/15$ $I_{y_c} = 16a^3b/175$ $I_y = 4a^3b/7$	$r_{x_c}^2 = r_x^2 = b^2/5$ $r_{y_c}^2 = 12a^2/175$ $r_y^2 = 3a^2/7$	$I_{x_c y_c} = 0$ $I_{xy} = 0$
 HALF A PARABOLA	$A = 2ab/3$ $x_c = 3a/5$ $y_c = 3b/8$	$I_x = 2ab^3/15$ $I_y = 2ba^3/7$	$r_x^2 = b^2/5$ $r_y^2 = 3a^2/7$	$I_{xy} = Aab/4 = a^2b^2$
 n^{th} DEGREE PARABOLA	$A = bh/(n+1)$ $x_c = \frac{n+1}{n+2}b$ $y_c = \frac{h}{2}\frac{n+1}{2n+1}$	$I_x = \frac{bh^3}{3(3n+1)}$ $I_y = \frac{hb^3}{n+3}$	$r_x^2 = \frac{h^2(n+1)}{3(3n+1)}$ $r_y^2 = \frac{n+1}{n+3}b^2$	
 n^{th} DEGREE PARABOLA	$A = \frac{n}{n+1}bh$ $x_c = \frac{n+1}{2n+1}b$ $y_c = \frac{n+1}{2(n+2)}h$	$I_x = \frac{n}{3(n+3)}bh^3$ $I_y = \frac{n}{3n+1}b^3h$	$r_x^2 = \frac{n+1}{3(n+1)}h^2$ $r_y^2 = \frac{n+1}{3n+1}b^2$	

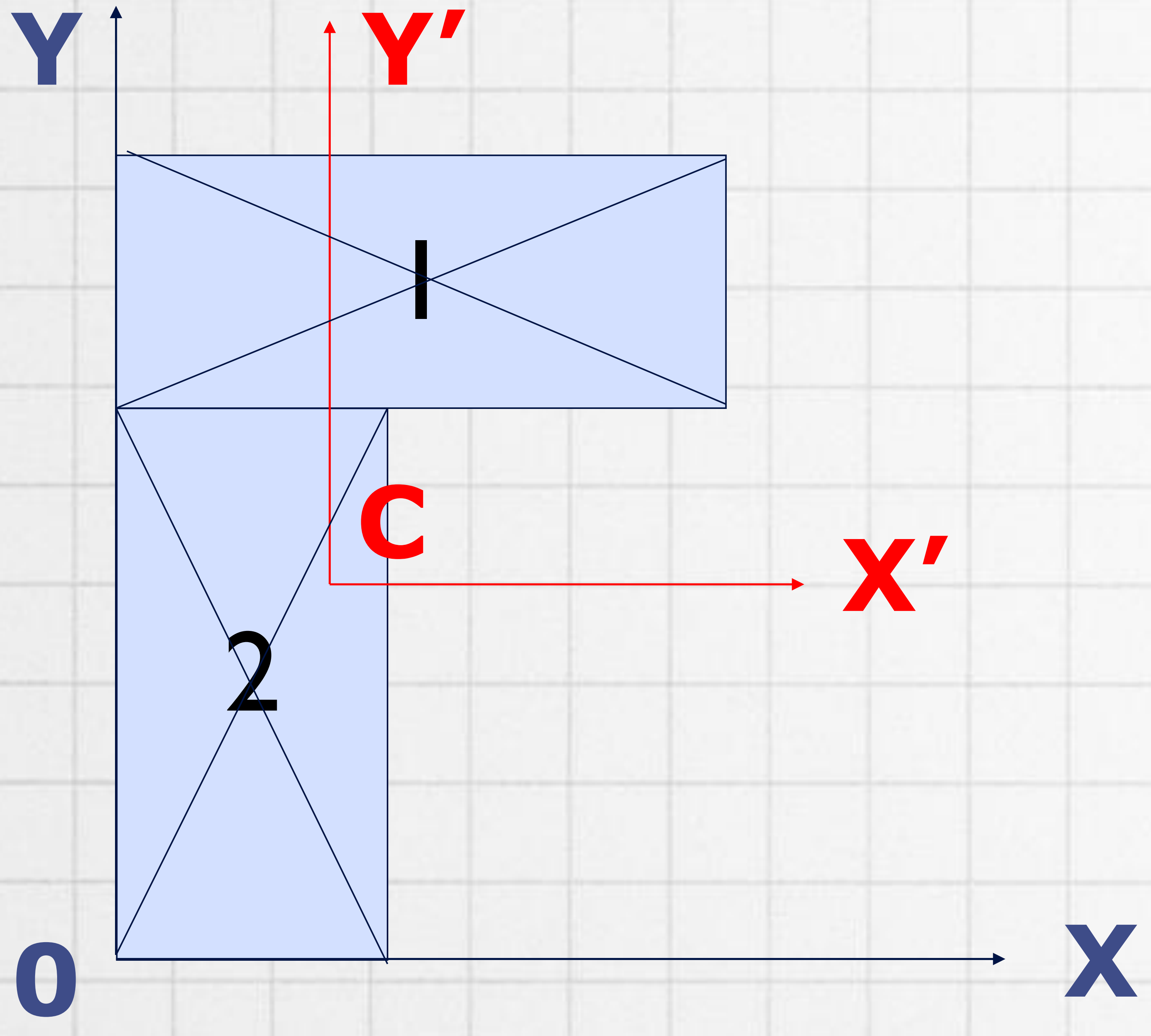
Housner, George W., and Donald E. Hudson, Applied Mechanics Dynamics, D. Van Nostrand Company, Inc., Princeton, NJ, 1959. Table reprinted by permission of G.W. Housner & D.E. Hudson.

SUMMARY (3)

Moments of inertia (I) in the previous slides are the moments of inertia **about the centroidal axis**.

$$I_{xc} = \int y^2 dA \qquad I_{yc} = \int x^2 dA$$

For Moments of Inertia of built up sections (Parallel Axis Theorem):



$$I'_x = I_{xc} + d_x^2 A \qquad I'_y = I_{yc} + d_y^2 A$$

where

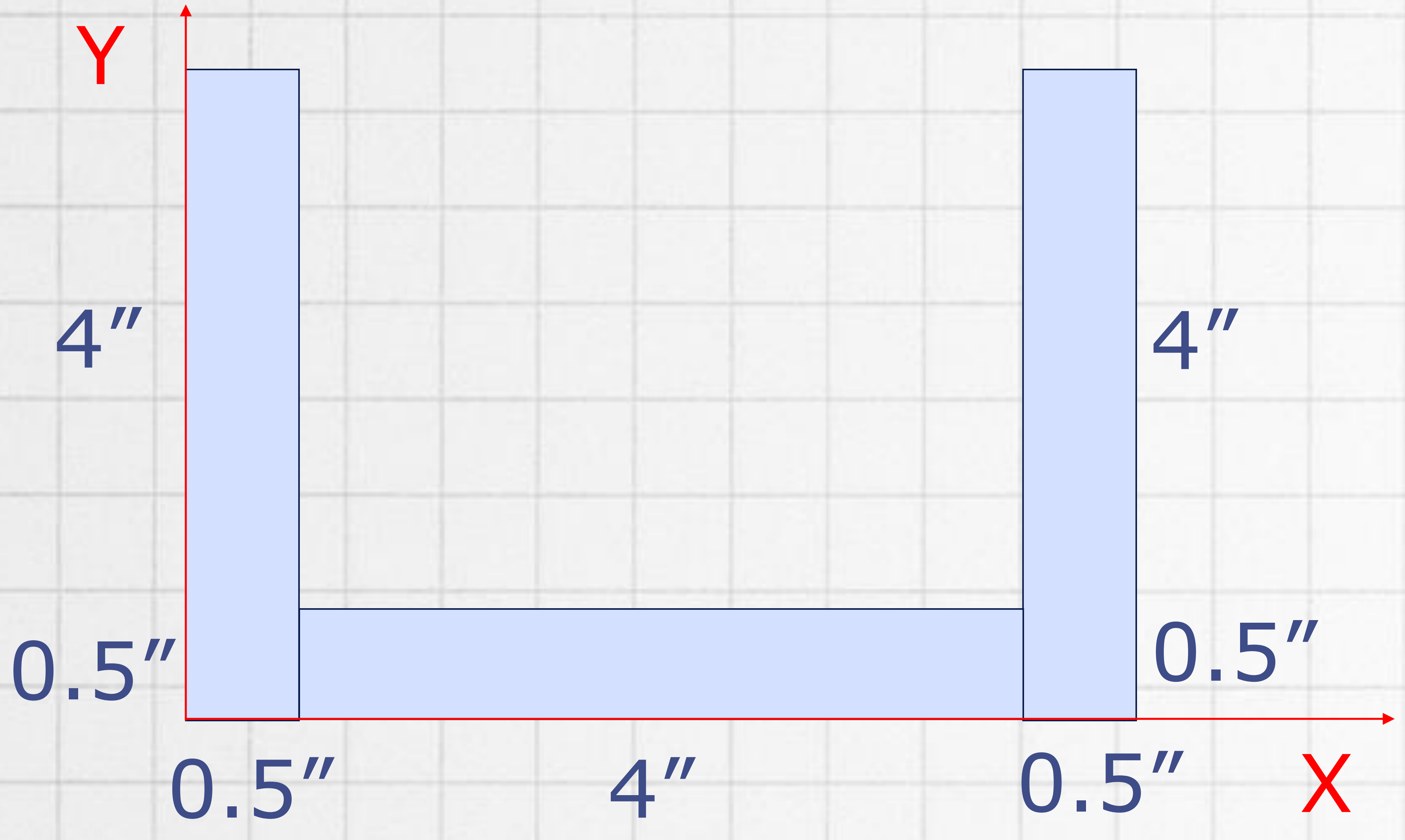
d_x = the distances between two axes in X direction

d_y = the distances between two axes in Y direction

MOMENT OF INERTIA

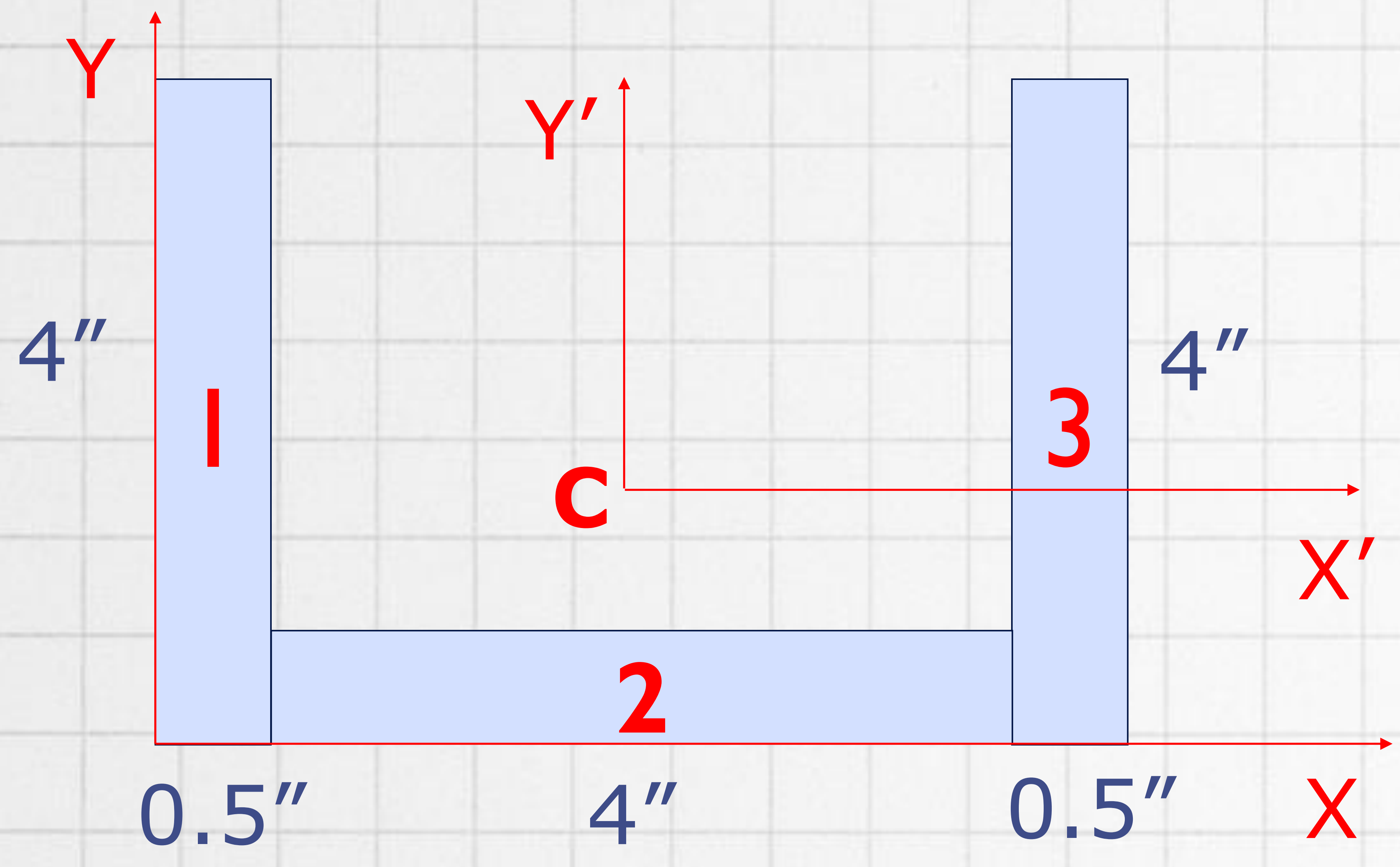
EXAMPLE #3

Find the moment of inertia about X-axis and Y-axis of the following section.



MOMENT OF INERTIA

EXAMPLE #3



1. Solve for the centroid of total area:

$$x_c = \frac{(0.25)(2)+(2.5)(2)+(4.75)(2)}{2+2+2} = 2.5"$$

$$y_c = \frac{(2)(2)+(0.25)(2)+(2)(2)}{2+2+2} = 1.4167"$$

2. Solve for the moment of inertia about each centroidal area:

$$I_{xc,1} = I_{xc,3} = \frac{1}{12}(0.5)(4)^3 = 2.667 \text{ in}^4$$

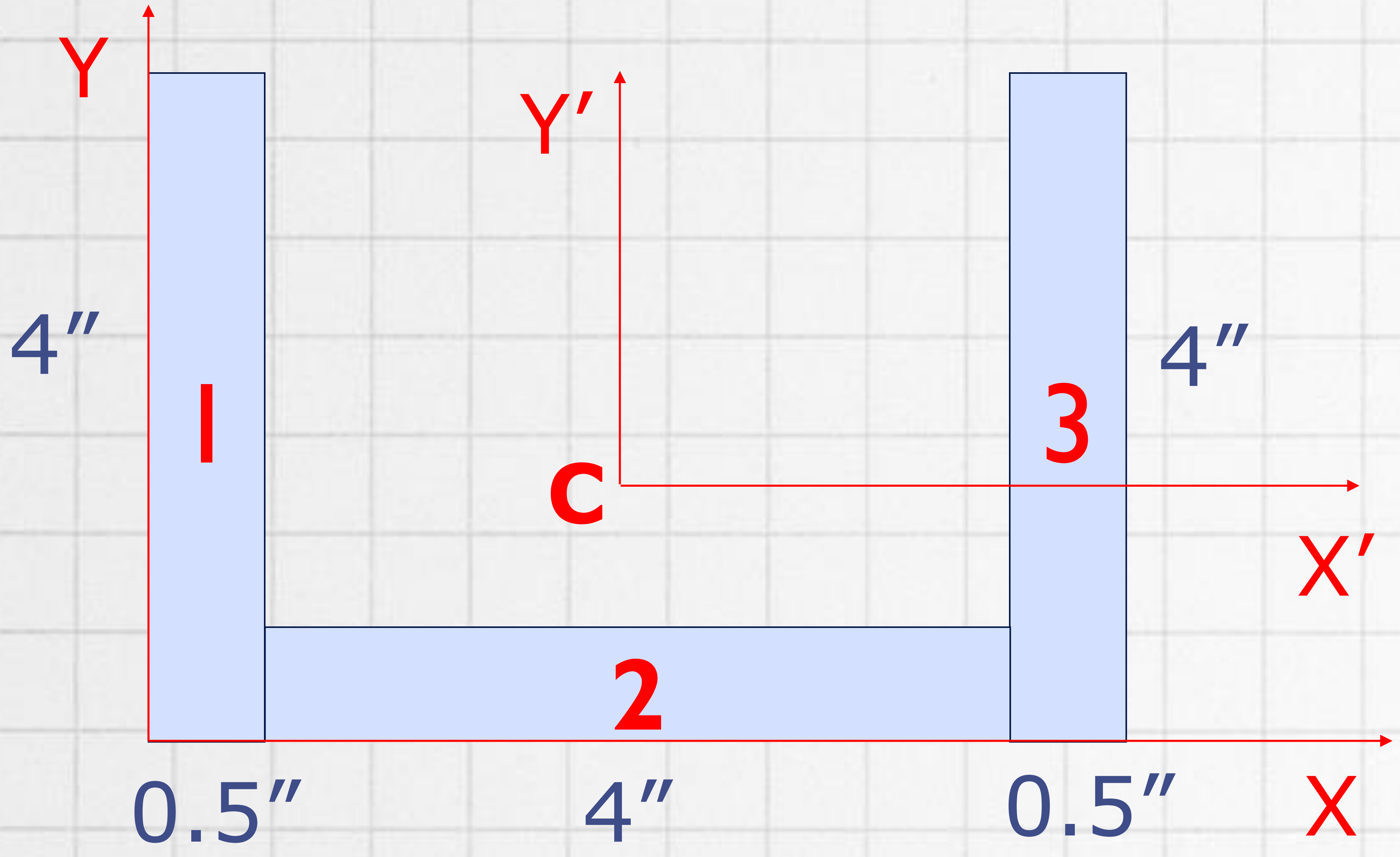
$$I_{xc,2} = \frac{1}{12}(4)(0.5)^3 = 0.041667 \text{ in}^4$$

$$I_{yc,1} = I_{yc,3} = \frac{1}{12}(4)(0.5)^3 = 0.041667 \text{ in}^4$$

$$I_{yc,2} = \frac{1}{12}(0.5)(4)^3 = 2.667 \text{ in}^4$$

MOMENT OF INERTIA

EXAMPLE #3



3. Solve for the moment of inertia about section's total area:

$$I'_x = 2\left(2.667 + (1.4167 - 2)^2(2)\right) + \left(0.041667 + (1.4167 - 0.25)^2(2)\right) = 9.459 \text{ in}^4$$

$$I'_y = \left(0.041667 + (2.5 - 0.25)^2(2)\right) + 2.667 + \left(0.041667 + (2.5 - 4.75)^2(2)\right) = 23 \text{ in}^4$$

MOMENT OF INERTIA

Static Friction:

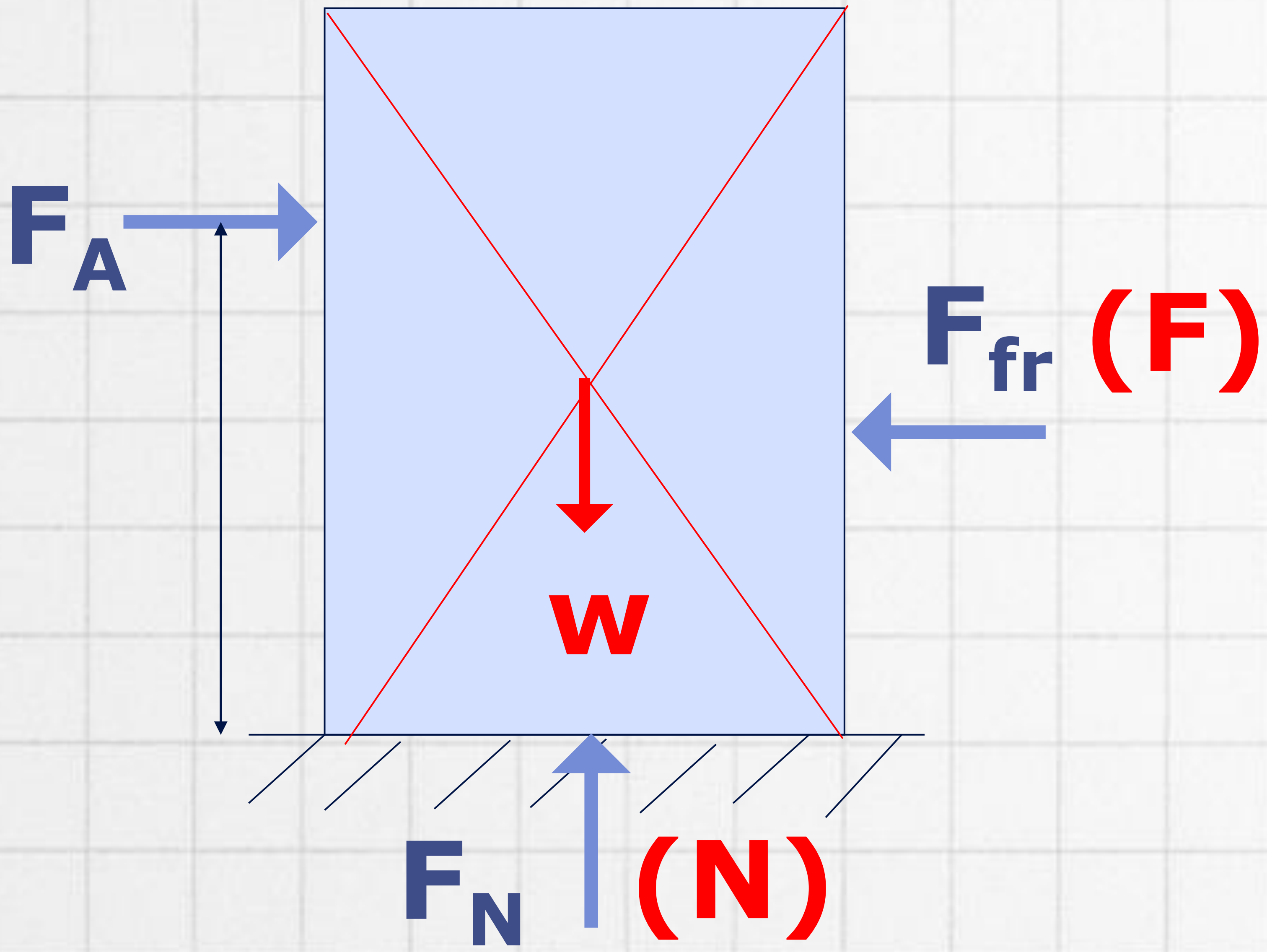
The largest friction force occurs right before an object moves

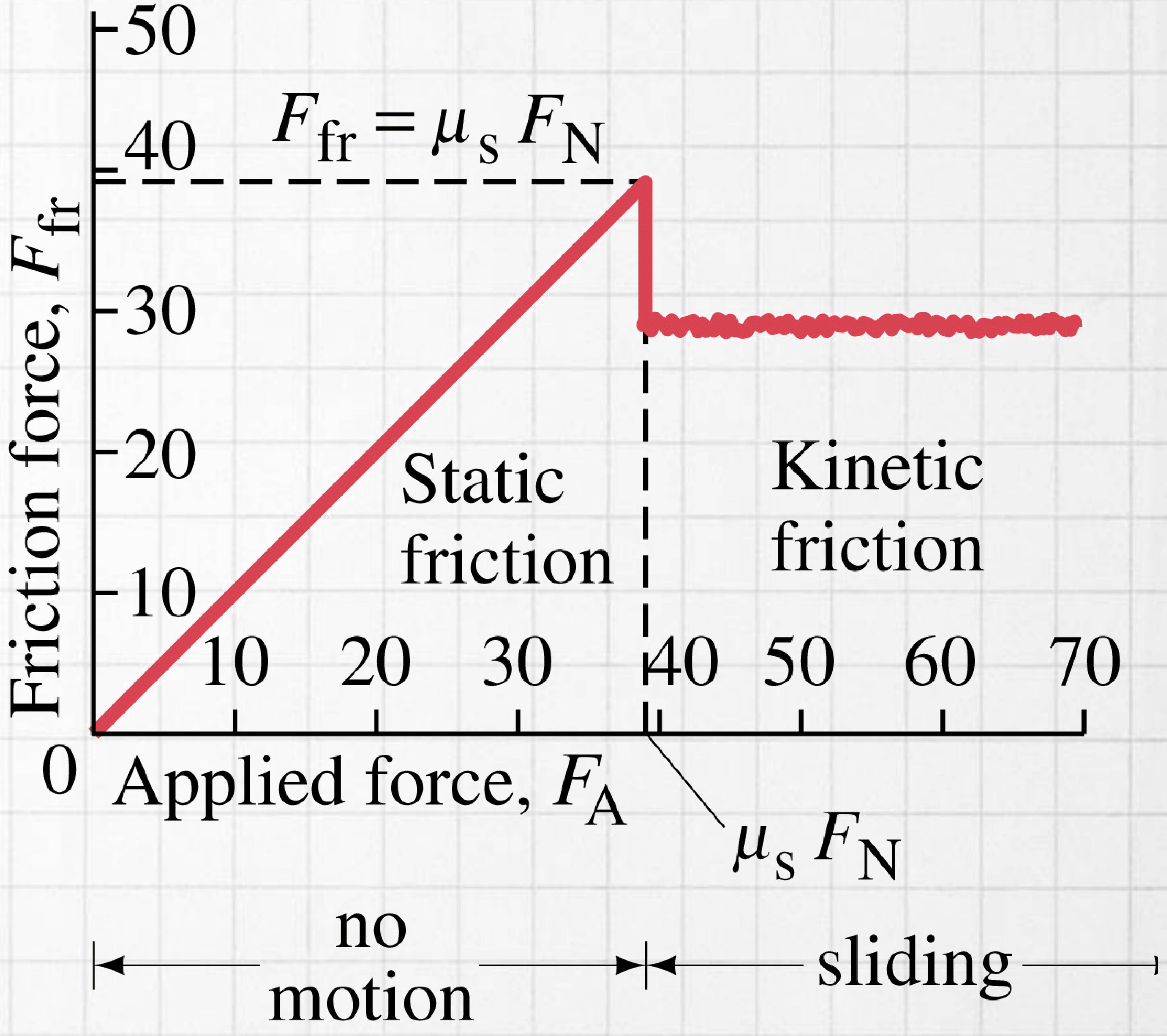
$$F \leq \mu_s N$$

where

μ_s = coefficient of static friction

N = normal force between surface in contact

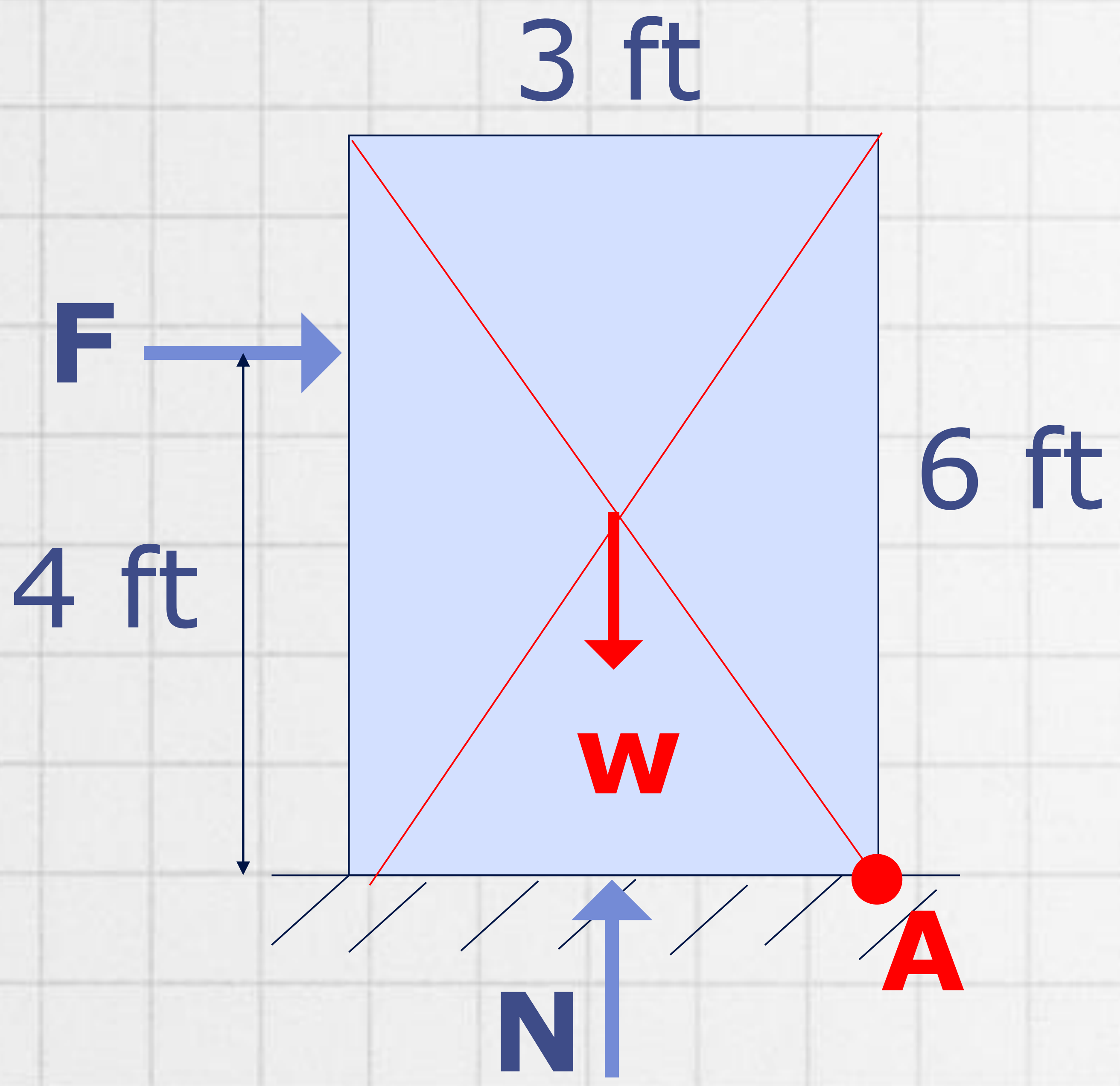




STATIC FRICTION

EXAMPLE #4

A large box will be moved by pushing it with force **F**. Suppose the weight of the box is 200 lb at its center of gravity. Find the required force **F** to move it. Also judge if the box will tip due to the force or not. Assume $\mu_s = 0.4$



Solution:

The required force is $F = \mu_s N = 0.4 \times 200\ \text{lb} = 80\ \text{lb}$

Check if the box will tip by checking the equilibrium of moment at point A:

Resisting moment = $w(1.5\ \text{ft}) = (200\ \text{lb})(1.5\ \text{ft}) = 300\ \text{lb-ft}$

Overturning moment = $F(4\ \text{ft}) = (80\ \text{lb})(4\ \text{ft}) = 320\ \text{lb-ft}$

Overturning moment > resisting moment,
so the box will tip.

STATIC FRICTION

Module 5

Dynamics

COMING UP NEXT…