AS#27 EXPONENTIAL MODELLING

AEM questions are taken from past exam papers - they have been carefully chosen to represent a typical exam question at each level of difficulty. If you can do these questions, you're ready to move onto past papers for this topic.

APPRENTICE

A radioactive substance decays exponentially, so that its mass M grams can be modelled by the equation $M = Ae^{-kt}$, where t is the time in years, and A and k are positive constants.

a. An initial mass of 100 g of substance decays to 50 g in 1500 years. Find A and k.

b. The substance becomes safe when $99\ \%$ of its initial mass has decayed. Find how long it will take before the substance becomes safe.

EXPERT

The mass of a substance is decreasing exponentially. Its mass is m grams at time t years.
The table below shows certain values of t and m.

a. Find values missing in the table.

b. Determine the value of t, correct to the nearest integer, for which the mass is 50 grams.

t	0	5	10	25
m	200	160		

MASTER

The mass of radioactive atoms in a substance can be modelled by the equation $m = m_0 k^t$
m_0 grams is its initial mass, m grams is the mass after t days and k is a constant. The value of k differs from one substance to another.

a. i. A sample of radioactive iodine reduced in mass from 24 grams to 12 grams in 8 days.
Show that the value of the constant k for this substance is 0.917004, correct to six decimal places.

ii. A similar sample of radioactive iodine reduced in mass to 1 gram after 60 days.
Calculate the initial mass of this sample, giving your answer to the nearest gram.

b. The half-life of a radioactive substance is the time it takes for a mass of m_0 to reduce to a mass of $\frac{1}{2}m_0$.
A sample of radioactive vanadium reduced in mass from exactly 10 grams to 8.106 grams in 100 days.

Find the half-life of radioactive vanadium, giving your answer to the nearest day.

AS#29 EXPONENTIALS & LOGS BASICS

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APPRENTICE

a. Find the exact solution, in its simplest form, of the equation $\mathrm{e}^{3\mathrm{x}-9} = 8$

b. Solve the equation $2^{3x-4} - 7 = 0$, giving your value of x to three significant figures.

c. Find the exact solution, in its simplest form, of the equation $3\ln(4 - 2x) = 1$

EXPERT

a. Solve the equation $4^{3x-9} = 2^{x+1}$ using logarithms

b. Solve the equation $4^{3x-9} = 2^{x+1}$ without using logarithms

MASTER

Given that $\log_3 c = m$ and $\log_{27} d = n$, express $\dfrac{\sqrt{c}}{d^2}$ in the form 3^y, where y is an expression in terms of m and n.

AS#28 EXPONENTIAL & LOG GRAPHS

AEM questions are taken from past exam papers - they have been carefully chosen to represent a typical exam question at each level of difficulty. If you can do these questions, you're ready to move onto past papers for this topic.

APPRENTICE

a. Sketch the graph of $y = 1.2^x$, showing the equations of any asymptotes & intersections with any axes.

b. Sketch the graph of $y = 0.2^x$, showing the equations of any asymptotes & intersections with any axes.

c. Sketch the graph of $y = \ln(2x)$, showing the equations of any asymptotes & intersections with any axes.

EXPERT

a. Sketch the graph of $y = 1 - 2e^{0.5x+3}$, showing the equations of any asymptotes & intersections with any axes.

b. Sketch the graph of $y = 3\ln(x - 2)$, showing the equations of any asymptotes & intersections with any axes.

MASTER

The amount of a drug, x mg, remaining in the bloodstream t hours after the dose is given will decay exponentially and is given by $x = 20e^{-0.7t}$.

A patient is given a 20 mg dose then 12 hours later is given a second 20 mg dose.

Sketch a graph to show the amount of the drug in the patient's bloodstream for the 24 hours after the first dose is given.

AS#30 RULES OF LOGS

AEM questions are taken from past exam papers - they have been carefully chosen to represent a typical exam question at each level of difficulty. If you can do these questions, you're ready to move onto past papers for this topic.

APPRENTICE

a. Express $2\log_a 3 + \log_a 11$ as a single logarithm to the base a.

b. Solve the equation $\ln(2y+5) = 2 + \ln(4-y)$

EXPERT

a. Show that the equation $\log_2(y+1) - 1 = 2\log_2 x$ can be written in the form $y = ax^2 + b$, where $a, b \in \mathbb{Z}$

b. Hence solve the simultaneous equations

$$\log_2(y+1) - 1 = 2\log_2 x$$
$$\log_x(y - 10x + 14) = 0$$

MASTER

Show that the equation $\log_4(2x+3) + \log_4(2x+15) = 1 + \log_4(14x+5)$ has only one solution and state its value.

AS#31 LOG-LOG GRAPHS

AEM questions are taken from past exam papers - they have been carefully chosen to represent a typical exam question at each level of difficulty. If you can do these questions, you're ready to move onto past papers for this topic.

APPRENTICE

a. Given that $y = ax^b$, show that the graph of $\log_{10} x$ against $\log_{10} y$ is a straight line.

b. Given that $y = ab^x$, show that the graph of x against $\log_{10} y$ is a straight line.

EXPERT

Two variables, x and t, are related by the equation $x = at^b$.

The graph of $\log_{10} x$ against $\log_{10} t$ has a line of best fit with gradient 1.48 passing through -0.60 on the vertical axis.

Estimate the values of a and b.

MASTER

The table shows population data for a country.

Year	1969	1979	1989	1999	2009
Population in millions (p)	58.81	80.35	105.27	134.79	169.71

The data may be represented by an exponential model of growth. Using t as the number of years after 1960, a suitable model is $p = a \times 10^{kt}$.

a. Derive an equation for $\log_{10} p$ in terms of a, k and t.

b. Plot the graph of $\log_{10} p$ against t, drawing a line of best fit by eye.

c. Use your line of best fit to express $\log_{10} p$ in terms of t and and hence find p in terms of t.

d. According to the model, what was the population in 1960?

e. According to the model, when will the population reach 200 million?