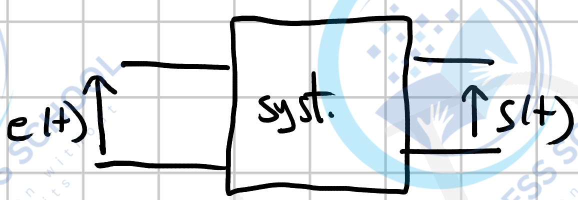
Electronique (spé)

Stabilité d'un syst.

a_i et β_i constantes.



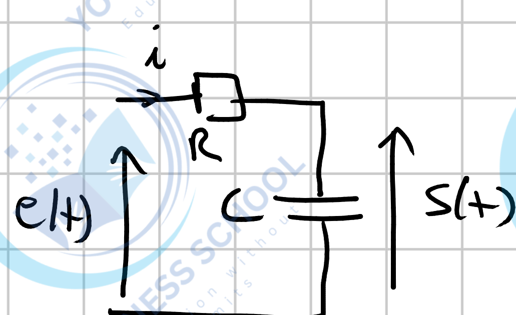
Linéaire : $e(t) + a_1 \frac{de}{dt} + a_2 \frac{d^2e}{dt^2} + \dots = \beta_0 s(t) + \beta_1 \frac{ds}{dt} + \beta_2 \frac{d^2s}{dt^2} + \dots$

$\Downarrow$

stable : $s(t) = s_h(t) + s_p(t)$

β_i sont de même signe $\iff$ $\lim_{t \to +\infty} s_h(t)$ finie

Exemple ①



$e(t) = Ri + s(t)$

$$e = RC\frac{ds}{dt} + s$$

eq. diff. linéaire $\Rightarrow$ syst. linéaire.

eq. homogène:

coef. >0 ⇒ stable.

$$RC\frac{ds_h}{dt}+s_h=0$$

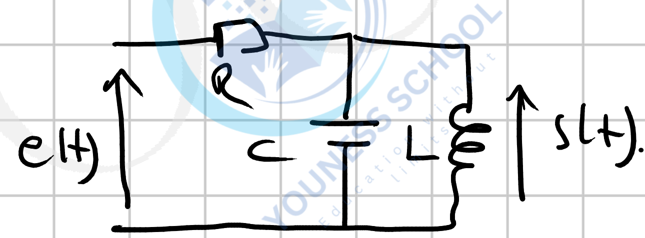
$$\frac{ds_h}{dt}=-\frac{s_h}{RC}\Rightarrow\int\frac{ds_h}{s_h}=\int-\frac{dt}{RC}$$

$$\ln(s_h)=-\frac{t}{RC}+cte$$

$$s_h=A\exp\left(-\frac{t}{RC}\right)$$

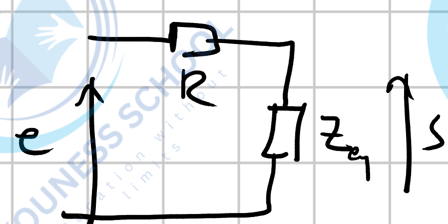
$$\lim_{t\to+\infty}s_h=0\Rightarrow$$ syst. stable

Exemple ②



Diviseur de tension

$$\underline{s}=\frac{Z_{eq}}{Z_{eq}+R}\,\underline{e}$$



$$\frac{1}{Z_{eq}}=\frac{1}{Z_C}+\frac{1}{Z_L}=jC\omega+\frac{1}{jL\omega}=\frac{-LC\omega^2+1}{jL\omega}$$

$$\underline{s}=\frac{\frac{jL\omega}{1-LC\omega^2}}{\frac{jL\omega}{1-LC\omega^2}+R}\,\underline{e}$$

$$\underline{s} = \frac{jL\omega}{jL\omega + R - RLC\omega^2}\,\underline{e}$$

$$j\omega \Leftrightarrow \frac{d}{dt}$$

$$-\omega^2 \Leftrightarrow \frac{d^2}{dt^2}$$

$$\frac{1}{j\omega} \Leftrightarrow \int dt$$

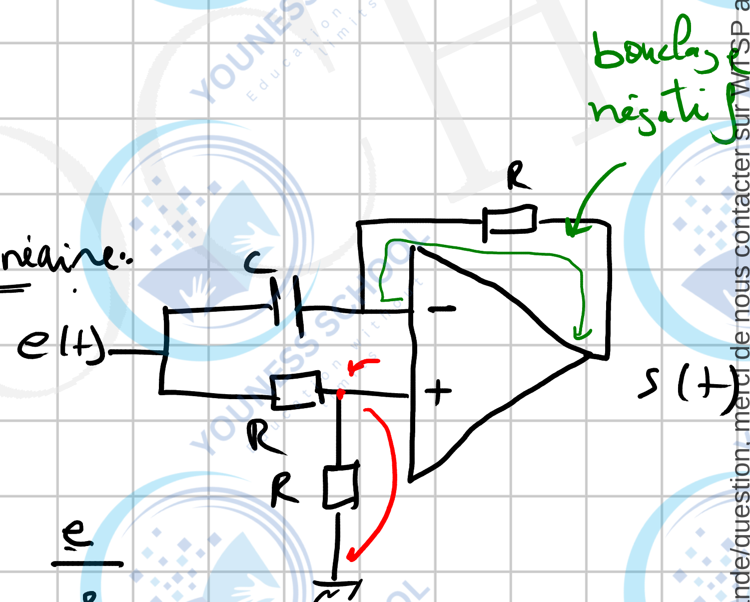
$$jL\omega\,\underline{s} + R\underline{s} - RLC\omega^2\,\underline{s} = jL\omega\,\underline{e}$$

$$L\frac{ds}{dt} + Rs + RLC\frac{d^2s}{dt^2} = L\frac{de}{dt}$$

$>0 \qquad >0 \qquad >0 \quad \Rightarrow$ stable.

L, R, LRC constantes $\Rightarrow$ Linéaire.

Amplificateur



bouclage négatif $\Rightarrow$ A.O linéaire.

$$\varepsilon = 0 \Rightarrow v^+ = v^-$$

$$v^+ = \frac{\frac{0}{R} + \frac{e}{R}}{\frac{1}{R} + \frac{1}{R}} = \frac{e}{2}$$

$$v^- = \frac{\frac{s}{R} + \frac{e}{Z_c}}{\frac{1}{R} + \frac{1}{Z_c}} = \frac{\frac{s}{R} + jC\omega\, e}{\frac{1}{R} + jC\omega} = \frac{\underline{s} + jRC\omega\,\underline{e}}{1 + jRC\omega}$$

$$v^+ = v^- \Rightarrow \frac{e}{2} = \frac{s + jRC\omega\, e}{1 + jRC\omega} \Rightarrow \underline{e}(1 + jRC\omega) = 2\underline{s} + 2jRC\omega\,\underline{e}$$

$$2\underline{s} = \underline{e}\left(1 + jRC\omega - 2jRC\omega\right)$$

$$2\underline{s} = \underline{e} - jRC\omega\,\underline{e}$$

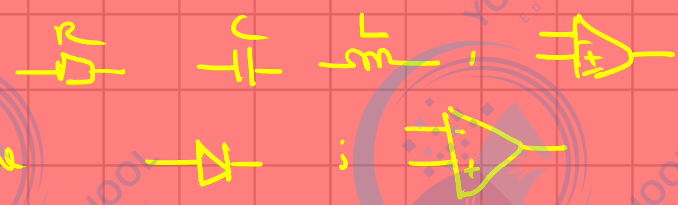
$$2s(t) = e(t) - RC\frac{de}{dt}$$

↳ coef. constante et >0 ⇒ Linéaire stable.

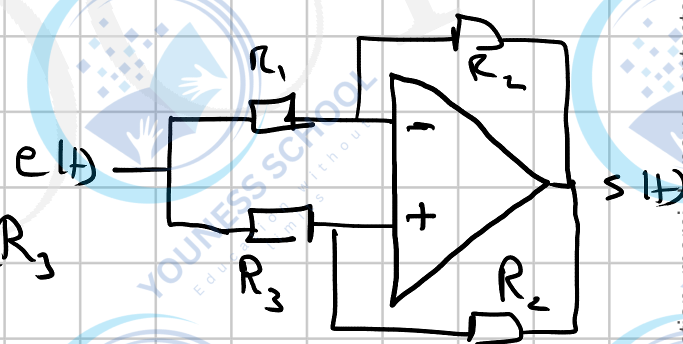
Composantes

Linéaire

non linéaire



Exemple 4



$$v^+ = \frac{\frac{e}{R_3} + \frac{s}{R_2}}{\frac{1}{R_3} + \frac{1}{R_2}} \times R_2R_3$$

$$= \frac{R_2 e + R_3 s}{R_2 + R_3}$$

$$v^- = \frac{\frac{e}{R_1} + \frac{s}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_2 e + R_1 s}{R_1 + R_2}.$$

On modélise l'A.O par un filtre passe bas:

!

$$\underline{s} = \underline{\mu}\,\underline{\varepsilon} = \frac{\mu_0}{1 + j\frac{\omega}{\omega_0}}\left(v^+ - v^-\right)$$

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$$\underline{s} + j\frac{w}{w_0}\underline{s} = \mu_0\left(\frac{R_2\underline{e} + R_3\underline{s}}{R_2+R_3} - \frac{R_2\underline{e} + R_1\underline{s}}{R_1+R_2}\right)$$

$$\underline{s}\left(1-\mu_0\left(\frac{R_3}{R_2+R_3} - \frac{R_1}{R_1+R_2}\right)\right) + j\frac{w}{w_0}\underline{s} = \mu_0 e\left(\frac{R_2}{R_2+R_3} - \frac{R_2}{R_1+R_2}\right)$$

$$\underbrace{s(t)\left[1-\frac{\mu_0 R_2}{(R_2+R_3)(R_1+R_2)}(R_3-R_1)\right]}_{?} + \underbrace{\frac{1}{w_0}}_{>0}\frac{ds}{dt} = \mu_0 e R_2\left(\frac{R_1-R_3}{(R_2+R_3)(R_1+R_2)}\right)$$

coef. constantes ⇒ Lineaire (circuit)

gain statique 10^5

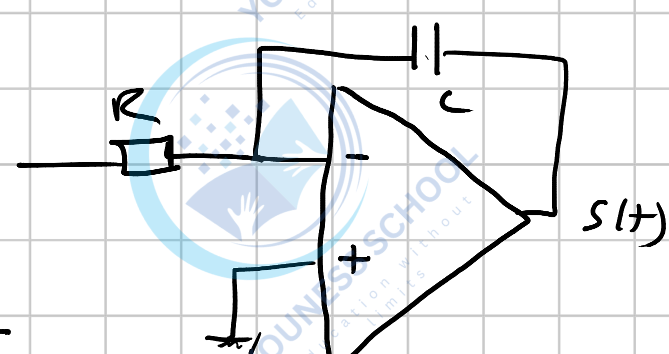
$$1-\frac{\mu_0 R_2}{(R_2+R_3)(R_1+R_2)}(R_3-R_1) \simeq \frac{R_2\mu_0}{(R_1+R_2)(R_2+R_3)}(R_1-R_3)$$

négligeable

- $R_1 > R_3 \Rightarrow > 0 \Rightarrow$ stable ⇒ A.O. Lineaire ⇒ $\varepsilon = 0 \Rightarrow v^+ = v^-$
- $R_1 < R_3 \Rightarrow < 0 \Rightarrow$ instable ⇒ A.O. non lineaire ⇒ $s = \text{signe}(\varepsilon)V_{sat}$

Exemple 6

bouclage négatif ⇒ A.O Lineaire



$\Rightarrow \varepsilon = 0 \Rightarrow v^+ = v^-$

$$v^+ = 0 \Rightarrow v^- = 0 = \frac{\frac{e}{R} + \frac{s}{Z_c}}{\frac{1}{R} + \frac{1}{Z_c}}$$

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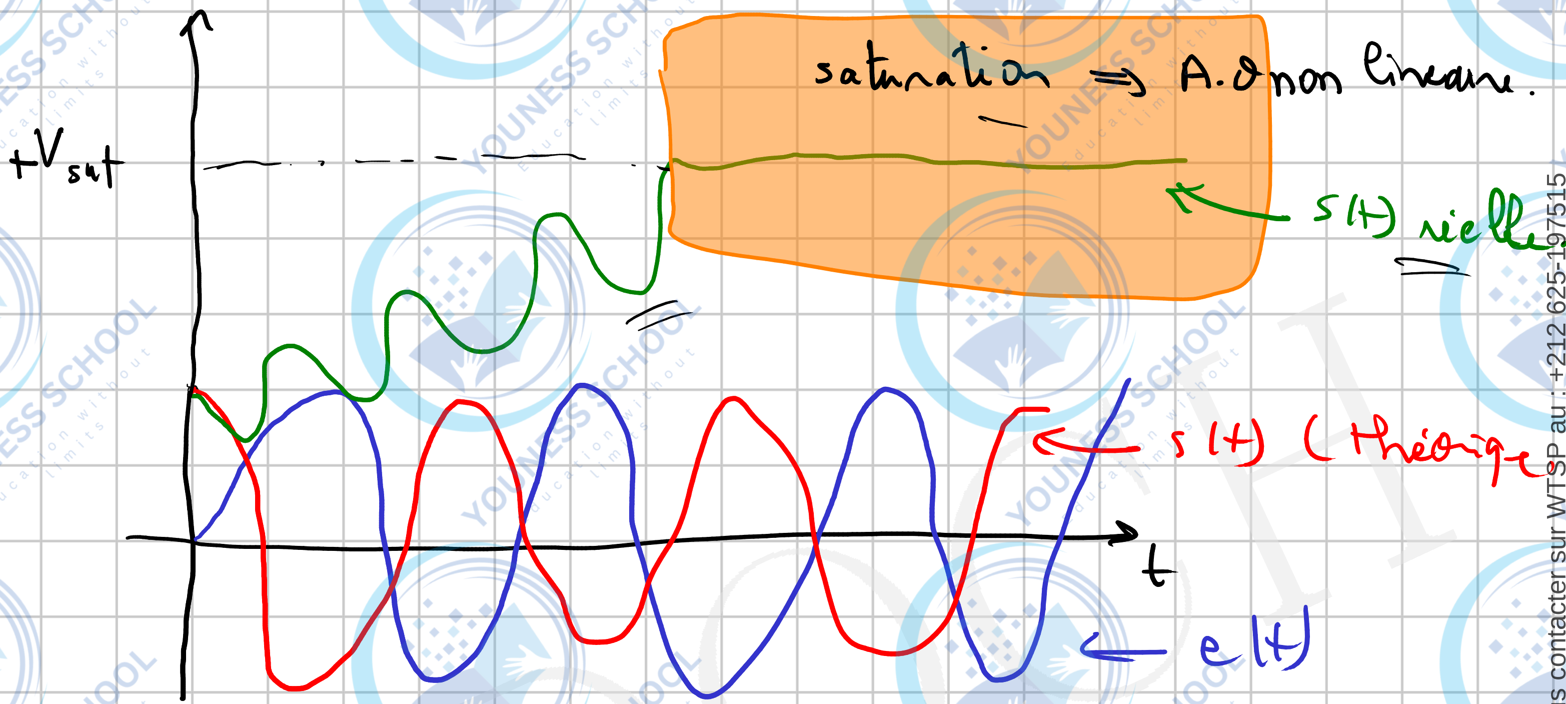
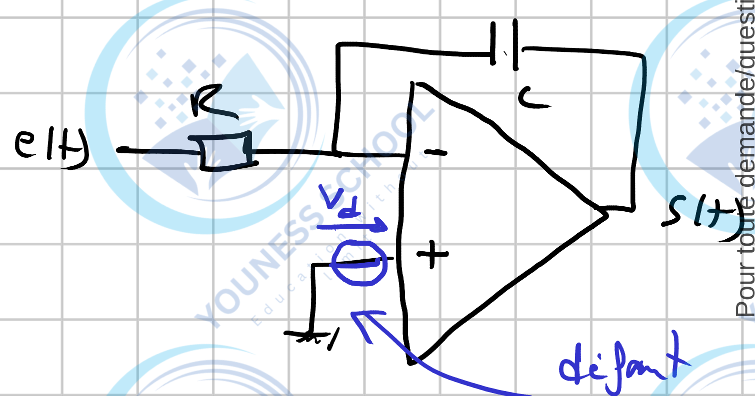
$$\frac{\underline{e}}{R} + \frac{\underline{s}}{\underline{Z}_c} = 0 \Rightarrow \underline{s}(t) = -\frac{\underline{Z}_c}{R}\,\underline{e} = -\frac{1}{jC\omega R}\,\underline{e}$$

$$s(t) = \frac{-1}{RC}\int e(t)\,dt \qquad \text{intégrateur.}$$



Explication : Tension de décalage de l'A.O.

$$v^- = \frac{\frac{\underline{e}}{R} + \frac{\underline{s}}{\underline{Z}_c}}{\frac{1}{R} + \frac{1}{\underline{Z}_c}} = \frac{\underline{e} + jRC\omega\,\underline{s}}{1 + jRC\omega}$$



$$v^+ = v_d = cte$$

$$v^+ = v^- \Rightarrow (1 + jRC\omega)\,V_d = \underline{e} + jRC\omega\,\underline{s}$$

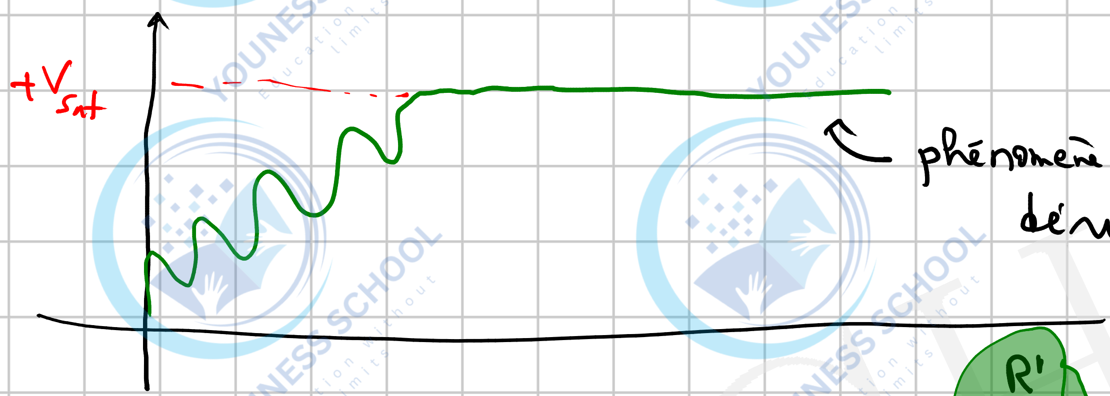
$$\underline{s} = \frac{V_d}{jRC\omega} + V_d - \frac{\underline{e}}{jRC\omega}$$

$$s(t) = \frac{1}{RC}\int V_d\,dt + V_d - \frac{1}{RC}\int e(t)\,dt$$

$$= \frac{V_d}{RC}\, t + V_d - \frac{1}{RC}\int e(t)\, dt.$$

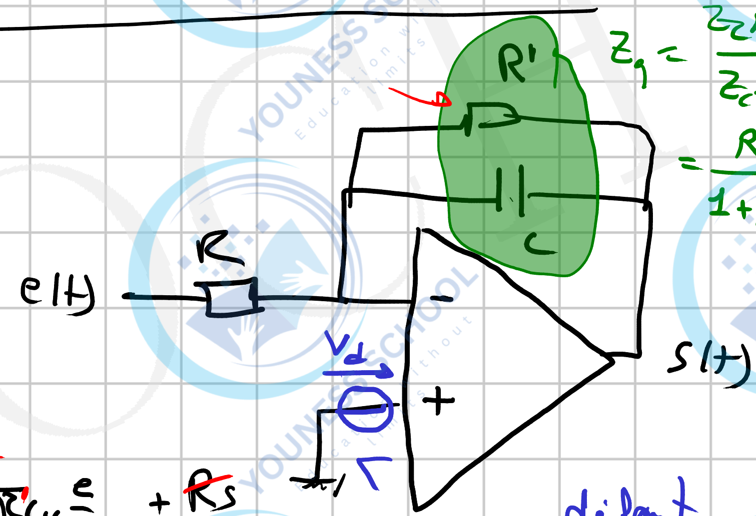
limitation en tension

$$e(t) = E\sin(\omega t) \Rightarrow s(t) = \frac{V_d}{RC}t + V_d + \frac{E}{RC\omega}\cos(\omega t) \leq V_{sat}$$



solution :

$$v^+ = V_d.$$



$$Z_q = \frac{Z_c * R'}{Z_c + R'} = \frac{R'}{1 + jR'C\omega}$$

$$v^- = \frac{\frac{e}{R} + \frac{s}{Z_q}}{\frac{1}{R} + \frac{1}{Z_q}} = \frac{\frac{R'}{1+jR'C\omega}e + Rs}{\frac{R'}{1+jR'C\omega} + R}$$

$$= \frac{R'e + R(1 + jR'C\omega)s}{R' + R + RjR'C\omega}$$

$$v^+ = v^- \Rightarrow V_d(R' + R) + jRR'C\omega V_d = R'e + (R + jRR'C\omega)s$$

$$(R' + R)V_d + R'RC\frac{dV_d}{dt} = R'e + Rs + RR'C\frac{ds}{dt}$$

e = 0

$$\frac{ds}{dt} + \frac{1}{R'C}s = \cancel{\frac{dV_d}{dt}} + \frac{R+R'}{RR'C}V_d$$

$$R' \ll \Rightarrow \frac{1}{R'C}s = \frac{dV_d}{dt} + \frac{1}{R'C}V_d \simeq \frac{1}{R'C}V_d.$$

$$\Rightarrow \boxed{s(t) = V_d}$$

$$R' \gg \Rightarrow \frac{ds}{dt} = \cancel{\frac{dV_d}{dt}} + \frac{1}{RC}V_d.$$

$$s(t) = V_d + \frac{1}{RC}\int V_d \Rightarrow \text{non linéaire.}$$

$V_d = 0$

$$0 = R'e + Rs + RR'C\frac{ds}{dt}$$

$$\frac{ds}{dt} + \frac{1}{R'C}s = -\frac{1}{RC}e(t).$$

$$e(t) = E\sin(wt) \Rightarrow s(t) = \frac{S}{w}\cos(wt)$$

$$-S\sin(wt) + \underbrace{\frac{1}{R'C}\frac{S}{w}\cos(wt)}_{\text{négligeable.}} = -\frac{E}{RC}\sin(wt).$$

$$R'Cw \to +\infty \Rightarrow R'Cw \gg 1 \Rightarrow R' \gg \frac{1}{Cw}.$$

$$\Rightarrow \frac{ds}{dt} = -\frac{1}{RC}e \Rightarrow \boxed{s = -\frac{1}{RC}\int e(t)dt}$$

$V_d \neq 0$ et $e \neq 0$

$$s(t) = V_d - \frac{1}{RC}\int e(t)\,dt$$

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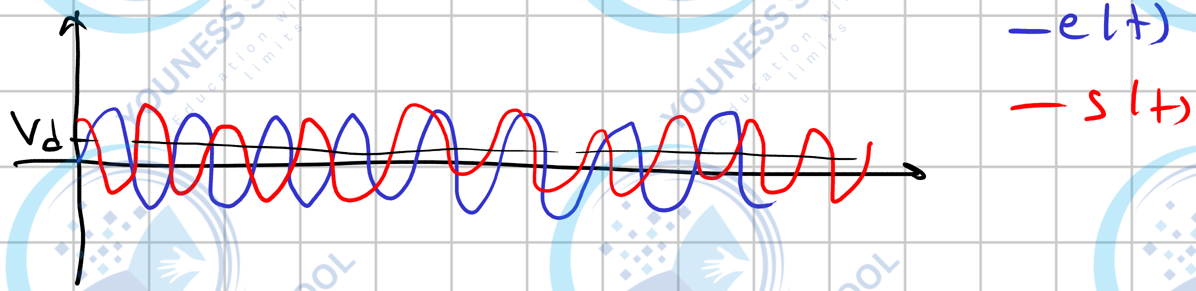
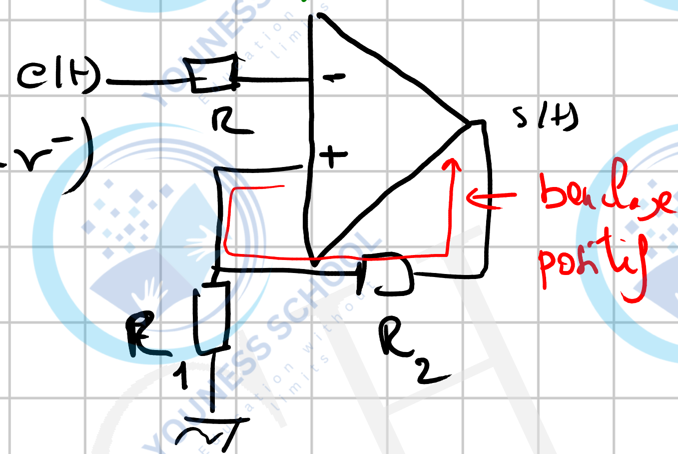


Exemple 7

Circuit linéaire / circuit stable / A.O linéaire



$$\underline{s} = \underline{\mu}\,\underline{\varepsilon} = \frac{\mu_0}{1+j\frac{\omega}{\omega_0}}(v^+ - v^-)$$

$$v^- = \frac{\frac{e}{R}}{\frac{1}{R}} = e \quad ; \quad v^+ = \frac{\frac{0}{R_1}+\frac{s}{R_2}}{\frac{1}{R_1}+\frac{1}{R_2}}$$

$$\underline{s} + j\frac{\omega}{\omega_0}\underline{s} = \mu_0\left(\frac{R_1\underline{s}}{R_1+R_2} - \underline{e}\right)$$

$$\underline{s}\left(1-\mu_0\frac{R_1}{R_1+R_2}\right) + j\frac{\omega}{\omega_0}\underline{s} = -\mu_0\underline{e}$$

$$s(t)\underbrace{\left[\cancel{1} - \overset{10^5}{\mu_0}\frac{R_1}{R_1+R_2}\right)}_{\text{const} < 0} + \underbrace{\frac{1}{\omega_0}}_{\text{constante} > 0}\frac{ds}{dt} = -\mu_0\, e(t)$$

Circuit ~~linéaire~~

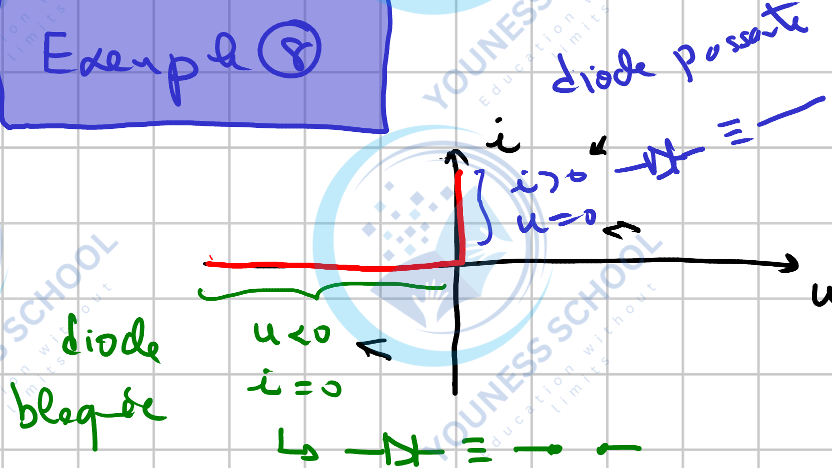
circuit instable

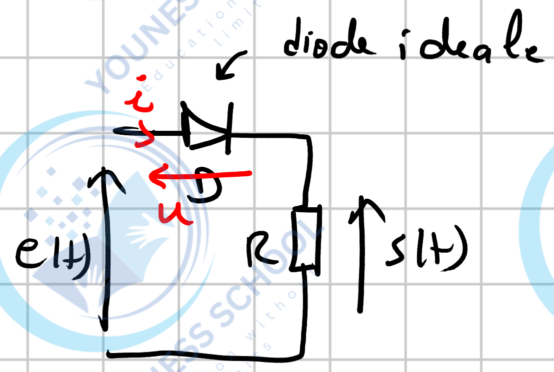
⇓

A.O. non linéaire

circuit non linéaire ←

Exemple 8





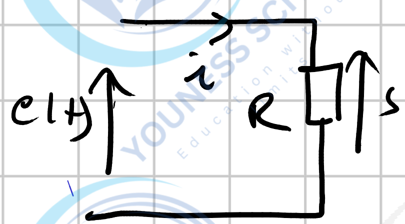
$e(t) = u + s(t)$

$s(t) = Ri$

Diode passante

$u = 0$

$\boxed{e(t) = s(t)}$



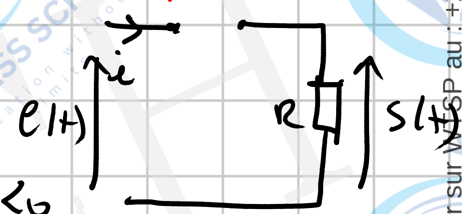
$i = \frac{s}{R} = \frac{e}{R} > 0$

si $e(t) > 0 \Rightarrow s(t) = e(t)$

Diode bloquée

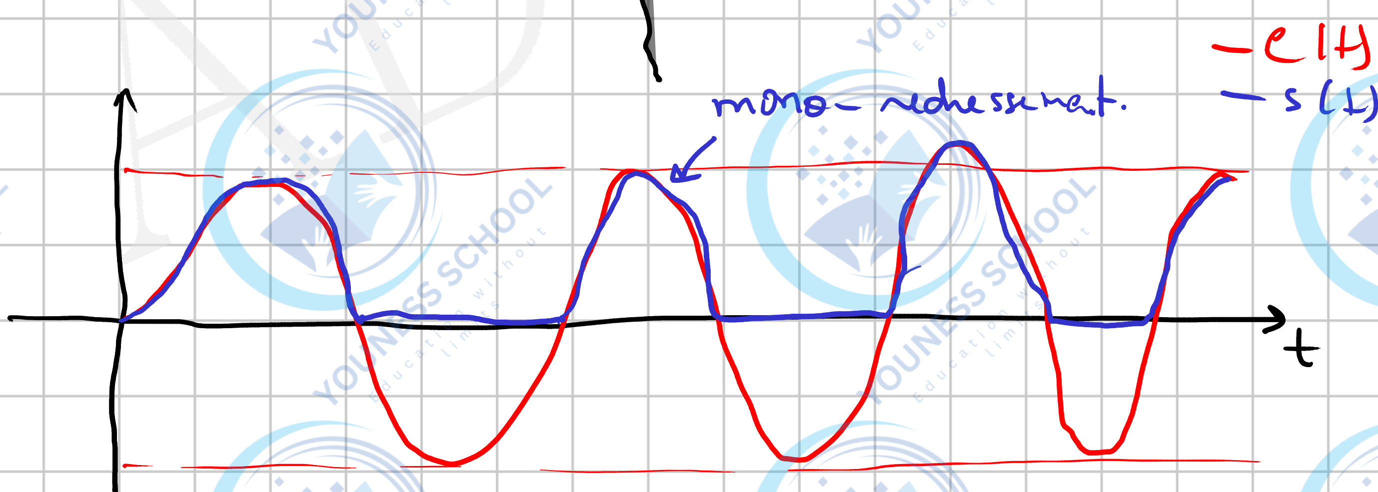
$i = 0$

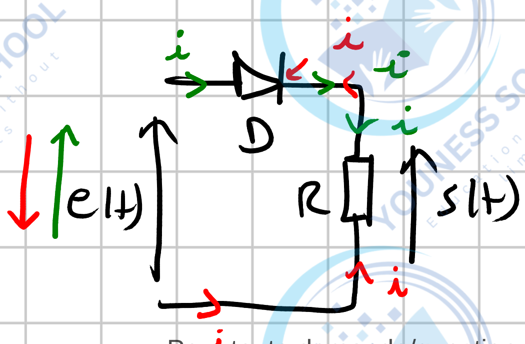
$\Rightarrow \boxed{s(t) = 0}$



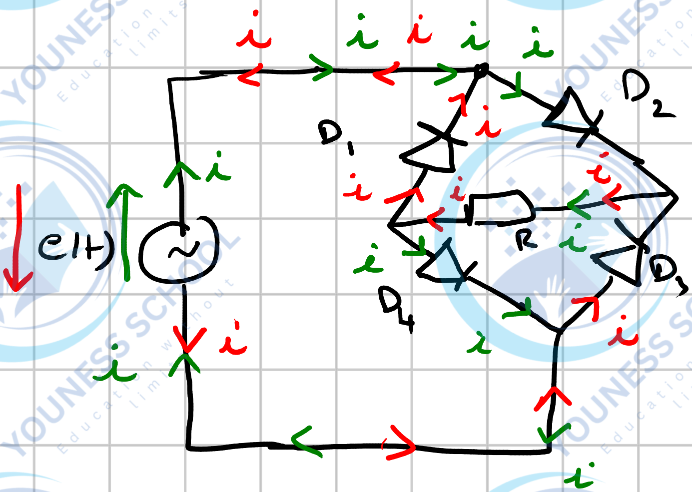
$e(t) = u < 0$

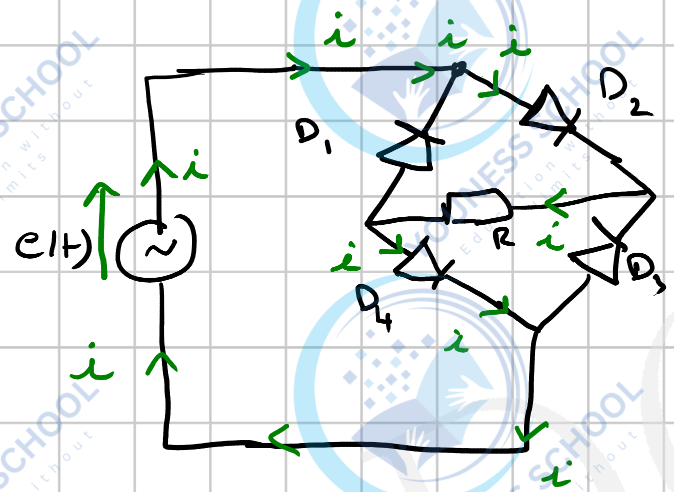
si $e(t) < 0 \Rightarrow s(t) = 0$

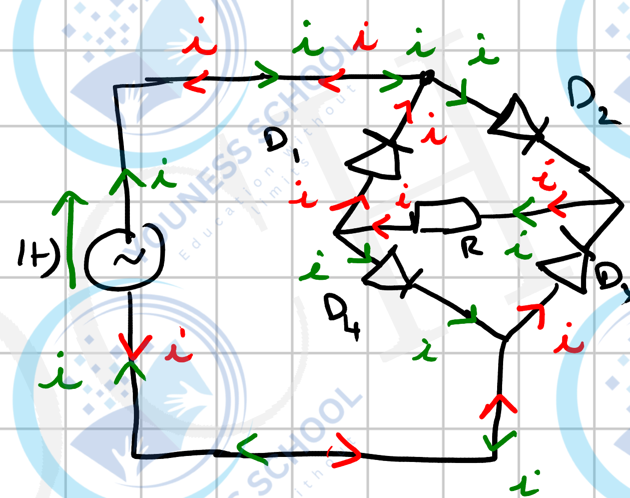


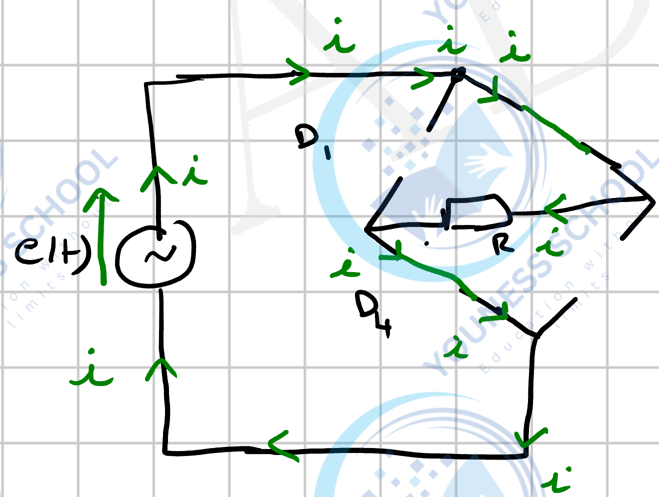


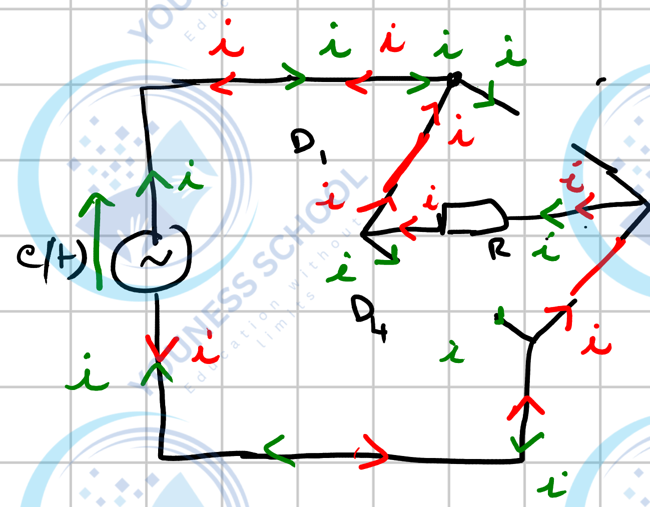
Exemple 9

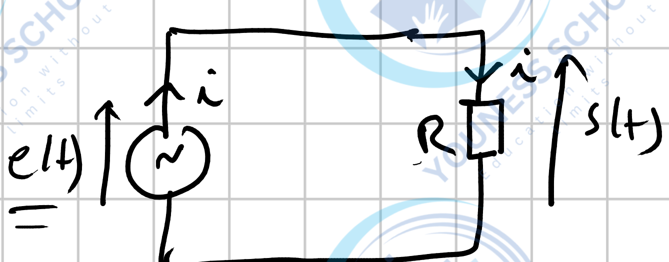


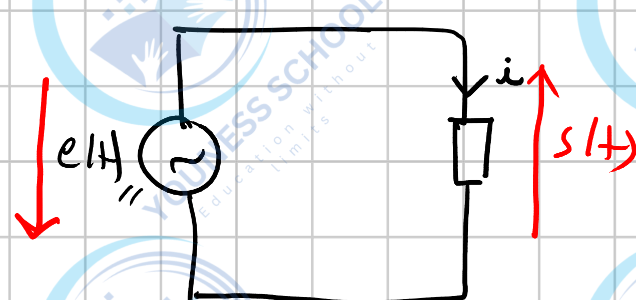








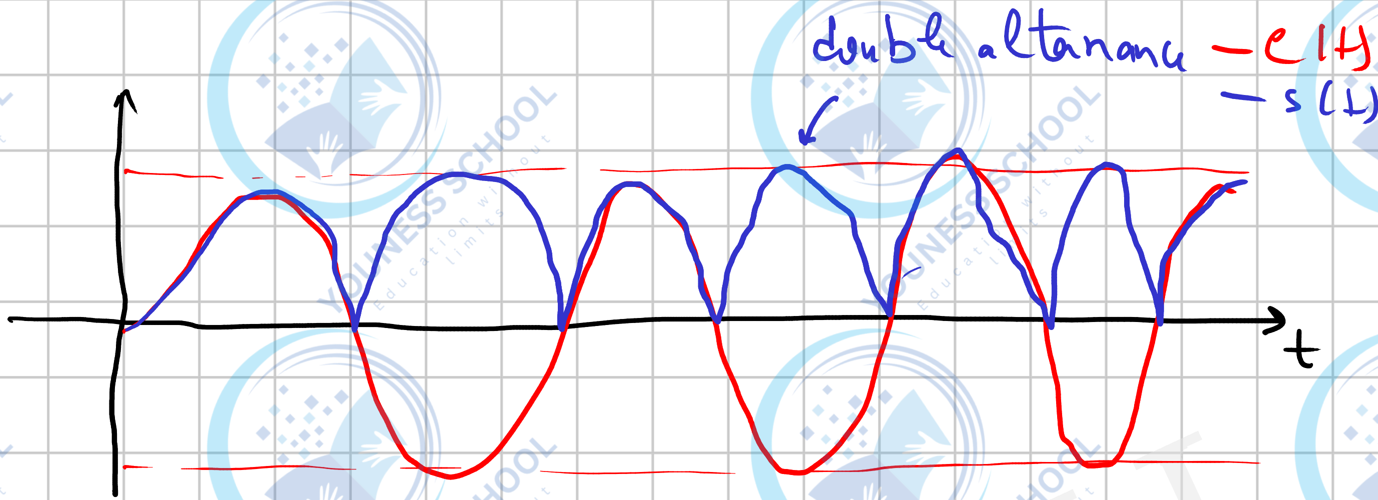




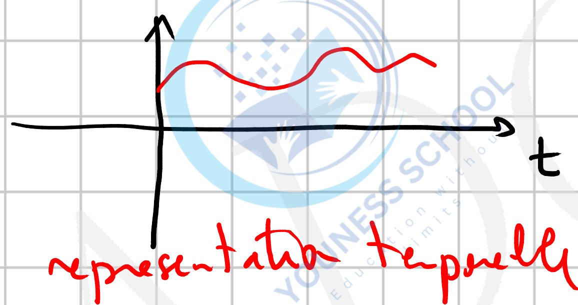
$s(t) = e(t)$

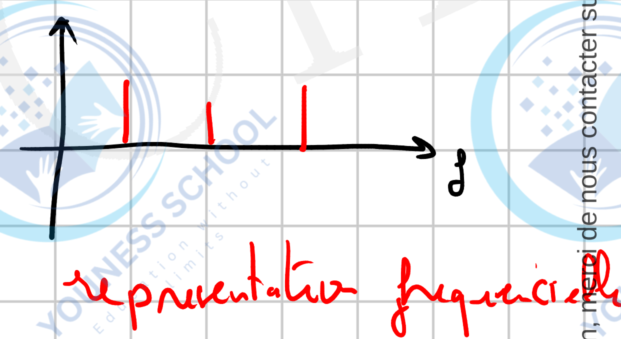
$s(t) = -e(t)$

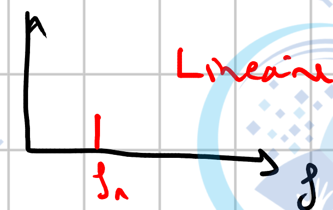
$$\boxed{s(t) = |e(t)|}$$



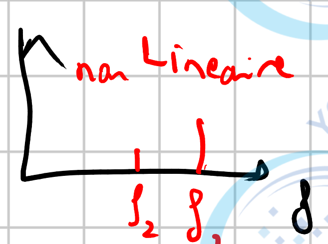
Relation avec Fourier







$$\underbrace{\left[\forall f \in \{f_1, f_2, f_3 \dots\}\right]}_{\text{sortie}} \in \underbrace{\{f_1, f_2, \dots\}}_{\text{entrée}} \quad \text{Linéaire}$$



$$\underbrace{\left[\exists f \in \{f_1, f_2, f_3 \dots\}\right]}_{\text{sortie}} \notin \underbrace{\{f_1, f_2, \dots\}}_{\text{entrée}} \quad \text{non linéaire}$$

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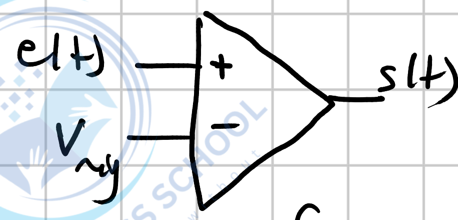
Exemple (11)

sans bouclage ⇒ non linéaire

$$s(t) = \text{signe}(\varepsilon)\, V_{sat}$$

$$= \text{signe}(e(t) - V_{réf})\, V_{sat}.$$

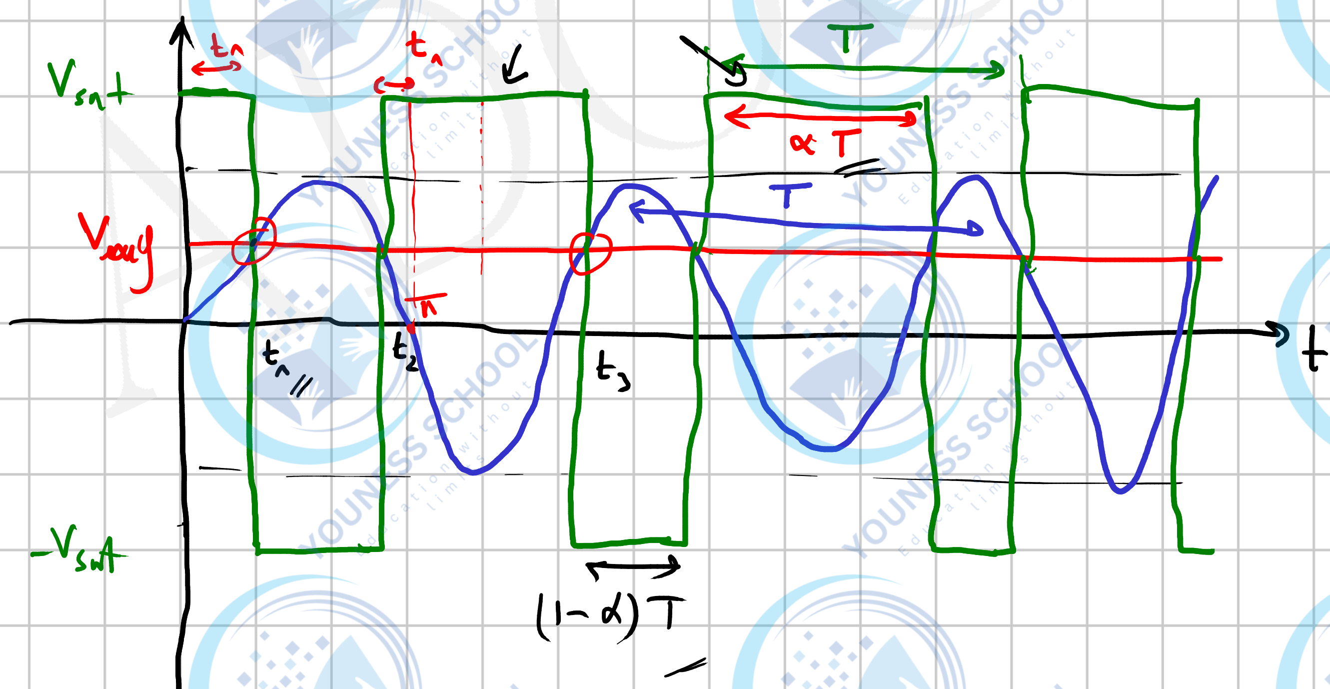


Comparateur simple.

$e(t) > V_{réf} \Rightarrow s(t) = +V_{sat}$

$e(t) < V_{réf} \Rightarrow s(t) = -V_{sat}.$

$e(t) = V_{réf} \Rightarrow \varepsilon = 0 \Rightarrow$ basculement.



— e(t) $= E \sin(\omega t)$ — $V_{réf}$. — s(t).

$t_3 - t_2 = \alpha T$ et $e(t_2) = e(t_3) = V_{réf}$

$\begin{cases} \text{période : } T \\ \text{rapport cyclique } \alpha \\ \text{amplitude : } V_{sat} \end{cases}$

$$e(t) = E\sin(\omega t) = V_{ref} \Rightarrow \sin(\omega t) = \frac{V_{ref}}{E}.$$

$$\omega t_1 = \arcsin\left(\frac{V_{ref}}{E}\right). \Rightarrow t_1 = \frac{1}{\omega}\arcsin\left(\frac{V_{ref}}{E}\right).$$

$$\omega t_3 - \omega t_1 = 2\pi \Rightarrow t_3 = \frac{2\pi}{\omega} + \frac{1}{\omega}\arcsin\left(\frac{V_{ref}}{E}\right)$$

$$\omega t_2 + \omega t_1 = \pi \Rightarrow t_2 = \frac{\pi}{\omega} - \frac{1}{\omega}\arcsin\left(\frac{V_{ref}}{E}\right)$$

$$\alpha T = t_3 - T_2 = \frac{\pi}{\omega} + \frac{2}{\omega}\arcsin\left(\frac{V_{ref}}{E}\right).$$

$$\alpha = \frac{\pi}{\omega T} + \frac{2}{\omega T}\arcsin\left(\frac{V_{ref}}{E}\right).$$

$$\omega = \frac{2\pi}{T} \Rightarrow \omega T = 2\pi$$

$$\boxed{\alpha = \frac{1}{2} + \frac{1}{\pi}\arcsin\left(\frac{V_{ref}}{E}\right)}$$

$$V_{ref} = 0 \Rightarrow \alpha = \frac{1}{2}.$$

$$V_{ref} = E \Rightarrow \alpha = \frac{1}{2} + \frac{1}{2} = 1 \Rightarrow s(t) = +V_{sat}$$

$$V_{ref} = -E \Rightarrow \alpha = \frac{1}{2} - \frac{1}{2} = 0 \Rightarrow s(t) = -V_{sat}$$

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$$P + \frac{1}{2}\rho v^2 + \rho g z = cte$$

$$\frac{\partial \rho S}{\partial t} + div(\rho \vec{v} S) = 0$$

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