# Test – Complex Numbers

## ■ Part I – no calculator on questions 1-5

**1.** Write each – completely simplified – expression in the form $a+b\mathrm{i}$, where $a, b \in \mathbb{R}$.

(a) $\dfrac{2-4\mathrm{i}}{1+3\mathrm{i}}$ (b) $\dfrac{2}{\mathrm{i}}\left(\dfrac{1}{\sqrt{3}}-\dfrac{3\mathrm{i}}{\sqrt{3}}\right)^2$

**2.** Given that $\dfrac{w+4}{w-2}=\mathrm{i}$, find $w$ in the form $a+b\mathrm{i}$, where $a, b \in \mathbb{R}$.

**3.** Given that $p, q \in \mathbb{R}$ and that $x=1+2\mathrm{i}$ is a solution of the equation $x^3+px^2+qx+20=0$ find the value of $p$ and the value of $q$.

**4.** Find all complex solutions to $z^5+32=0$. Express the solutions in modulus-argument form, i. e. in the form $z=r\,\mathrm{cis}\,\theta$

**5.** Find $(-1-\mathrm{i})^{11}$ in Cartesian form $a+b\mathrm{i}$, where $a, b \in \mathbb{R}$.

## ■ Part II – calculator allowed on questions 6-10

**6.** If $w_1=k-k\sqrt{3}\,\mathrm{i}$ and $w_2=4\mathrm{i}$, where $k$ is a real constant, express $w_1$ and $w_2$ in the form $r\,\mathrm{cis}\,\theta$, and hence find an expression for $\left(\dfrac{w_1}{w_2}\right)^3$ in terms of $k$ and i.

**7.** Find the four 4th roots of $-16$ and express them exactly in Cartesian form $a+b\mathrm{i}$, where $a, b \in \mathbb{R}$

**8.** (a) Show that $(a-1)(a^2+a+1)=a^3-1$

(b) Given that $z=e^{\mathrm{i}\left(\frac{2\pi}{3}\right)}$

- show that $z^3=1$ and $1+z+z^2=0$; and
- express each of the following expressions in terms of $z$ (write in **simplest** form)

(i) $z^8$ (ii) $(1-z)^2+4z$

**9.** (a) By using de Moivre's theorem, or otherwise, find the roots of the equation $z^4+4=0$ and express them in Cartesian form.

(b) Hence, or otherwise, express $z^4+4$ as the product of two quadratic polynomials in $z$ with real coefficients.

**10.** (a) Use de Moivre's theorem to prove that $\sin 3\theta=3\sin\theta-4\sin^3\theta$.

(b) Hence, find the exact solutions of the equation $3x-4x^3=\dfrac{1}{2}$.

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## Answers

**1.** (a) $-1-\mathrm{i}$ (b) $-4+\frac{16}{3}\mathrm{i}$

**2.** $w=-1-3\mathrm{i}$

**3.** $p=2,\ q=-3$

**4.** five solutions: $z=-2,\ z=2\mathrm{cis}\frac{\pi}{5},\ z=2\mathrm{cis}\frac{3\pi}{5},\ z=2\mathrm{cis}\left(-\frac{3\pi}{5}\right),\ z=2\mathrm{cis}\left(-\frac{\pi}{5}\right)$

**5.** $32-32\mathrm{i}$

**6.** $-\frac{1}{8}k^3\mathrm{i}$

**7.** $\sqrt{2}\pm\mathrm{i}\sqrt{2}$, $-\sqrt{2}\pm\mathrm{i}\sqrt{2}$

**8.** (b) (i) $z^2$ (ii) $z$

**9.** (a) $1+\mathrm{i},\ -1+\mathrm{i},\ -1-\mathrm{i},\ 1-\mathrm{i}$ (b) $\left(x^2-2x+2\right)\left(x^2+2x+2\right)$

**10.** (b) $x=\sin\left(\frac{\pi}{18}\right),\ \sin\left(\frac{5\pi}{18}\right),\ \sin\left(-\frac{7\pi}{18}\right)$