Probability & Counting Principles

[**answers** on next page]

Calculator allowed on all questions.

1. Using all six letters of the word THEORY, how many different arrangements of the letters can be made?

2. Using all eleven letters of the word INSPIRATION,

   (a) how many different arrangements of the letters can be made?

   (b) how many of these arrangements begin and end with the letter N?

3. A coin is biased so that $P(H) = \dfrac{1}{4}$ and $P(T) = \dfrac{3}{4}$. If the coin is tossed seven times find the probability of obtaining:

   (a) exactly six tails;   (b) at least two tails;   (c) at least one head.

4. The events A and B are such that $P(A \cup B) = \dfrac{3}{5}$, $P(B) = \dfrac{2}{5}$ and $P(B|A) = \dfrac{3}{7}$.

   (a) Find $P(A)$

   (b) State, with a reason, whether A and B are independent events.

5. In a bucket there are 12 white balls and 3 green balls. Without replacement three balls are randomly selected from the bucket.
   (a) The first two balls selected are green. What is the probability that the third ball is white?
   (b) What is the probability that exactly two of the three balls selected are white?

6. A box contains 36 apples of which 7 are rotten. If 8 apples are chosen at random, what is the probability that

   (a) none of them are rotten;   (b) exactly one of them is rotten.

7. Every morning Joe either cycles or walks to school with equal probability. If he walks the probability that he is late to school is 0.2 and if he cycles then the probability that he is late is 0.4.
   (a) What is the probability that Joe cycles and is late to school?
   (b) What is the probability that Joe is late to school?
   (c) Given that Joe is late on a particular day, what is the probability that he walked?

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ANSWERS

1. $6! = 720$

2. (a) $\dfrac{11!}{3!\,2!} = 3,326,400$ (b) $\dfrac{9!}{3!} = 60,480$

3. (a) ≈ 0.311 (b) ≈ 0.999 (c) ≈ 0.867

4. (a) $P(A) = \dfrac{7}{20}$

    (b) events A and B are not independent because $P(B \cap A) \neq P(B) \cdot P(A)$

    $$P(B \cap A) = \dfrac{3}{20},\ \ P(B) \cdot P(A) = \dfrac{7}{50}$$

5. (a) $\dfrac{12}{13} \approx 0.923$ (b) ≈ 0.435

6. (a) ≈ 0.142 (b) ≈ 0.361

7. (a) $\dfrac{1}{5} = 0.2$ (b) $\dfrac{3}{10} = 0.3$ (c) $P(\text{walked} \mid \text{late}) = \dfrac{1}{3}$