Quiz2 - Probability of Events SOLUTION KEY

[total marks on quiz: 35 marks]

Part I - No GDC for questions 1 & 2

1. Two independent events A and B are such that $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{3}$ and $P(A|B)=\frac{1}{6}$.

(a) Find $P(A\cap B)$. **[2 marks]**

(b) Find $P(A\cup B)$. **[3 marks]**

(c) Find the probability that only one of the events occurs. **[3 marks]**

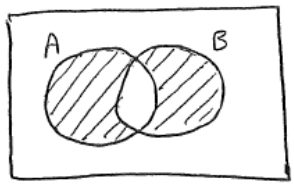
(d) Find the probability that neither event occurs. **[2 marks]**

(a) $P(A\cap B) = P(A)\cdot P(B) = \frac{1}{2}\cdot\frac{1}{3} = \frac{1}{6}$

(b) $P(A\cup B) = P(A)+P(B)-P(A\cap B)$

$= \frac{1}{2}+\frac{1}{3}-\frac{1}{6} = \frac{3}{6}+\frac{2}{6}-\frac{1}{6} = \frac{4}{6} = \frac{2}{3}$

(c)

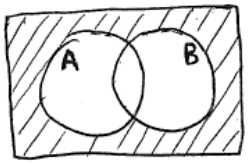


shaded region shows representation of only one of the events - A or B - occurring

$P(\text{only A or only B}) = P(A\cup B)-P(A\cap B)$

$= \frac{2}{3}-\frac{1}{6} = \frac{4}{6}-\frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

(d) $P(\text{neither A or B}) = P\left((A\cup B)'\right)$



$= 1-P(A\cup B)$

$= 1-\frac{2}{3} = \frac{1}{3}$

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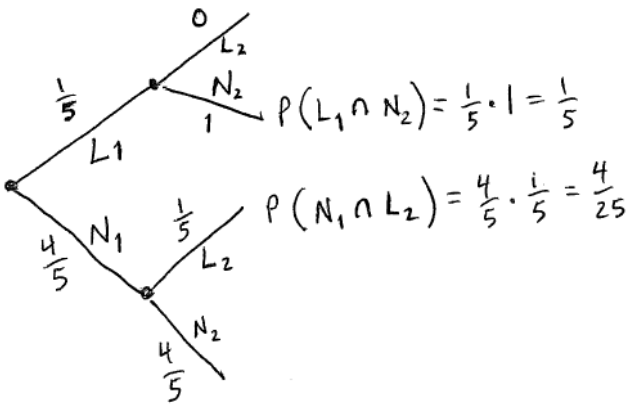
2. The probability that a student will leave his calculator in any classroom is $\frac{1}{5}$. Suppose the student has visited two classrooms in succession and leaves his calculator in one of the classrooms. What is the probability that the student left his calculator in the first classroom? **[6 marks]**

L: Leave calculator N: Not leave calculator



$$P(L_1 \mid L_1 \text{ OR } L_2) = \frac{P(L_1 \cap (L_1 \text{ OR } L_2))}{P(L_1 \text{ OR } L_2)}$$

$$= \frac{P(L_1)}{P(L_1 \cap N_2) + P(N_1 \cap L_2)} = \frac{\frac{1}{5}}{\frac{1}{5} + \frac{4}{25}} =$$

$$= \frac{\frac{1}{5}}{\frac{5}{25} + \frac{4}{25}} = \frac{\frac{1}{5}}{\frac{9}{25}} = \frac{25}{45} = \frac{5}{9}$$

Part II – GDC is allowed for questions 3–5

3. A biased coin is flipped five times. The probability of it landing 'heads' is 0.6.

(a) What is the probability that the coin never lands 'heads' ? **[2 marks]**

(b) What is the probability that exactly two of the tosses land 'heads' ? **[3 marks]**

(c) What is the probability that at least two of the tosses land 'heads' ? **[3 marks]**

$P(H) = 0.6 \qquad P(T) = 0.4$

(a) $P(5T) = (0.4)^5 = 0.01024$

(b) $P(2H) = {}_5C_2 (0.6)^2 (0.4)^3 = 0.2304$

(c) $P(H \geq 2) = 1 - P(H \leq 1)$

$= 1 - [P(0H) + P(1H)] = 1 - [0.01024 + 5(.6)(.4)^4]$

$= 0.91296$

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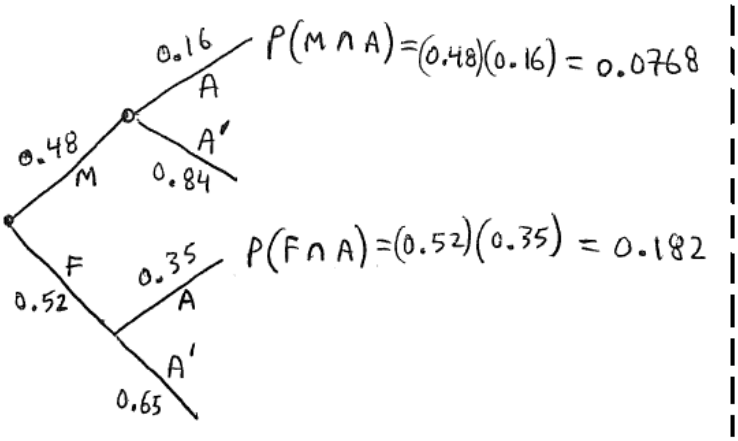
4. The students in a school are all vaccinated for measles. 48% of the students are male – and 16% of the males have an allergic reaction to the vaccination. 35% of the girls also have an allergic reaction to the vaccination. A student is randomly chosen. Find the probability that this student:

(a) has an allergic reaction; **[3 marks]**

(b) is female given that a reaction occurs. **[3 marks]**



$P(M \cap A) = (0.48)(0.16) = 0.0768$

$P(F \cap A) = (0.52)(0.35) = 0.182$

(a) $P(A) = P(M \cap A) + P(F \cap A)$

$= 0.0768 + 0.182$

$= 0.2588$

(b) $P(F|A) = \dfrac{P(F \cap A)}{P(A)}$

$= \dfrac{0.182}{0.2588} \approx 0.703$

5. In a group of 18 students, eight are females. What is the probability of choosing five students from the group of 18 such that:

(a) they are all female; **[2 marks]**

(b) there are three girls and two boys. **[3 marks]**

(a) $P(5F) = \dfrac{{}^{8}C_5}{{}^{18}C_5} = \dfrac{56}{8568} \approx 0.00654$

(b) $P(3F \text{ and } 2M) = \dfrac{{}^{8}C_3 \cdot {}^{10}C_2}{{}^{18}C_5} = \dfrac{56 \cdot 45}{8568} = \dfrac{2520}{8568} \approx 0.294$