H

National
Qualifications
2015

2015 Mathematics

New Higher Paper 1

Finalised Marking Instructions

General Comments

These marking instructions are for use with the 2015 Higher Mathematics Examination.

For each question the marking instructions are in two sections, namely **Illustrative Scheme** and **Generic Scheme**. The **Illustrative Scheme** covers methods which are commonly seen throughout the marking. The **Generic Scheme** indicates the rationale for which each mark is awarded. In general, markers should use the **Illustrative Scheme** and only use the **Generic Scheme** where a candidate has used a method not covered in the **Illustrative Scheme**.

All markers should apply the following general marking principles throughout their marking:

1 Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than deducted for what is wrong.

2 One mark is available for each •. There are **no** half marks.

3 Working subsequent to an error **must be followed through**, with possible full marks for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working for a follow through mark has been eased, the follow through mark cannot be awarded.

4 As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Throughout this paper, unless specifically mentioned in the marking instructions, a correct answer with no working receives no credit.

5 In general, as a consequence of an error perceived to be trivial, casual or insignificant, e.g. $6\times 6=12$, candidates lose the opportunity of gaining a mark. But note the second example in comment 7.

6 Where a transcription error (paper to script or within script) occurs, the candidate should be penalised, eg

This is a transcription error and so the mark is not awarded.

Eased as no longer a solution of a quadratic equation.

$$x^2+5x+7=9x+4$$
$$x-4x+3=0$$
$$x=1$$

2

Exceptionally this error is not treated as a transcription error as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt.

$$x^2+5x+7=9x+4$$
$$\cancel{x}-4x+3=0$$
$$(x-3)(x-1)=0$$
$$x=1 \text{ or } 3$$

7 **Vertical/horizontal marking**
Where a question results in two pairs of solutions, this technique should be applied, but only if indicated in the detailed marking instructions for the question.

Example: Point of intersection of line with curve

Illustrative Scheme: $\bullet^5$ $x = 2,\ x = -4$

$\bullet^6$ $y = 5,\ y = -7$

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x = 2$	$y = 5$
$\bullet^6$	$x = -4$	$y = -7$

Markers should choose whichever method benefits the candidate, but **not** a combination of both.

8 In final answers, numerical values should be simplified as far as possible, unless specifically mentioned in the detailed marking instructions.

Examples: $\frac{15}{12}$ should be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\qquad$ $\frac{43}{1}$ should be simplified to 43

$\frac{15}{0{\cdot}3}$ should be simplified to 50 $\qquad$ $\frac{4/5}{3}$ should be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8

The square root of perfect squares up to and including 100 must be known.

9 Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

10 Unless specifically mentioned in the marking instructions, the following should not be penalised:

- Working subsequent to a **correct** answer;
- Correct working in the wrong part of a question;
- Legitimate variations in numerical answers, eg angles in degrees rounded to nearest degree;
- Omission of units;
- Bad form (bad form only becomes bad form if subsequent working is correct), e.g.
  $(x^3 + 2x^2 + 3x + 2)(2x + 1)$
  written as
  $(x^3 + 2x^2 + 3x + 2) \times 2x + 1$
  $2x^4 + 4x^3 + 6x^2 + 4x + x^3 + 2x^2 + 3x + 2$
  $2x^4 + 5x^3 + 8x^2 + 7x + 2$ gains full credit;
- Repeated error within a question, but not between questions.

11 In any ‘Show that . . .’ question, where the candidate has to arrive at a required result, the last mark of that part is not available as a follow through from a previous error unless specifically stated otherwise in the detailed marking instructions.

12 All working should be carefully checked, even where a fundamental misunderstanding is apparent early in the candidate's response. Marks may still be available later in the question so reference must be made continually to the marking instructions.
All working must be checked: the appearance of the correct answer does not necessarily indicate that the candidate has gained all the available marks.

13 If you are in serious doubt whether a mark should or should not be awarded, consult your Team Leader (TL).

14 Scored out working which **has not been replaced** should be marked where still legible. However, if the scored out working **has been replaced**, only the work which has not been scored out should be marked.

15 Where a candidate has made multiple attempts using the same strategy, mark all attempts and award the lowest mark.
Where a candidate has tried different strategies, apply the above ruling to attempts within each strategy and then award the highest resultant mark. For example:

Strategy 1 attempt 1 is worth 3 marks	Strategy 2 attempt 1 is worth 1 mark
Strategy 1 attempt 2 is worth 4 marks	Strategy 2 attempt 2 is worth 5 marks
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

16 In cases of difficulty, covered neither in detail nor in principle in these instructions, markers should contact their TL in the first instance.

Detailed Marking Instructions for each question

<table>
<tr><th colspan="2">Question</th><th>Generic Scheme</th><th>Illustrative Scheme</th><th>Max Mark</th></tr>
<tr><td colspan="2">1.</td><td colspan="2"></td><td></td></tr>
<tr><td></td><td></td><td>$\bullet^1$ equate scalar product to zero

$\bullet^2$ state value of t</td><td>$\bullet^1$ $-24+2t+6=0$

$\bullet^2$ $t=9$</td><td>2</td></tr>
<tr><td colspan="5">Notes:</td></tr>
<tr><td colspan="5"></td></tr>
<tr><td colspan="5">Commonly Observed Responses:</td></tr>
<tr><td colspan="5">Candidate A
$-24+2t+6=-1$ $\bullet^1$ ✗
$t=\frac{17}{2}$ or $8\frac{1}{2}$ $\bullet^2$ $\boxed{✓1}$</td></tr>
<tr><td colspan="5"></td></tr>
<tr><td colspan="2">2.</td><td colspan="2"></td><td></td></tr>
<tr><td></td><td></td><td>$\bullet^1$ know to and differentiate

$\bullet^2$ evaluate $\frac{dy}{dx}$

$\bullet^3$ evaluate y-coordinate

$\bullet^4$ state equation of tangent</td><td>$\bullet^1$ $6x^2$

$\bullet^2$ 24

$\bullet^3$ -13

$\bullet^4$ $y=24x+35$</td><td>4</td></tr>
<tr><td colspan="5">Notes:</td></tr>
<tr><td colspan="5">1. $\bullet^4$ is only available if an attempt has been made to find the gradient from differentiation.
2. At mark $\bullet^4$ accept $y+13=24(x+2)$, $y-24x=35$ or any other rearrangement of the equation.</td></tr>
<tr><td colspan="5">Commonly Observed Responses:</td></tr>
<tr><td colspan="5"></td></tr>
</table>

Question		Generic Scheme	Illustrative Scheme	Max Mark
3.				
		•[1] know to use $x=-3$ •[2] interpret result and state conclusion	**Method 1** •[1] $(-3)^3-3(-3)^2-10(-3)+24$ •[2] $=0 \therefore (x+3)$ is a factor. **Method 2** •[1] $\begin{array}{r\|rrrr} -3 & 1 & -3 & -10 & 24 \\ & & -3 & & \\ \hline & 1 & & & \end{array}$ •[2] $\begin{array}{r\|rrrr} -3 & 1 & -3 & -10 & 24 \\ & & -3 & 18 & -24 \\ \hline & 1 & -6 & 8 & 0 \end{array}$ remainder $=0 \therefore (x+3)$ is a factor. **Method 3** •[1] $x+3 \overline{\smash{)}x^3-3x^2-10x+24}$ with quotient x^2, x^3+3x^2 •[2] $=0 \therefore (x+3)$ is a factor.	**4**
		•[3] state quadratic factor •[4] factorise completely	•[3] x^2-6x+8 stated or implied by •[4] •[4] $(x+3)(x-4)(x-2)$	

Notes:

1. Communication at •[2] must be consistent with working at that stage ie a candidate's working must arrive legitimately at 0 before •[2] is awarded.
2. Accept any of the following for •[2]:
'$f(-3)=0$ so $(x+3)$ is a factor'
'since remainder is 0, it is a factor'
the 0 from the table linked to the word 'factor' by eg 'so', 'hence', '$\therefore$', '$\rightarrow$', '$\Rightarrow$'
3. Do not accept any of the following for •[2]:
double underlining the zero or boxing the zero without comment
'$x=3$ is a factor', '$(x-3)$ is a factor', '$x=-3$ is a root', '$(x-3)$ is a root', "$(x+3)$ is a root"
the word 'factor' **only,** with no link
4. At •[4] the expression may be written in any order.
5. An incorrect quadratic correctly factorised may gain •[4]
6. Where the quadratic factor obtained is irreducible, candidates must clearly demonstrate that $b^2-4ac<0$ to gain •[4]
7. $=0$ must appear at •[1] or •[2] for •[2] to be awarded.
8. For candidates who do not arrive at 0 at the •[2] stage •[2]•[3]•[4] not available.
9. Do not penalise candidates who attempt to solve a cubic equation. However, within this working there may be evidence of the correct factorisation of the cubic.

Commonly Observed Responses:

Candidate A	Candidate B
$\begin{array}{r\|rrrr} 2 & 1 & -3 & -10 & 24 \\ & & 2 & -2 & -24 \\ \hline & 1 & -1 & -12 & 0 \end{array}$ $\Rightarrow x-2$ is a factor $\bullet^2$ ✓1 $\bullet^3$ ✓	$\begin{array}{r\|rrrr} 2 & 1 & -3 & -10 & 24 \\ & & 2 & -2 & -24 \\ \hline & 1 & -1 & -12 & 0 \end{array}$ $\Rightarrow x-2$ is a factor $\bullet^1$ ✓2 $\bullet^2$ ✓1
$(x-2)(x^2-x-12)$ $\bullet^4$ ✓	
$(x-2)(x-4)(x+3) \Rightarrow x+3$ is a factor $\bullet^1$ ✓	

4.			
	$\bullet^1$ state the value of p	$\bullet^1$ $p=3$	**3**
	$\bullet^2$ state the value of q	$\bullet^2$ $q=4$	
	$\bullet^3$ state the value of r	$\bullet^3$ $r=1$	

Notes:

1. These are the only acceptable responses for p, q and r.

Commonly Observed Responses:

5(a).			
	$\bullet^1$ let $y=6-2x$ and rearrange.	$\bullet^1$ $x=\dfrac{6-y}{2}$ or $y=\dfrac{6-x}{2}$	**2**
	$\bullet^2$ state expression.	$\bullet^2$ $g^{-1}(x)=\dfrac{6-x}{2}$ or $3-\dfrac{x}{2}$ or $\dfrac{x-6}{-2}$	
	Method 2	**Method 2**	
	$\bullet^3$ equates composite function to x	$g(g^{-1}(x))=x$ this gains $\bullet^3$	
	$\bullet^1$ start to rearrange.	$6-2g^{-1}(x)=x$	
	$\bullet^2$ state expression.	$g^{-1}(x)=\dfrac{6-x}{2}$ or $3-\dfrac{x}{2}$ or $\dfrac{x-6}{-2}$	

Notes:

1. At $\bullet^1$ accept any equivalent expression with any 2 distinct variables.

Commonly Observed Responses:

5(b).			
	$\bullet^3$ state expression	$\bullet^3$ x	**1**

Notes:

2. Candidates using method 2 may be awarded $\bullet^3$ at line one.
3. For candidates who attempt to find the composite function $g\left(g^{-1}(x)\right)$, accept

   $6-2\left(\dfrac{6-x}{2}\right)$ for $\bullet^3$.
4. In this case $\bullet^3$ may be awarded as follow through where an incorrect $g^{-1}(x)$ is found at $\bullet^2$, provided it includes the variable x.

Commonly Observed Responses:

Question		Generic Scheme	Illustrative Scheme	Max Mark
6.				
		•¹ use laws of logs •² use laws of logs ³ evaluate log	•¹ $\log_6 27^{\frac{1}{3}}$ •² $\log_6\left(12\times 27^{\frac{1}{3}}\right)$ •³ 2	**3**

Notes:

Commonly Observed Responses:

Candidate A	**Candidate B**
$\log_6 12+\log_6 9$ •¹ ✗ $\log_6(12\times 9)$ •² ✓1 $\log_6 108$ •³ ✓2	$\frac{1}{3}\log_6(12\times 27)$ $\frac{1}{3}\log_6 324$ $\log_6 324^{\frac{1}{3}}$ **Award 1 out of 3** ^,^ ✓1

Question		Generic Scheme	Illustrative Scheme	Max Mark
7.				
		•¹ write in differentiable form •² differentiate first term •³ differentiate second term •⁴ evaluate derivative at $x=4$	•¹ $3x^{\frac{3}{2}}-2x^{-1}$ •² $\frac{9}{2}x^{\frac{1}{2}}+\ldots$ •³ $\ldots+2x^{-2}$ •⁴ $9\frac{1}{8}$	**4**

Notes:

1. •² must involve a fractional index.
2. •³ must involve a negative index.
3. •⁴ is only available as a consequence of substituting into a 'derivative' containing a fractional or negative index.
4. If no attempt has been made to expand the bracket at •¹ then •² & •³ are not available. •⁴ is still available as follow through.

Commonly Observed Responses:

Candidate A

$$f(x)=3x^{\frac{1}{2}}-2x^{-\frac{1}{4}}$$

$$f'(x)=\frac{3}{2}x^{-\frac{1}{2}}+\frac{1}{2}x^{-\frac{5}{4}}$$

$$=\frac{3}{2\sqrt{x}}+\frac{1}{2\sqrt[4]{x^5}}$$

$$f'(4)=\frac{3}{2\sqrt{4}}+\frac{1}{2\sqrt[4]{4^5}}$$

$$=\frac{3}{4}+\frac{1}{8\sqrt{2}}$$

•¹ ✗
•² ✓1
•³ ✓1
•⁴ ✓1

Question	Generic Scheme	Illustrative Scheme	Max Mark
8.			
	•1 interpret information	•1 $x(x-2)<15$	4
	•2 express in standard quadratic form	•2 $x^2-2x-15<0$	
	•3 factorise	•3 $(x-5)(x+3)<0$	
	•4 state range	•4 $2<x<5$	

Notes:

Commonly Observed Responses:

Candidate A $x(x-2)=15$ •1 ✗ $x^2-2x-15=0$ •2 $\boxed{✓ 2}$ $x=-3,\ 5$ •3 $\boxed{✓ 1}$ •4 ^	**Candidate B - Mistaking perimeter for area** $4x-4<15$ $x<\frac{19}{4}$ **Award 1/4**
Candidate C $x^2-2x<15$ $x>2$ **Award 1/4**	**Candidate D** $x^2-2x<15$ $x>2$ $x<5$ **Inequalities not linked by 'and'** **Award 2/4**
Candidate E $x^2-2x<15$ $x>2$ **and** $x<5$ **Inequalities linked by 'and'** **Award 4/4**	

Question		Generic Scheme	Illustrative Scheme	Max Mark
9.				
		•[1] find gradient of AB •[2] calculate gradient of BC •[3] interpret results and state conclusion	•[1] $m_{AB} = -\sqrt{3}$ •[2] $m_{BC} = -\frac{1}{\sqrt{3}}$ •[3] $m_{AB} \neq m_{BC} \Rightarrow$ points are not collinear. **Method 2** •[1] $m_{AB} = -\sqrt{3}$ •[2] AB makes 120° with positive direction of the x-axis. •[3] $120 \neq 150$ so points are not collinear.	**3**

Notes:

1. The statement made at •[3] must be consistent with the gradients or angles found for •[1] and •[2].

Commonly Observed Responses:

Question		Generic Scheme	Illustrative Scheme	Max Mark
10(a).				
		•[1] state value of $\cos 2x$	•[1] $\frac{4}{5}$	**1**

Notes:

Commonly Observed Responses:

Candidate A		Candidate B	
$\cos 2x = \frac{3}{5}$	•[1] ✗	$\cos 2x = 4$	•[1] ✗
$2\cos^2 x - 1 = ...$	•[2] $\boxed{✓1}$	$2\cos^2 x - 1 = 4$	•[2] $\boxed{✓1}$
$\cos x = \frac{2}{\sqrt{5}}$	•[3] $\boxed{✓1}$	$\cos^2 x = \frac{5}{2}$	
		$\cos x = \sqrt{\frac{5}{2}}$	•[3] ✗ invalid answer

Question		Generic Scheme	Illustrative Scheme	Max Mark
10(b).				
		•[2] use double angle formula •[3] evaluate $\cos x$	•[2] $2\cos^2 x - 1 = ...$ •[3] $\frac{3}{\sqrt{10}}$	**2**

Notes:

1. Ignore the inclusion of $-\frac{3}{\sqrt{10}}$.
2. At •[2] the double angle formula must be equated to the candidates answer to part (a).

Commonly Observed Responses:

Question		Generic Scheme	Illustrative Scheme	Max Mark
11(a).				
		•[1] state coordinates of centre •[2] find gradient of radius •[3] state perpendicular gradient •[4] determine equation of tangent	•[1] $(-8,-2)$ •[2] $-\frac{1}{2}$ •[3] 2 •[4] $y=2x-1$	**4**

Notes:

1. •[4] is only available as a consequence of trying to find and use a perpendicular gradient.

2. At mark •[4] accept $y+5=2(x+2)$, $y-2x=-1$, $y-2x+1=0$ or any other rearrangement of the equation.

Commonly Observed Responses:

Question	Generic Scheme	Illustrative Scheme	Max Mark
11(b).			
	Method 1	**Method 1**	**6**
	•5 arrange equation of tangent in appropriate form and equate y_{tangent} to y_{parabola}	•5 $2x-1=-2x^2+px+1-p$	
	•6 rearrange and equate to 0	•6 $2x^2+(2-p)x+p-2=0$	
	•7 know to use discriminant and identify $a,\ b,$ and c	•7 $(2-p)^2-4\times2\times(p-2)$	
	•8 simplify and equate to 0	•8 $p^2-12p+20=0$	
	•9 start to solve	•9 $(p-10)(p-2)=0$	
	•10 state value of p	•10 $p=10$	
	Method 2	**Method 2**	
	•5 arrange equation of tangent in appropriate form and equate y_{tangent} to y_{parabola}	•5 $2x-1=-2x^2+px+1-p$	
	•6 find $\frac{dy}{dx}$ for parabola	•6 $\frac{dy}{dx}=-4x+p$	
	•7 equate to gradient of line and rearrange for p	•7 $2=-4x+p$ $p=2+4x$	
	•8 substitute and arrange in standard form	•8 $0=2x^2-4x$	
	•9 factorise and solve for x	•9 $0=2x(x-2)$ $x=0,\ x=4$	
	•10 state value of p	•10 $p=10$	

Notes:

1. At •6 accept $2x^2+2x-px+p-2=0$.
2. At •7 accept $a=2,\ b=(2-p),$ and $c=(p-2)$.

Commonly Observed Responses:

Just using the parabola

$a=-2 \quad b=p \quad c=1-p$	•5 ^
$b^2-4ac=p^2-4\times(-2)(1-p)$	•6 ^
$=p^2-8p+8=0$	•7 ✓1
$p=4\pm\sqrt{8}$	•8 ✓2
$p=4+\sqrt{8}$ as $p>3$	•9 ✓1
	•10 ✓1

Question		Generic Scheme	Illustrative Scheme	Max Mark
12.				
		•¹ interpret integral below $x - \text{axis}$ •² evaluate	•¹ -1 (accept area below $x - \text{axis}$ $= 1$) •² $-\dfrac{1}{2}$	**2**

Notes:

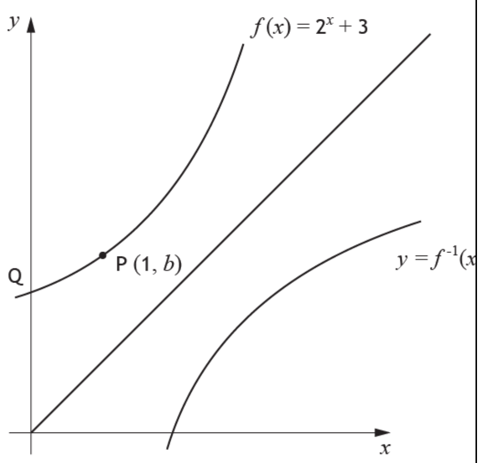
1. For candidates who calculate the area as $\dfrac{3}{2}$ award 1 out of 2.

Commonly Observed Responses:

Question		Generic Scheme	Illustrative Scheme	Max Mark
13(a)				
		•¹ calculate b	•¹ 5	**1**

Notes:

Commonly Observed Responses:

Question		Generic Scheme	Illustrative Scheme	Max Mark
13 (b)(i)				
		• ² reflecting in the line $y = x$	•² 	**1**

Notes:

1. If the reflected graph cuts the $y - \text{axis}$, •² is not awarded.

Commonly Observed Responses:

Question		Generic Scheme	Illustrative Scheme	Max Mark
13(b)(ii)				
		•[3] calculate y intercept •[4] state coordinates of image of Q •[5] state coordinates of image of P	•[3] 4 •[4] $(4,\ 0)$ see note 2 •[5] $(5,\ 1)$	3

Notes:

2. •[4] can only be awarded if $(4,0)$ is clearly identified either by their labelling or by their diagram.
3. •[3] is awarded for the appearance of 4, or $(4,0)$ or $(0,4)$.
4. •[5] is awarded for the appearance of $(5,1)$. Ignore any labelling attached to this point.

Commonly Observed Responses:

Candidate A		Candidate B	
$y = f(x)$ reflected in $x-$axis		$y = f(x)$ reflected in $y-$axis	
4	•[3] ✓	4	•[3] ✓
(0,-4)	•[4] ✓2	(0,4)	•[4] ✓2
(1,-5)	•[5] ✓1	(-1,5)	•[5] ✓2

Question		Generic Scheme	Illustrative Scheme	Max Mark
13(c)				
		•[6] state x coordinate of R •[7] state y coordinate of R	•[6] $x = 2$ •[7] $y = -7$	2

Notes:

Commonly Observed Responses:

Question		Generic Scheme	Illustrative Scheme		Max Mark
14.					
		•[1] identify length of radius •[2] determine value of k	$y-$axis tangent to circle •[1] $r = 6$ •[2] $k = 25$	Circle passes through origin $r = \sqrt{61}$ $k = 0$	2

Question		Generic Scheme	Illustrative Scheme	Max Mark
15.				
		•¹ know to integrate	•¹ $\int$	6
		•² integrate a term	•² $\frac{1}{50}t^2 \ldots$ or $\ldots - kt$	
		•³ complete integration	•³ $\ldots - kt$ or $\frac{1}{50}t^2 \ldots$	
		•⁴ find constant of integration	•⁴ $c = 100$	
		•⁵ find value of k	•⁵ $k = 2$	
		•⁶ state expression for T	•⁶ $T = \frac{1}{50}t^2 - 2t + 100$	

Notes:

1. Accept unsimplified expressions at •² and •³ stage.
2. •⁴, •⁵ and •⁶ are not available for candidates who have not considered the constant of integration.
3. •¹ may be implied by •².

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]