PRACTICE PROBLEMS

CENTER OF MASS

Center of Mass

1. The center of mass of an object or system is described using which SI unit?
   - A m
   - B kg
   - C cm
   - D g

2. When can we assume the center of mass of an object is located at its center? (Select all that apply)
   - A If the object has uniform density (mass per unit of volume)
   - B If the object is a sphere with uniform density
   - C If the object is balanced on a pivot point
   - D If the object is a rectangular block with uniform density

3. The center of mass of a system describes which of the following?
   - A The total mass of the system
   - B The average mass of each object in the system (the total mass divided by the number of objects)
   - C The mass-weighted average position of the objects in the system
   - D The point that is an equal distance from the center of mass of each object

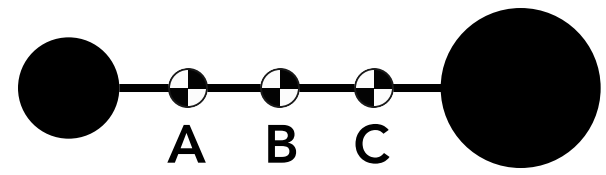
4. Which of the following is true about the center of mass of an object?
   - A The center of mass is always located on the physical object
   - B The center of mass is never located on the physical object
   - C The center of mass is always located on the physical obect if it's symmetrical with uniform density
   - D None of the above

5. Which of the following may affect the center of mass of a system of objects? (Select all that apply)
   - A The mass of each object
   - B The shape of each object
   - C The position of each object
   - D The orientation (rotational position) of each object

6. Which of the following is true about the center of mass of a system of two objects? (Select all that apply)
   - A It cannot be located at the center of mass of one object
   - B It is an equal distance from the closest edges of the two objects
   - C It is an equal distance from the center of masses of the two objects
   - D It must be located on an imaginary line connecting the center of masses of the two objects

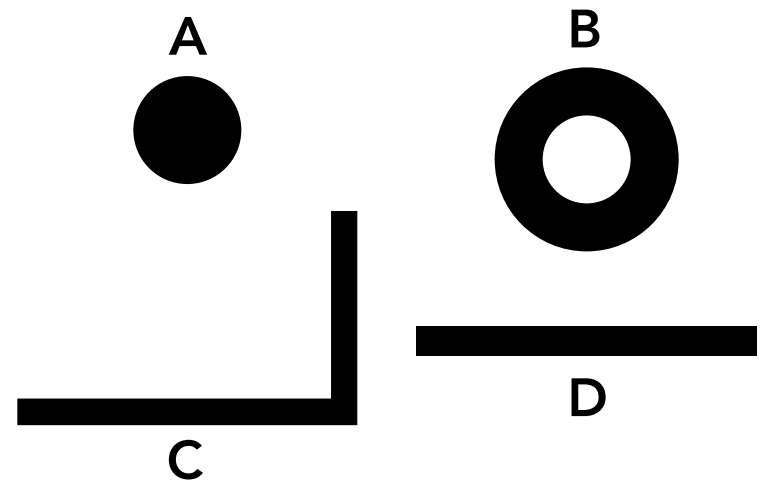
7. When calculating the center of mass of a system of objects, what do we use for the coordinates of each object?
   - A The center of each object
   - B The center of mass of each object
   - C The edge of the object closest to the center of the system
   - D The edge of the object farthest from the center of the system

8. Two spheres with the same uniform density are connected by a rod with some mass as shown on the right. Which of the points shown is most likely the center of mass of the system of the rod and two spheres?



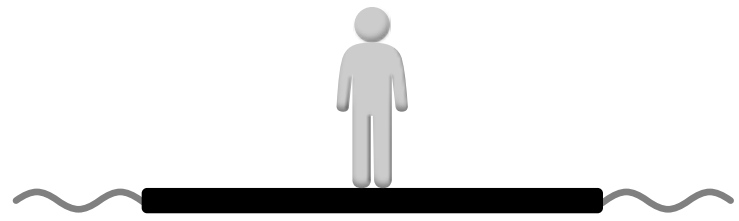
A Point A
B Point B
C Point C
D None of the above

9. Four solid objects with uniform density are shown on the right. Which of the objects has a center of mass that is not located on a physical part of the object? (Select all that apply)



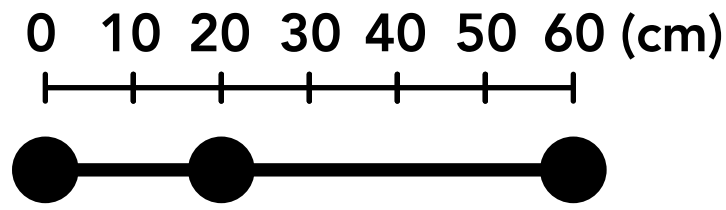
A Object A
B Object B
C Object C
D Object D

10. A person is standing in the middle of a large raft with some mass which is floating in the water. The person and the raft are initially at rest when the person walks to the right end of the raft. Assuming that the raft can slide through the water without friction and the center of mass of the person-raft system does not move, which of the following is true?



A The raft does not move
B The raft moves to the right
C The raft moves to the left
D It cannot be determined if the raft moves

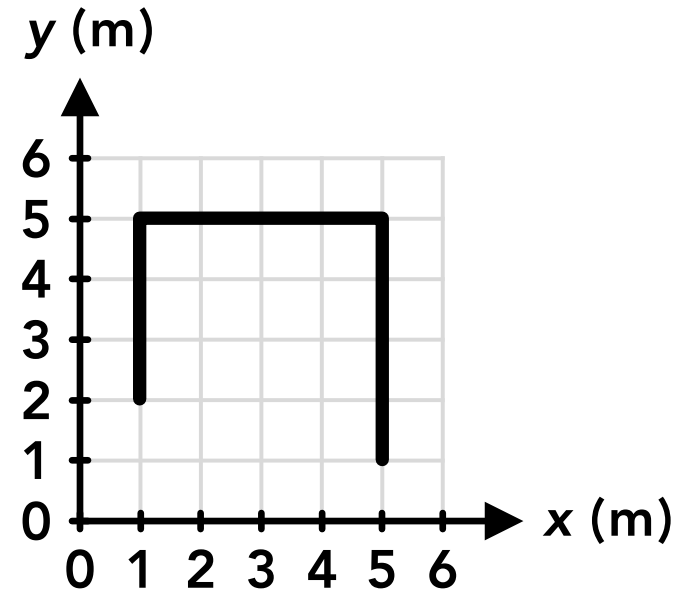
11. Three 3 kg spheres are attached to a 3 kg rod which is 60 cm long as shown on the right. What is the position of the system's center of mass, measured from the left end of the rod?



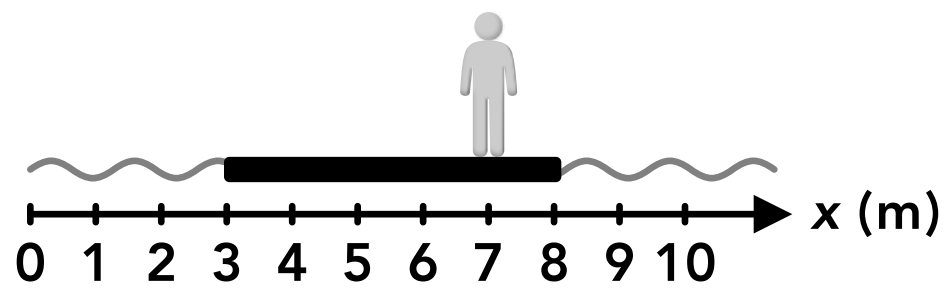
12. A system consists of three masses which are small enough to be treated as point masses. The position of each mass is shown on the right. What are the coordinates of the system's center of mass?

Mass:	Coordinates:
5 kg	(3, 4)
12 kg	(10, 6)
8 kg	(5, 1)

13. A metal wire is bent into the shape shown on the right. The wire has a uniform density. What are the coordinates of its center of mass?



14. An 80 kg person is standing on a 160 kg raft which is at rest as shown on the right. The person then walks to the left end of the raft. The raft can slide across the water without friction. After the person reaches the left end of the raft, what is the position of the left end of the raft on the axis shown?



Answers

1. A
2. B, D
3. C
4. D
5. A, B, C, D
6. A, D
7. B
8. C
9. B, C
10. C
11. 27.5 cm
12. (7, 4)
13. (3.18, 3.86)
14. 4.33 m

Answers - Center of Mass

1. **Answer: A**
   The center of mass of an object or system is a point or location in space, which would be described using the SI unit of meters (m). The mass of the object(s) are used to calculate the center of mass, but the center of mass is only a position and does not describe the amount of mass in an object or system (unlike rotational inertia, which does describe the amount of mass in an object or system as well as how that mass is distributed).

2. **Answer: B, D**
   If the object is symmetrical in shape (like a sphere or a rectangular block) the center of mass is located at the object's center (the middle of its width, length and height) if it has a uniform density, which means the mass is the same per unit of volume throughout the object. An object with a uniform density that is not symmetrical will not have its center of mass at its dimensional center if it's not symmetrical.

3. **Answer: C**
   The center of mass of an object or system describes the mass-weighted average position of the particles in the object or the objects in the system. It does not describe the total or average mass of the system, and it is not an equal distance from the center of mass of each object (because the objects can have different masses).

4. **Answer: D**
   The center of mass of an object can be located on the physical object or in the space surrounding the object, which depends on its shape. Even if the object is symmetrical with uniform density, the center of mass can still be located in empty space (for example, if the object is a hollow sphere or a ring).

5. **Answer: A, B, C, D**
   The center of mass of a system of objects is calculated based on the mass of each object and the center of mass of each object. The center of mass of each object may depend on its shape, position and orientation (if it's not symmetrical with a uniform density).

6. **Answer: A, D**
   If a system consists of two masses, the center of mass of the system cannot be located at the center of mass of one of the objects because the second object will move or "pull" the center of mass towards it (the second object has mass, so its position will be weighted by some amount). The center of mass of the system must be located on an imaginary line connecting the center of masses of each object, which may be easier to think about if we place the objects on a single axis (like the x axis). The center of mass of the system may not be an equal distance from the edges or the center of masses of each object because the objects may have different masses.

7. **Answer: B**
   When calculating the center of mass of a system, we use the coordinates of the center of mass of each object.

8. **Answer: C**
   The spheres have the same, uniform density which means the larger sphere on the right has more mass. The center of mass of the system is closer to the larger mass than the center of the rod.

9. **Answer: B, C**
   Objects A and D are symmetrical and their center of masses are located at the center of each object which is on a physical part of the object. Object B is a symmetrical ring whose center of mass is located at its center which is not on a physical part of the object. Object C is not symmetrical and its center of mass is located somewhere in the empty space between the centers of each of the two parts.

10. **Answer: C**
    Since there is no friction force between the water and the raft, there is no external horizontal force acting on the person-raft system so the center of mass of the system does not accelerate (Newton's 1st law of motion). Since the system is initially at rest the center of mass of the system does not move. If the person's mass moves to the right, the raft must move to the left.

11. **Answer: 27.5 cm**
    We can calculate the center of mass of the system using the mass of each object and the position of each object's center of mass. The center of mass of the rod is in the middle of the rod, 30 cm from the left end (we assume the rod is uniform).

$$x_{COM} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_{rod}x_{rod}}{m_1 + m_2 + m_3 + m_{rod}} = \frac{(3\text{ kg})(0\text{ cm}) + (3\text{ kg})(20\text{ cm}) + (3\text{ kg})(60\text{ cm}) + (3\text{ kg})(30\text{ cm})}{(3\text{ kg}) + (3\text{ kg}) + (3\text{ kg}) + (3\text{ kg})}$$

$$x_{COM} = 27.5\text{ cm}$$

12. **Answer: (7, 4)**
    We can calculate the x and y coordinates of the system's center of mass separately using the masses and coordinates of each mass.

$$x_{COM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{(5\text{ kg})(3) + (12\text{ kg})(10) + (8\text{ kg})(5)}{(5\text{ kg}) + (12\text{ kg}) + (8\text{ kg})} = 7$$

$$y_{COM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{(5\text{ kg})(4) + (12\text{ kg})(6) + (8\text{ kg})(1)}{(5\text{ kg}) + (12\text{ kg}) + (8\text{ kg})} = 4$$

$$(x_{COM}, y_{COM}) = (7, 4)$$

13. **Answer: (3.18, 3.86)**
    We can treat the wire as 3 separate sections which each have their own mass and center of mass. We don't know the mass of the wire, but the wire has a uniform density and we can use a variable (M) to represent the mass of 1 m of length of wire. We'll find that the variable for mass cancels out and does not actually affect the location of the center of mass because the wire has a uniform density.

Section 1 (left): $m = 3M$ center of mass $= (1, 3.5)$
Section 2 (middle): $m = 4M$ center of mass $= (3, 5)$
Section 3 (right): $m = 4M$ center of mass $= (5, 3)$

$$x_{COM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{(3M)(1) + (4M)(3) + (4M)(5)}{(3M) + (4M) + (4M)} = 3.18$$

$$y_{COM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{(3M)(3.5) + (4M)(5) + (4M)(3)}{(3M) + (4M) + (4M)} = 3.86$$

14. **Answer: 4.33 m**

Since there is no friction acting on the raft, there is no external horizontal force acting on the person-raft system so the center of mass of the system does not move as the person walks across the raft (the system's center of mass does not accelerate, which is Newton's 1st law of motion). One way to solve this is to start by finding the initial center of mass of the system relative to the origin. Then we can find the new center of mass of the system relative to the left end of the raft, then find the distance between the left end of the raft and the center of mass relative to the origin (which doesn't change).

COM relative to the origin (initial and final): $x_{COM} = \dfrac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \dfrac{(80\text{ kg})(7\text{ m}) + (160\text{ kg})(5.5\text{ m})}{(80\text{ kg}) + (160\text{ kg})} = 6\text{ m}$

COM relative to the left end of the raft (final): $x_{COM} = \dfrac{(80\text{ kg})(0\text{ m}) + (160\text{ kg})(2.5\text{ m})}{(80\text{ kg}) + (160\text{ kg})} = 1.67\text{ m}$

After the person moves, the left end of the raft is 1.67 m to the left of the system's center of mass, which is still at a position of 6 m relative to the origin, so the left end of the raft is at a position of 4.33 m from the origin.