3

Determining Height of *KM*

In actual practice the ship's officer is not called upon to calculate the height of the *KM* (transverse metacenter) above the keel. Accurate calculations of *KM* cannot be made except by the naval architect after much involved work. Nevertheless, a purpose may be served by presenting approximate methods for the determination of *KM*.

You must remember that metacentric height, *GM*, is not only determined by the position of *G*, but also by the position of *M*. A knowledge of how and why the value of *KM* changes with draft and the form of the vessel is indispensable for understanding and using transverse stability properly. Fortunately, today, vessels are required to be supplied with a stability booklet approved by the United States Coast Guard. By entering a curve on a graph or a table with the ship's displacement from the stability booklet, the *KM* for the vessel is readily available to the ship's officer.

What is *KM*?

The distance that the transverse metacenter, *M*, measured in feet, is above the keel, *K*, is designated *KM*. *KM* is the sum of *KB*, the height of the center of buoyancy above the keel, and *BM*, the metacentric radius, or the distance from *B* to *M*. Any change in the value of *KB* or *BM* changes the value of *KM*.

Calculating *KB*

The position of *B*, the center of buoyancy, naturally depends upon the immersed shape of the hull. If the hull has a rectangular shape like a barge, *B* will be at half the draft. The greater the flare of the ship's sides or the dead rise of the ship's bottom, the higher *B* will be located above the half-draft point. For the usual merchant ship form, the height of *B* above the keel is very close to 0.53 times the draft; in other words, slightly above the half-draft point.

Another widely used approximation of *KB* is Morrish's formula which states:

$$KB = 1/3\,(5/2\,D - V/A)$$

where: D = draft in feet
V = volume of displacment in cubic feet
A = area of water plane in square feet

The ratio of volume of displacement to a block having length, breadth, and draft of the vessel is known as a vessel's block coefficient. For merchant form hulls with block coefficients which range between 0.68 and .75, the results of the approximation of "0.53 × draft" and Morrish's approximation are for all practical purposes valid. Figure 17 shows that finer ends do not always mean an increase in the value of *KB*. Flare and dead rise of the hull determine this value and not the block coefficient.

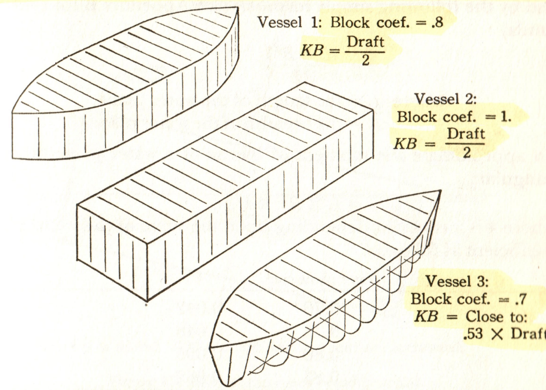


Figure 17. Reducing the block coefficient does not necessarily mean increasing the value of *KB*.

Calculating *BM*

BM is the distance from the center of buoyancy to the transverse metacenter. It is called metacentric radius. It is so called because it is the radius of a circle which has *M* at its center and a small arc of the circle formed by the movement of *B* as the vessel inclines through small angles. See Figure 11 in Chapter 1.

The formula commonly used for calculating *BM* is:

$$BM = I / V$$

where: I = the moment of inertia (about a longitudinal axis of the waterplane, in units of (feet)^4.
V = the volume of displacement, in units of (feet)^3.

Moment of inertia is a difficult term to define simply. Some texts state that the moment of inertia of a water plane is a measure of a vessel's resistance to rolling motion about the vessel's longitudinal axis. For example, the moment of inertia of a log would be very small, because there is little resistance to rolling motion about its longitudinal axis. A broad-beamed barge, on the other hand, would offer a great deal of resistance to rolling motion about its longitudinal axis. It would have a large moment of inertia. This moment which resists motion actually is made up of an infinite number of moments, each of which are composed of the product of the elementary area and the square of the distance from the axis.

The moment of inertia of a rectangular water plane area can be easily found by the following simple formula: (See Appendix B for evolving of formula)

$$I = \frac{(L \times B^3)}{12}$$

where: L = the length of the water plane.
B = the breadth of the water plane.

To approximate the moment of inertia of a water plane other than rectangular:

$$I = L \times B^3 \times k$$

where k is a constant depending upon the value of the water plane coefficient as follows:

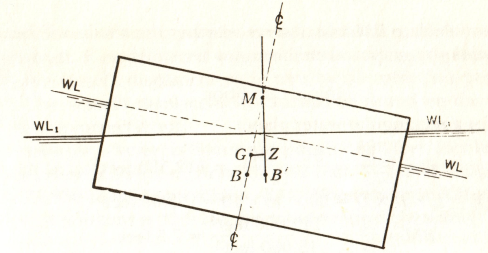
Water plane coef.	k
0.70	0.042
0.75	0.048
0.80	0.055
0.85	0.062

Note: Water plane coefficient is the ratio of the area of the vessel's water plane to the product of the length and breadth of the vessel.

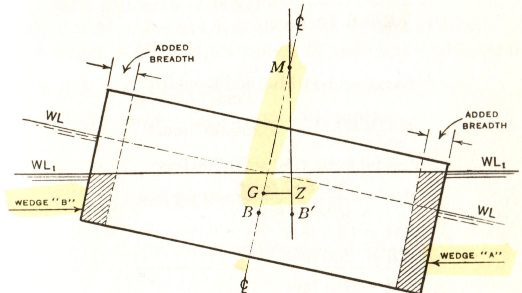
By observing both of the above formulas, it is evident at once that I is almost wholly dependent on the breadth of the vessel. Any small increase in beam will increase I tremendously, thus increasing the value of BM. Keep this fact firmly in mind. *One of the most important factors of initial stability is the breadth of the vessel.* Knowing the importance of breadth, we can see the reason for outriggers, sponsons, etc.

Let us consider the effect of additional breadth on the barge in Figure 18. The dimensions of the barge are:

	Before	After
Length	50 feet	50 feet
Breadth	30 feet	40 feet
Draft	10 feet	10 feet



Original barge



Breadth is added: *GM* and righting moment are increased.

Figure 18. Effect of increased breadth on *KM*.

The barge in the initial condition is inclined to the new waterline, WL_1, causing a shift of B to the low side, thus producing a righting moment and BM as shown. The barge after additional breadth is inclined to the same angle, producing a BM and righting moment approximately twice the original BM and righting moment. This is due to the increased buoyancy on the submerged side. The increase is equal to a wedge A minus wedge B. The center of buoyancy naturally moves over farther because of this new buoyancy, thus creating a greater distance between the lines of force through G and B, in other words, a greater righting moment.

(The function of V in $BM = I / V$ can be visualized by grasping that with a larger V the movement of B will be smaller, thus reducing BM. Or, a decrease in volume of displacement produces greater movement of B and consequent increase in BM.)

Now let us calculate BM and observe whether the results will bear out our observations by graphical method.

Initial Condition

(Using I for a rectangular water plane)

$$I = \frac{LB^3}{12} = \frac{50 \times (30)^3}{12} = 112{,}500 \text{ feet}^4$$

$$V = L \times B \times D = 50 \times 30 \times 10 = 15{,}000 \text{ feet}^3$$

$$BM = \frac{I}{V} = \frac{112{,}000 \text{ feet}^4}{15{,}000 \text{ feet}^3} = 7.5 \text{ feet}$$

$$\begin{aligned} BM &= 7.5 \text{ feet} \\ +KB &= 5.0 \text{ feet} \\ \hline KM &= 12.5 \text{ feet} \end{aligned}$$

Barge with Additional Breadth

$$I = \frac{50 \times (40)^3}{12} = 266{,}667 \text{ feet}^4$$

$$V = 50 \times 40 \times 10 = 20{,}000 \text{ feet}^3$$

$$BM = \frac{266{,}667 \text{ feet}^4}{20{,}000 \text{ feet}^3} = 13.3 \text{ feet}$$

$$\begin{aligned} BM &= 13.3 \text{ feet} \\ +KB &= 5.0 \text{ feet} \\ \hline KM &= 18.3 \text{ feet} \end{aligned}$$

By the above calculations we have proved that, with the addition of 10 feet (one-third the breadth of the barge) we have almost doubled the original BM. The metacenter has been raised 5.8 feet, thus increasing the initial stability of the barge tremendously.

Approximating *BM* for Curved Water Planes

In using the $BM = I / V$ formula for a curved water plane, the I is found by multiplying length by breadth cubed by the constant k. k depends upon the value of the water plane coefficient p which, if not available directly from the ship's stability booklet, can be calculated by the following formula:

$$p = \frac{\text{Area of water plane}}{L \times B}$$

The area of the water plane can be calculated by multiplying TPI (tons per inch) by 420. TPI can be obtained from the deadweight scale on the scale on the vessel's capacity plan or located in the stability booklet for the correct draft. V or displaced volume of the vessel can be found by multiplying the vessel's displacement in tons by 35 cubic feet per ton of salt water. The vessel's displacement can also be found on the deadweight scale for the correct draft.

For example, a vessel is floating at a draft of 28 feet. Her length on the load waterline is 444 feet. Breadth is 62 feet. TPI is 51. Displacement in tons is 14,850 tons. Approximate KM. (Note: the true KM at a displacement of 14,850 tons is 25.9 feet from the deadweight scale.)

$$Awp = \text{TPI} \times 420 = 51 \times 420 = 21{,}420 \text{ feet}^2$$

$$p = Awp \,/\, L \times B = 21{,}420 \text{ feet}^2 \,/\, 27{,}528 \text{ feet}^2 = 0.78 \text{ (Using } p \text{ of } 0.78 \text{ pick off } k = 0.052)^*$$

$$I = L \times B^3 \times k = 444 \times 62^3 \times 0.052 = 5{,}502{,}517 \text{ feet}^4$$

$$V = \text{Displ.} \times 35 = 14{,}850 \text{ tons} \times 35 \text{ feet}^3/\text{ton} = 519{,}750 \text{ feet}^3$$

$$BM = I \,/\, V = \frac{5{,}502{,}517 \text{ feet}^4}{519{,}750 \text{ feet}^3} = 10.6 \text{ feet}$$

$$KB = 0.53 \times \text{Draft} = 0.53 \times 28 \text{ feet} = 14.8 \text{ feet}$$

$$\begin{aligned} BM &= 10.6 \text{ feet} \\ +KB &= 14.8 \text{ feet} \\ \hline \text{Approximate } KM &= 25.4 \text{ feet} \\ \text{True } KM &= 25.9 \text{ feet} \\ \hline \text{Approximation Error} &= 0.5 \text{ foot} \end{aligned}$$

Analysis of Vertical Movement of *KM*

A vessel in a light displacement condition is stiff. This means a great value of GM, transverse metacentric height. Is this value due to a high position of M or to a low position of G? The common belief is the latter. Actually, it is due to the former. Since the height of the transverse metacenter above the keel, KM, is determined by the values of KB and BM, it is necessary to analyze the changes in these distances caused by the change of draft. In a merchant form vessel, B rises approximately half the draft change. To illustrate:

For a typical merchant form ship KB values are as follows:

Light Condition

Draft	*KB*	Rise of *B*
12 feet	6.35 feet	
13 feet	6.89 feet	0.54 foot
14 feet	7.41 feet	0.52 foot

*See table for k on page 32.

Deep Condition

Draft	*KB*	Rise of B
25 feet	13.25 feet	
26 feet	13.78 feet	0.53 foot
27 feet	14.31 feet	0.53 foot

For both light drafts and deep drafts, the rise of *B* for a one foot increase of draft is a little over one-half foot. A constant *rate of change* is maintained (0.53 foot for each foot of increase).

It is obvious, also, that at light drafts the value of *KB* is small. The great value of *KM* (30.8 feet at 12 feet draft for a typical merchant form vessel) therefore must be due to a great value of *BM* rather than *KB*.

In order to explain this great value of *BM* at light drafts it is necessary to use the *BM* formula, $BM = I / V$. For the sake of argument we will assume our beam to draft ratio for our typical merchant ship is six. At light displacement drafts, the volume of displacement is naturally small, whereas the value of *I*, which depends almost solely on the breadth of the vessel, is only slightly less than it will be at deep drafts. This is due to the fact that a merchant vessel's breadth is the same at light drafts as at full load condition which is because of its wall-sided mid-body section. This means that the fraction *I* / *V* is changing rapidly in value. Also, the rate of change is changing rapidly with increase of draft. To illustrate:

Light Condition

Draft	*I / V (BM)*	Decrease of *BM*
12 feet	$\frac{4,859,260}{199,150} = 24.4$ feet	
13 feet	$\frac{4,915,260}{217,875} = 22.56$ feet	1.84 feet
14 feet	$\frac{4,987,006}{237,025} = 21.04$ feet	1.52 feet

Deep Condition

Draft	*I / V (BM)*	Decrease of *BM*
25 feet	$\frac{5,606,470}{456,925} = 12.27$ feet	
26 feet	$\frac{5,669,892}{477,750} = 11.87$ feet	0.40 foot
27 feet	$\frac{5,729,637}{498,750} = 11.49$ feet	0.38 foot

It can be seen from the above table that *BM* has great value at light drafts due to small value of *V*. Also, as *V* increases rapidly with draft, *I* is increasing very slowly. Therefore, the fraction *I* / *V* decreases in value

very rapidly at first, with the rate of decrease becoming less as the denominator *V* bears a greater and greater proportion to the numerator *I*. This results, as shown above, in a decrease of *BM* of 1.84 feet as the draft increases from 12 feet to 13 feet while the decrease of *BM* is only 0.38 foot in increasing draft from 26 to 27 feet.

It is now well to observe the effect that this constant rate of increase of *KB*, and the changing rate of decrease of *BM* with increase of draft, have upon the value of *KM* as the draft changes for the merchant hull form vessel.

Draft	*KB*	Increase in *KB*	*BM*	Decrease of *BM*	Sum	*KM*
12 feet	6.35		24.4			30.75
13 feet	6.89	.54	22.56	1.84	−1.3	29.45
14 feet	7.41	.52	21.04	1.52	−1.0	28.45
25 feet	13.25		12.27			25.52
26 feet	13.78	.53	11.87	0.40	+.13	25.65
27 feet	14.31	.53	11.49	0.38	+.15	25.80

The following observations can be made:

1. At low drafts the increase of *KB* is much smaller than the decrease of *BM*; therefore, the value of *KM* decreases rapidly with increase of draft.
2. At deep drafts the increase of *KB* finally overcomes the smaller decrease of *BM*, thus causing the value of *KM* to rise slowly.
3. The relationship of *KB* and *BM* to the value of *KM*, as shown by the table, can be graphically illustrated by a typical curve of metacenter as found on the hydrostatic curves of a vessel. See Figure 19.

By inspection of the curves in Figure 19 we note:

1. The curve of transverse *KB* is nearly a straight line. Graphically this means that the increase of *KB* bears a ratio which is almost direct with an increase in draft.
2. The curve of transverse *KM* slopes slowly to the left, then increases its slope, and finally slopes slowly to the right.

This rapid decrease in the value of *KM* at low drafts is of course due to the rapid decrease of *BM*. At the point where the *KM* curve starts to slope to the right, the increase of *KB* is finally greater than the decrease of *BM*. Study Figure 19. As an interesting exercise you should be able to find *BM* by subtracting *KB* from transverse *KM*, and construct a curve of *BM* directly on Figure 19.

Effect of Vertical Movement of *M*, on Beam to Draft Ratio

The foregoing discussion of the vertical movement of *M* for a typical merchant form hull is applicable only to a vessel of moderate beam to

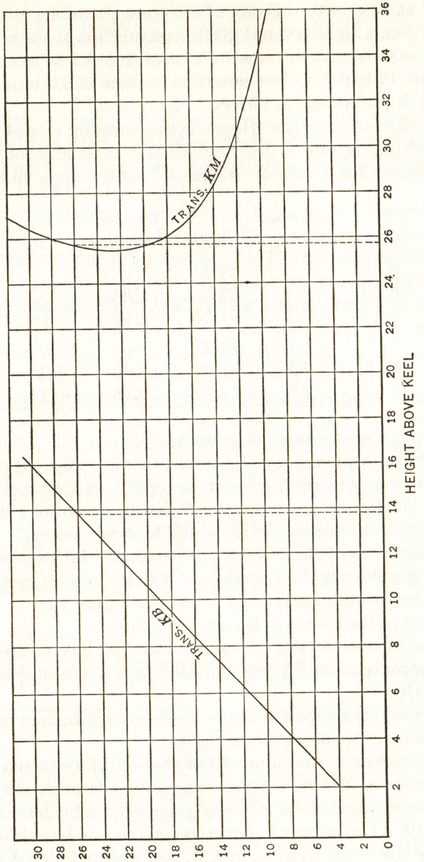


Figure 19. Transverse *KB* and *KM* curves for a typical ship of the merchant hull form.

draft ratio (a beam to draft ratio of 6 can be considered moderate). Beamier and narrower vessels would have larger and smaller ratios, respectively. Naturally this change of beam affects the moment of inertia

for these vessels. Also, the volume of displacement will bear a different ratio to *I*. *I*/*V* will therefore have considerably different values.

A broad-beamed vessel will have a tremendous metacentric radius at low drafts, which value will drop sharply with increase of draft due to the great increase of *V*. *B* meanwhile is not rising enough to overcome the decrease of *BM*. A very low or negative metacentric height, *GM*, at load drafts may be the result.

On the other hand, a deep, narrow vessel may have a negative metacentric height at low drafts due to her small moment of inertia. With increase of draft, however, the metacenter will rise rather than fall, as the increase of *KB* is greater. At load drafts, the metacentric height is adequate or, if ballast has been added low to give positive stability for the light draft, it may be excessive.

The effect of the beam to draft ratio must be kept in mind, therefore, in any analysis of the vertical movement of *M*. By studying Figure 20 you can readily see how a change in the beam to draft ratio will alter the location of *M*.

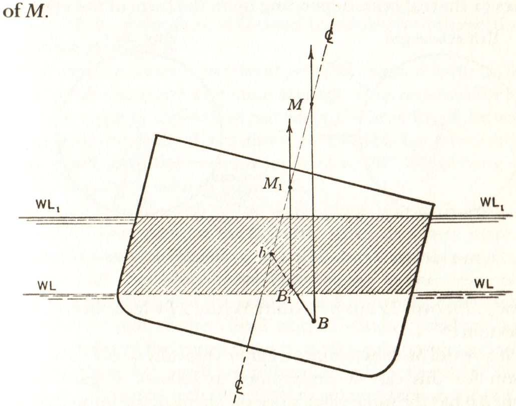


Figure 20. Graphic illustration of the drop in *M* due to an increase of displacement, thus creating a change in the beam to draft ratio. The condition is assumed for an infinitely small angle of inclination. Remember *B* always acts up through *M*.

Summation of *KM* Vertical Movement

The well-trained officer knows that, with every change of draft, the position of *M* changes. For the usual merchant form vessel, *M* drops rapidly with increase of draft, gradually reducing the speed of its drop, until drafts of nearly loaded condition are reached, thereafter slowly rising to load draft and beyond. This movement of *M* is directly due to the changes in value of *KB* and *BM*. You should inspect the Hydrostatic Properties, sheet 3 of the trim and stability booklet (See Appendix D) to note the change in *KM*.

Movement of *M* With Transverse Inclination

In Chapter 1 we explained that *M* remains on the centerline of the vessel only for small angles of inclination and therefore can be used conveniently only in conjunction with initial stability. Now we propose to explain the effect of inclination on this movement of *M*.

The term meta was selected as a prefix for center because its Greek meaning implies movement. The metacenter therefore is a moving center. Theoretically, *M* starts to move off the centerline as soon as a vessel inclines but, practically, the movement is negligible for inclinations up to 10 degrees or thereabouts, depending upon the form of the vessel.

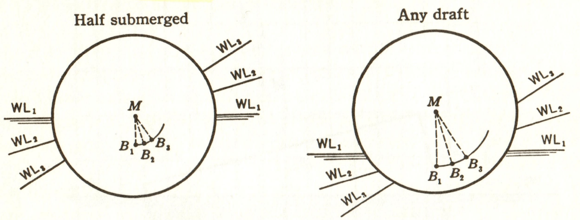


Figure 21. Position of *M* for a vessel of circular cross section. When is a metacenter not a metacenter?

Why does *M* move? Let us first study Figure 21 which shows a vessel of circular section.

For such a vessel *M* does not move either vertically or off the centerline. The reason for this can be determined as follows: Regardless of the inclination, *KB* has the same value since the shape of the immersed section remains the same. *BM* also retains the same value. This can be proved analytically by showing that I/V does not change. *I*, which is equal to $L \times B^3 \times k$, cannot change since the length, breadth, and water plane coefficient remain the same. *V*, the volume of displacement, is unchanged since inclination cannot change this volume.

If *KB* and *BM* retain their values at all inclinations, it is undeniable that *KM* retains its value. This is true for the vessel at any draft.

Moreover, *M* will be at the center of the circle since, for any angle of inclination, the vertical lines through *B* all pass through the center of the

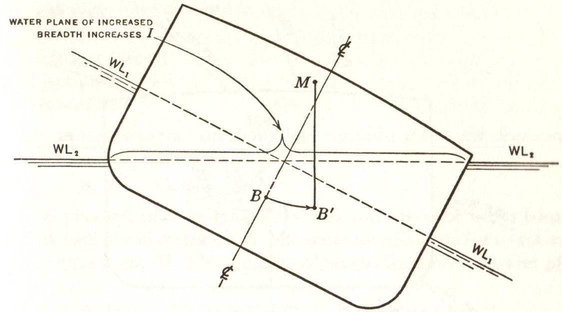


Figure 22. The metacenter moves off the centerline since inclination changes breadth.

circle, and *M*, by definition, is formed by successive intersections of the lines of force through *B*.

In what respects does a merchant vessel of normal form differ from a vessel of circular section? The same breadth is not constant for all inclinations. As the angle of inclination increases, the breadth of the water plane increases until the deck edge is about, or a little below, the waterline and then decreases until the vessel is inclined to 90°. This change of water plane breadth can be seen in Figure 22.

As we know, the breadth of the water plane is directly associated with the value of *I*. Therefore, as the breadth increases, *I* must increase. Since *V* remains the same, the fraction I/V will increase in value until deck edge immersion, and then decreases. This change in the value of *BM*, associated with the change in the position of *B* due to the change in shape of the immersed form of the vessel will, of course, produce a changing position of *M*. Since the vessel is not one of circular cross section, it moves off the centerline. For small inclination, a merchant vessel of normal hull form can be compared with this vessel of circular section, since *B* moves in the arc of a circle, and breadth does not change considerably. In other words *KB* and *BM* do not change markedly, therefore *KM* does not change markedly for small angles of inclination.

How does *M* move for large angles of inclination? Figure 23 shows a typical locus of metacenters and centers of buoyancy for an average form or typical merchant vessel.

In actual calculations of stability for large angles of inclination, the metacenter, therefore, is not used, and a resort to the use of righting arms is made. Stability at large angles of heel will be discussed in Chapter 6.

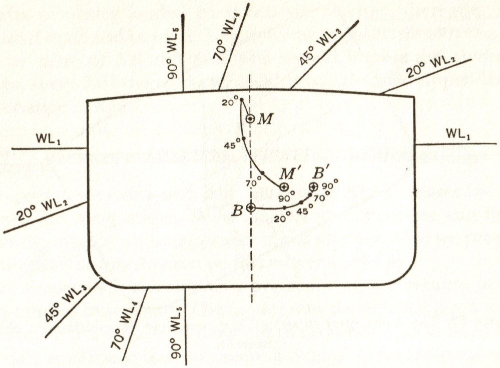


Figure 23. Typical locus of metacenters and centers of buoyancy for an average form merchant vessel.

Questions

1. The distance that the transverse metacenter is above the keel is designated as:

A. *KG* B. *KB* C. *KM* D. *KG* + *BM*

2. *KM* can be obtained by the ship's officer from: **I.** The ship's stability booklet. **II.** The ship's capacity plan.

A. I B. II C. Either I or II D. Neither I nor II

3. The value of *KM* is calculated by: **I.** Adding *BM* to *KB*. **II.** Subtracting *BM* from *KB*.

A. I B. II C. Either I or II D. Neither I nor II

4. In general, on what does the position of *B* depend?

A. Length B. Depth C. Freeboard D. Draft

5. If you were looking for the *KB* curve on a typical ship's hydrostatic curves plan, you would expect the shape of the *KB* curve to be: **I.** A straight line for all practical purposes. **II.** A gently curving line.

A. I B. II C. Either I or II D. Neither I nor II

6. An increase in flare will produce an increase in *KB*.

A. True B. False

7. A decrease in block coefficient necessarily means an increase in flare.

A. True B. False

8. To calculate *BM* you would:

A. Add *V* to *I*
B. Subtract *I from V*
C. Multiply *V* by *I*
D. Divide I by *V*

9. The resistance to motion about the longitudinal axis of a ship's water plane is:

A. *I* B. *V* C. *B* D. *M*

10. The product of a length of a vessel and the cube of its breadth divided by twelve will result in: **I.** The moment of inertia for a rectangular water plane area. **II.** The moment of inertia for a curved water plane area.

A. I B. II C. Either I or II D. Neither I nor II

11. Which of the following is the most significant dimension of the ship when calculating *KM*?

A. Length B. Breadth C. Draft D. Freeboard

12. By multiplying displacement in tons by 35 you will be able to calculate:

A. Water plane coefficient
B. Tons per inch immersion
C. Volume of displacement
D. None of the above

13. By multiplying the area of the water plane by 420 you will be able to calculate:

A. Water plane coefficient
B. Tons per inch immersion
C. Volume of displacement
D. None of the above

14. By dividing the area of the water plane by the product of length and breadth you will be able to calculate:

A. Water plane coefficient
B. Tons per inch immersion
C. Volume of displacement
D. None of the above

15. As a ship's displacement changes such that its beam to draft ratio increases *KM* will: **I.** Increase. **II.** Decrease.

A. I B. II C. Either I or II D. Neither I nor II

As draft increases in a vessel, the values of *KB* and *BM* are changing. In questions 16 and 17 below explain how combination of *KB* and *BM* affects *KM* as: