CENTER OF MASS

Center of Mass

1. The center of mass of an object or system is described using which SI unit?
   - A m
   - B kg
   - C cm
   - D g

2. When can we assume the center of mass of an object is located at its center? (Select all that apply)
   - A If the object has uniform density (mass per unit of volume)
   - B If the object is a sphere with uniform density
   - C If the object is balanced on a pivot point
   - D If the object is a rectangular block with uniform density

3. The center of mass of a system describes which of the following?
   - A The total mass of the system
   - B The average mass of each object in the system (the total mass divided by the number of objects)
   - C The mass-weighted average position of the objects in the system
   - D The point that is an equal distance from the center of mass of each object

4. Which of the following is true about the center of mass of an object?
   - A The center of mass is always located on the physical object
   - B The center of mass is never located on the physical object
   - C The center of mass is always located on the physical obect if it's symmetrical with uniform density
   - D None of the above

5. Which of the following may affect the center of mass of a system of objects? (Select all that apply)
   - A The mass of each object
   - B The shape of each object
   - C The position of each object
   - D The orientation (rotational position) of each object

6. Which of the following is true about the center of mass of a system of two objects? (Select all that apply)
   - A It cannot be located at the center of mass of one object
   - B It is an equal distance from the closest edges of the two objects
   - C It is an equal distance from the center of masses of the two objects
   - D It must be located on an imaginary line connecting the center of masses of the two objects

7. In which of the following scenarios would an object have a center of gravity? (Select all that apply)
   - A An object is on the moon where gravity is weaker than it is on earth
   - B An object is falling near the surface of the earth
   - C An object hanging from a string by one end
   - D An object sitting at rest on the ground

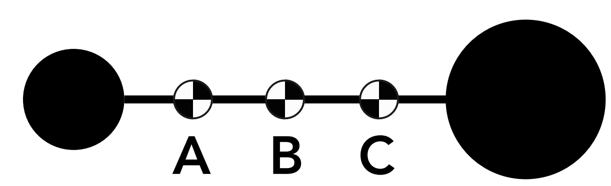
8. We always treat an object's weight force as acting on which point? (Select all that apply)
   - A The object's center of mass
   - B The object's center of gravity
   - C The object's center of mass if it's within a uniform gravitational field
   - D The object's center (the middle of its width, length and height)

9. Two spheres of different masses are attached to the ends of a rod. A string is attached to the rod at the center of mass of the system of the rod and two spheres, and the rod is suspended below the string. When the rod is released, which direction will the rod rotate?
   - A The rod will rotate so that the sphere with more mass is lower
   - B The rod will rotate so that the sphere with less mass is lower
   - C The rod will not rotate
   - D Cannot be determined

10. When calculating the center of mass of a system of objects, what do we use for the coordinates of each object?
    - A The center of each object
    - B The center of mass of each object
    - C The edge of the object closest to the center of the system
    - D The edge of the object farthest from the center of the system

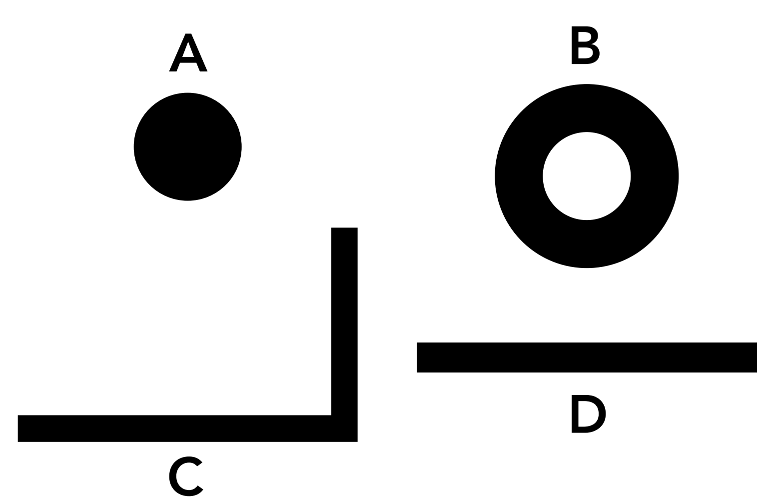
11. Which of the following explains why an object or system is balanced when pivoting above or below its center of gravity? (Select all that apply)
    - A The center of gravity is located at the same position as the center of mass
    - B There is no weight force acting on the object or system when pivoting above or below its center of gravity
    - C The weight forces acting on each part of the object or system can be treated as a single weight force which acts at the center of gravity, which does not produce a torque because it acts in line with the pivot point
    - D The individual torques produced by the weight forces on each part of the object or system add up to zero

12. Two spheres with the same uniform density are connected by a rod with some mass as shown on the right. Which of the points shown is most likely the center of mass of the system of the rod and two spheres?



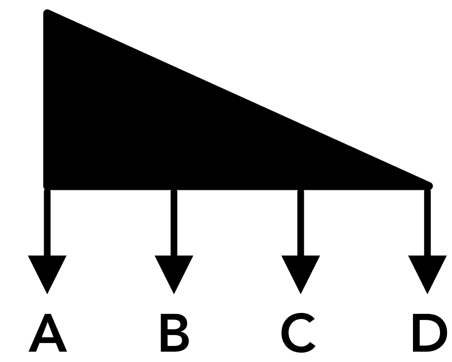
    - A Point A
    - B Point B
    - C Point C
    - D None of the above

13. Four solid objects with uniform density are shown on the right. Which of the objects has a center of mass that is not located on a physical part of the object? (Select all that apply)



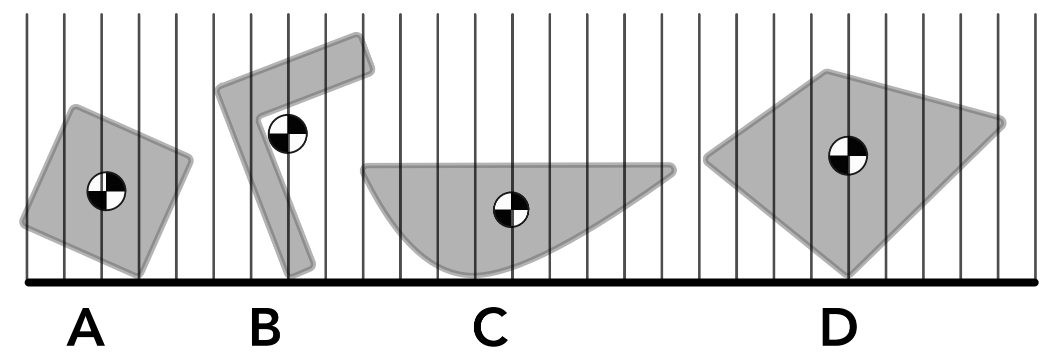
    - A Object A
    - B Object B
    - C Object C
    - D Object D

14. A solid triangular object is shown on the right. Which of the vectors shown is most likely in line with the object's weight force?



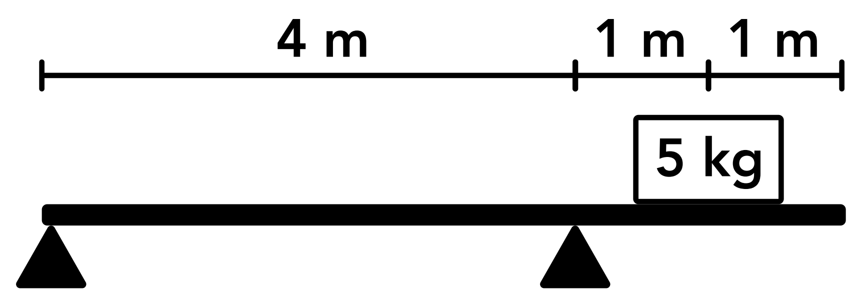
- A Vector A
- B Vector B
- C Vector C
- D Vector D

15. Four solid objects are in contact with the ground as shown on the right. The center of mass of each object is shown. Which of the objects are balanced? (Select all that apply)



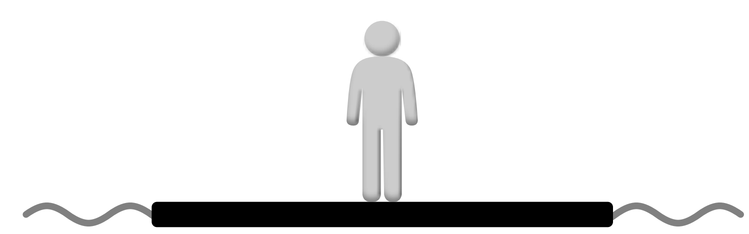
- A Object A
- B Object B
- C Object C
- D Object D

16. A 5 kg block is placed on a 6 kg uniform beam, which is placed on two pivots as shown on the right. If the beam and the block are released from rest, which of the following will happen?



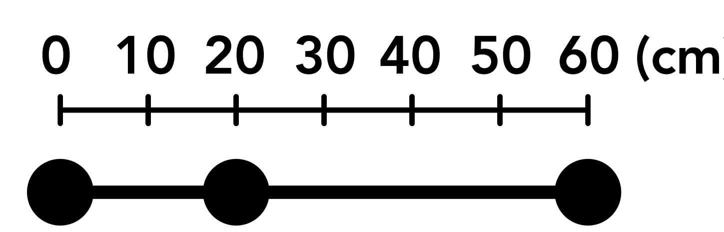
- A The beam will not rotate
- B The beam will rotate clockwise
- C The beam will rotate counterclockwise
- D Cannot be determined

17. A person is standing in the middle of a large raft with some mass which is floating in the water. The person and the raft are initially at rest when the person walks to the right end of the raft. Assuming that the raft can slide through the water without friction and the center of mass of the person-raft system does not move, which of the following is true?

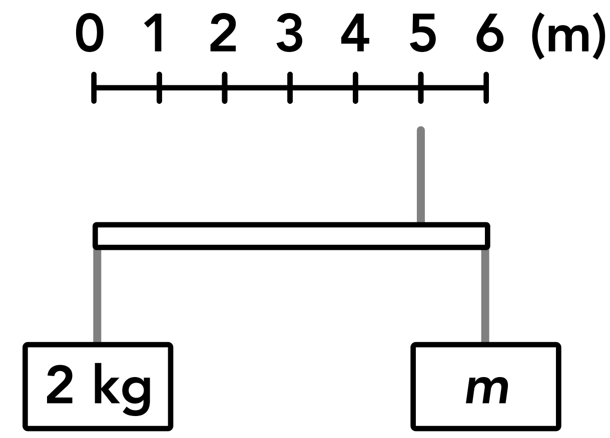


- A The raft does not move
- B The raft moves to the right
- C The raft moves to the left
- D It cannot be determined if the raft moves

18. Three 3 kg spheres are attached to a 3 kg rod which is 60 cm long as shown on the right. What is the position of the system's center of mass, measured from the left end of the rod?



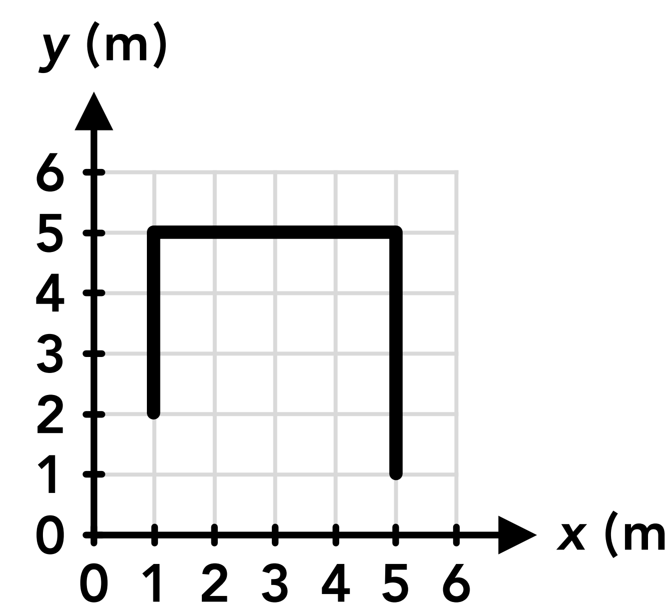
19. Two blocks are hanging from the ends of an 8 kg uniform rod which is hanging from a string as shown on the right. If the system is balanced (the rod is horizontal and not rotating), what is the mass ***m***?



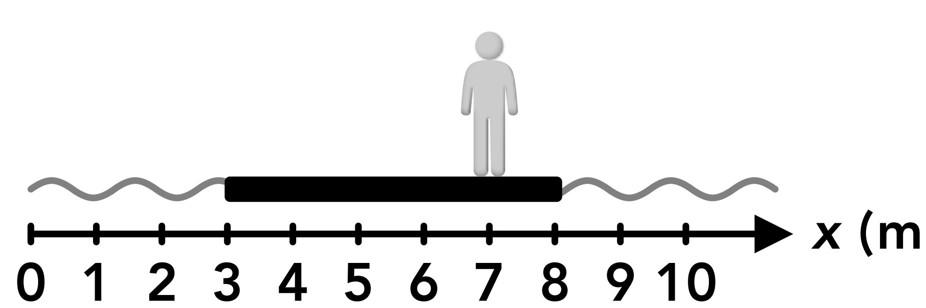
20. A system consists of three masses which are small enough to be treated as point masses. The position of each mass is shown on the right. What are the coordinates of the system's center of mass?

Mass:	Coordinates:
5 kg	(3, 4)
12 kg	(10, 6)
8 kg	(5, 1)

21. A metal wire is bent into the shape shown on the right. The wire has a uniform density. What are the coordinates of its center of mass?



22. An 80 kg person is standing on a 160 kg raft which is at rest as shown on the right. The person then walks to the left end of the raft. The raft can slide across the water without friction. After the person reaches the left end of the raft, what is the position of the left end of the raft on the axis shown?



Answers

1. A	6. A, D	11. C, D	16. A	21. (3.18, 3.86)
2. B, D	7. A, B, C, D	12. C	17. C	22. 4.33 m
3. C	8. B, C	13. B, C	18. 27.5 cm	
4. D	9. C	14. B	19. 26 kg	
5. A, B, C, D	10. B	15. B, D	20. (7, 4)	

Answers - Center of Mass

1. **Answer: A**
   The center of mass of an object or system is a point or location in space, which would be described using the SI unit of meters (m). The mass of the object(s) are used to calculate the center of mass, but the center of mass is only a position and does not describe the amount of mass in an object or system (unlike rotational inertia, which does describe the amount of mass in an object or system as well as how that mass is distributed).

2. **Answer: B, D**
   If the object is symmetrical in shape (like a sphere or a rectangular block) the center of mass is located at the object's center (the middle of its width, length and height) if it has a uniform density, which means the mass is the same per unit of volume throughout the object. An object with a uniform density that is not symmetrical will not have its center of mass at its dimensional center if it's not symmetrical.

3. **Answer: C**
   The center of mass of an object or system describes the mass-weighted average position of the particles in the object or the objects in the system. It does not describe the total or average mass of the system, and it is not an equal distance from the center of mass of each object (because the objects can have different masses).

4. **Answer: D**
   The center of mass of an object can be located on the physical object or in the space surrounding the object, which depends on its shape. Even if the object is symmetrical with uniform density, the center of mass can still be located in empty space (for example, if the object is a hollow sphere or a ring).

5. **Answer: A, B, C, D**
   The center of mass of a system of objects is calculated based on the mass of each object and the center of mass of each object. The center of mass of each object may depend on its shape, position and orientation (if it's not symmetrical with a uniform density).

6. **Answer: A, D**
   If a system consists of two masses, the center of mass of the system cannot be located at the center of mass of one of the objects because the second object will move or "pull" the center of mass towards it (the second object has mass, so its position will be weighted by some amount). The center of mass of the system must be located on an imaginary line connecting the center of masses of each object, which may be easier to think about if we place the objects on a single axis (like the x axis). The center of mass of the system may not be an equal distance from the edges or the center of masses of each object because the objects may have different masses.

7. **Answer: A, B, C, D**
   An object has a center of gravity if it's in a gravitational field (there is a gravitational force acting on it) which is true in each of the scenarios given.

8. **Answer: B, C**
We treat an object's weight force as acting on its center of gravity. The center of gravity is located at the same position as the object's center of mass if it's within a uniform gravitational field (the strength of the gravitational field, or the acceleration due to gravity, is assumed to be the same across the entire object, which we assume is true for objects near the surface of the earth).

9. **Answer: C**
If an object or system is suspended from or balanced on its center of mass (specifically its center of gravity), the object or system will be balanced in rotational equilibrium and it will not rotate (assuming it started at rest).

10. **Answer: B**
When calculating the center of mass of a system, we use the coordinates of the center of mass of each object.

11. **Answer: C, D**
Options C and D are both valid ways to analyze the torques on a balanced object or system. Option A does not explain why the object or system is balanced, and option B is not true.

12. **Answer: C**
The spheres have the same, uniform density which means the larger sphere on the right has more mass. The center of mass of the system is closer to the larger mass than the center of the rod.

13. **Answer: B, C**
Objects A and D are symmetrical and their center of masses are located at the center of each object which is on a physical part of the object. Object B is a symmetrical ring whose center of mass is located at its center which is not on a physical part of the object. Object C is not symmetrical and its center of mass is located somewhere in the empty space between the centers of each of the two parts.

14. **Answer: B**
An object's weight force acts at the object's center of gravity which is also the center of mass if the object is in a uniform gravitational field (which we assume it is). The center of mass and center of gravity are near the center of the object but are closer to the areas with more mass, so the center of gravity is left of the center.

15. **Answer: B, D**
An object will be balanced when its center of mass is directly above its pivot point (or directly below the point it's suspended from). The pivot point of each object is the point which is in contact with the ground. The center of mass of objects B and D are directly above their pivot points.

16. **Answer: A**
By looking at the beam, the block and the pivots we can see that the beam may either rotate clockwise about the right pivot or it will not rotate. The right pivot would prevent the beam from rotating clockwise about the left pivot. The left pivot would prevent the beam from rotating counterclockwise about the right pivot. To determine if the beam will rotate clockwise about the right pivot, we need to find the direction of the net torque about the right pivot. There is a clockwise torque produced by the weight force of the block which acts at the center of mass of the block, 1 m to the right of the right pivot. There is a counterclockwise torque produced by the weight force of the beam which acts at the center of mass of the beam. The beam is 6 m long so the center of the beam is 3 m from either end, which is 1 m to the left of the right pivot. Using counterclockwise as positive:
$\sum\tau = \tau_{\text{beam}} - \tau_{\text{block}} = (1\text{ m})(6\text{ kg})g - (1\text{ m})(5\text{ kg})g = 9.8\text{ N}\cdot\text{m}$
The net torque is counterclockwise (positive) so the beam does not rotate (because of the left pivot).

17. **Answer: C**
Since there is no friction force between the water and the raft, there is no external horizontal force acting on the person-raft system so the center of mass of the system does not accelerate (Newton's 1st law of motion). Since the system is initially at rest the center of mass of the system must be in the same position after the person moves to the right end of the raft. If person's mass moves to the right, the raft's mass must move to the left so that the center of mass of the system does not move.

18. **Answer: 27.5 cm**
We can calculate the center of mass of the system using the mass of each object and the position of each object's center of mass. The center of mass of the rod is in the middle of the rod, 30 cm from the left end (we assume the rod is uniform).

$$x_{COM} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_{rod}x_{rod}}{m_1 + m_2 + m_3 + m_{rod}} = \frac{(3\text{ kg})(0\text{ cm}) + (3\text{ kg})(20\text{ cm}) + (3\text{ kg})(60\text{ cm}) + (3\text{ kg})(30\text{ cm})}{(3\text{ kg}) + (3\text{ kg}) + (3\text{ kg}) + (3\text{ kg})}$$

$x_{COM} = 27.5$ cm

19. **Answer: 26 kg**
If the system of the rod and two blocks is balanced, it's in rotational equilibrium and the net torque on the system about the pivot point is zero. The pivot point of the rod is the point where the upper string is attached, 5 m from the left end of the rod. There is a torque produced by the weight force of the 2 kg block, a torque produced by the weight force of the rod, which acts at its center of mass (3 m from the left end), and a torque produced by the weight force on the right block. Using counterclockwise as positive:

$$\sum\tau = 0 \qquad \tau_{2\text{ kg}} + \tau_{rod} - \tau_m = 0 \qquad (5\text{ m})(2\text{ kg})g + (2\text{ m})(8\text{ kg})g - (1\text{ m})mg = 0 \qquad m = 26\text{ kg}$$

20. **Answer: (7, 4)**
We can calculate the ***x*** and ***y*** coordinates of the system's center of mass separately using the masses and coordinates of each mass.

$$x_{COM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{(5\text{ kg})(3) + (12\text{ kg})(10) + (8\text{ kg})(5)}{(5\text{ kg}) + (12\text{ kg}) + (8\text{ kg})} = 7$$

$$y_{COM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{(5\text{ kg})(4) + (12\text{ kg})(6) + (8\text{ kg})(1)}{(5\text{ kg}) + (12\text{ kg}) + (8\text{ kg})} = 4$$

$(x_{COM}, y_{COM}) = (7, 4)$

21. **Answer: (3.18, 3.86)**
We can treat the wire as 3 separate sections which each have their own mass and center of mass. We don't know the mass of the wire, but the wire has a uniform density and we can use a variable (M) to represent the mass of 1 m of length of wire. We'll find that the variable for mass cancels out and does not actually affect the location of the center of mass because the wire has a uniform density.

Section 1 (left): $m = 3M$ center of mass $= (1, 3.5)$

Section 2 (middle): $m = 4M$ center of mass $= (3, 5)$

Section 3 (right): $m = 4M$ center of mass $= (5, 3)$

$$x_{COM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{(3M)(1) + (4M)(3) + (4M)(5)}{(3M) + (4M) + (4M)} = 3.18$$

$$y_{COM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{(3M)(3.5) + (4M)(5) + (4M)(3)}{(3M) + (4M) + (4M)} = 3.86$$

22. **Answer: 4.33 m**

Since there is no friction acting on the raft, there is no external horizontal force acting on the person-raft system so the center of mass of the system does not move as the person walks across the raft (the system's center of mass does not accelerate, which is Newton's 1st law of motion). One way to solve this is to start by finding the initial center of mass of the system relative to the origin. Then we can find the new center of mass of the system relative to the left end of the raft, then find the distance between the left end of the raft and the center of mass relative to the origin (which doesn't change).

COM relative to the origin (initial and final): $x_{COM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{(80\text{ kg})(7\text{ m}) + (160\text{ kg})(5.5\text{ m})}{(80\text{ kg}) + (160\text{ kg})} = 6\text{ m}$

COM relative to the left end of the raft (final): $x_{COM} = \frac{(80\text{ kg})(0\text{ m}) + (160\text{ kg})(2.5\text{ m})}{(80\text{ kg}) + (160\text{ kg})} = 1.67\text{ m}$

After the person moves, the left end of the raft is 1.67 m to the left of the system's center of mass, which is still at a position of 6 m relative to the origin, so the left end of the raft is at a position of 4.33 m from the origin.