Probability Exam
(Exam P)

สอนโดย

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1. You are given $P(A \cup B) = 0.7$ and $P(A \cup B') = 0.9$ Calculate $P(A)$.

2. An urn contain 10 balls: 4 red and 6 blue. A second urn contains 16 red balls and an unknown number of blue balls. A single ball is drawn from each urn. The probability that both balls are the same color is 0.44

calculate the number of blue balls in the second urn.

- Mutually independent

$$P(A \cap B) = P(A)P(B)$$

- Mutually exclusive

$$P(A \cap B) = 0$$

3. An actuary studying the insurance preferences of automobile owners makes the following conclusion

(i) An automobile owner is twice as likely to purchase collision coverage as disability coverage.

(ii) The event that an automobile owner purchases collision coverage is independent of the event that he or she purchases disability coverage.

(iii) The probability that an automobile owner purchases both collision and disability coverages is 0.15.

Calculate the probability that an automobile owner purchases neither collision nor disability coverage.

4. An insurance agent offers his clients auto insurance, homeowners insurance and renters insurance. The purchase of homeowners insurance and the purchase of renters insurance are mutually exclusive. The profile of the agent's clients is as follows

i) 17% of the clients have none of these three products.

ii) 64% of the clients have auto insurance

iii) Twice as many of the clients have homeowners insurance as have renters insurance.

iv) 35% of the clients have two of these three products.

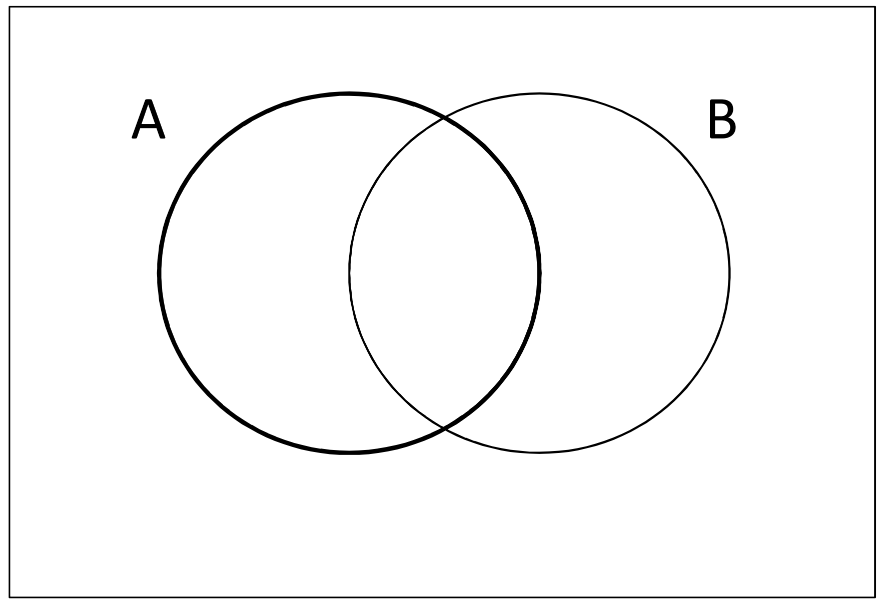
v) 11% of the clients have homeowners insurance, but not auto insurance.

Calculate the percentage of the agent's clients that have both auto and renters insurance

Bayes Theorem / Law of total probability

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

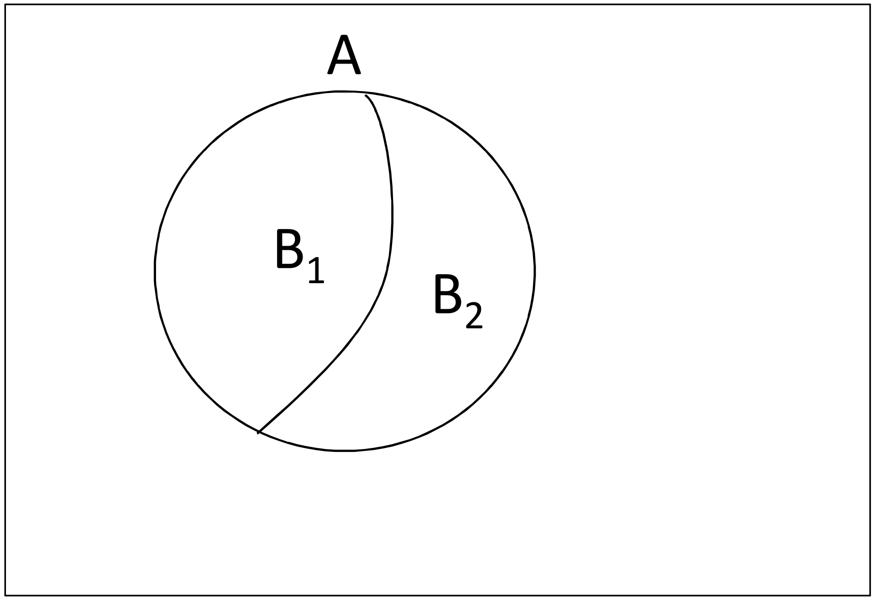
$$=$$



Bayes Theorem / Law of total probability

$$P(A) = P(A \cap B_1) + P(A \cap B_2)$$

$$= \boxed{}$$



Bayes Theorem / Law of total probability

$$P(B_1 \mid A) = \frac{P(A \cap B_1)}{P(A \cap B_1) + P(A \cap B_2)}$$

$$= \boxed{}$$

5. A health study tracked a group of persons for five years. At the beginning of the study, 20% were classified as heavy smokers, 30% as light smoker, and 50% as nonsmokers.

Results of the study showed that light smokers were twice as likely as non smokers. to die during the five-year study, but only half as likely as heavy smokers.

A randomly selected participant from the study died during the five-year period. Calculate the probability that the participant was a heavy smoker

6. An auto insurance company insures drivers of all ages. An actuary compiled the following statics on the company's insured drivers

Age of Driver	Probability of Accident	Portion of Company's Insured Drivers
16-20	0.06	0.08
21-30	0.03	0.15
31-65	0.02	0.49
66-99	0.04	0.28

A randomly selected driver that the company insures has an accident. Calculate the probability that the driver was age 16-20.

Random variable

- Discrete Random Variable
  - Expected

$$E(X) = \sum_{x=0}^{\infty} xp(x)$$

$$E(g(x)) = \sum_{x=0}^{\infty} g(x)p(x)$$

  - Variance

$$V(X) = E(X^2) - (E(X))^2$$

7. An insurance policy pays 100 per day for up to three days of hospitalization and 50 per day for each day of hospitalization thereafter.

The number of days of hospitalization, X is a discrete random variable with probability function

$$P[X = k] = \begin{cases} \dfrac{6-k}{15}, & k = 1, 2, 3, 4, 5 \\ 0, & \text{otherwise.} \end{cases}$$

Determine the expected payment for hospitalization under this policy.

- discrete distribution

Distribution	P(N=n)	Expected	Variance	Definition
Binomial				
Hypergeometric				
Geometric				
Negative Binomial				
Poisson				
Uniform				

8. An actuary has discovered that policyholders are three times as likely to file two claims as to file four claims.

The number of claim filed has a Poisson distribution.

Calculate the variance of the number of claims filed

9. A company prices its hurricane insurance using the following assumptions:

(i) In any calendar year, there can be at most one hurricane.

(ii) in any calendar year, the probability of a hurricane is 0.05

(iii) The numbers of hurricanes in different calendar years are mutually independent.

Using the company's assumptions, calculate the probability that there are fewer than 3 hurricanes in a 20 year period.

10. A company takes out an insurance policy to cover accidents that occur at its manufacturing plant. The probability that one or more accidents will occur during any given month is 0.6. The numbers of accidents that occur in different months are mutually independent. Calculate the probability that there will be at least four months in which no accidents occur before the fourth month in which at least one accident occurs.

11. In a hospital ward there are 16 patients, 4 whom have AIDS. A doctor is assigned to 6 of these patients at random. What is the probability that he gets 2 of the AIDS patients?

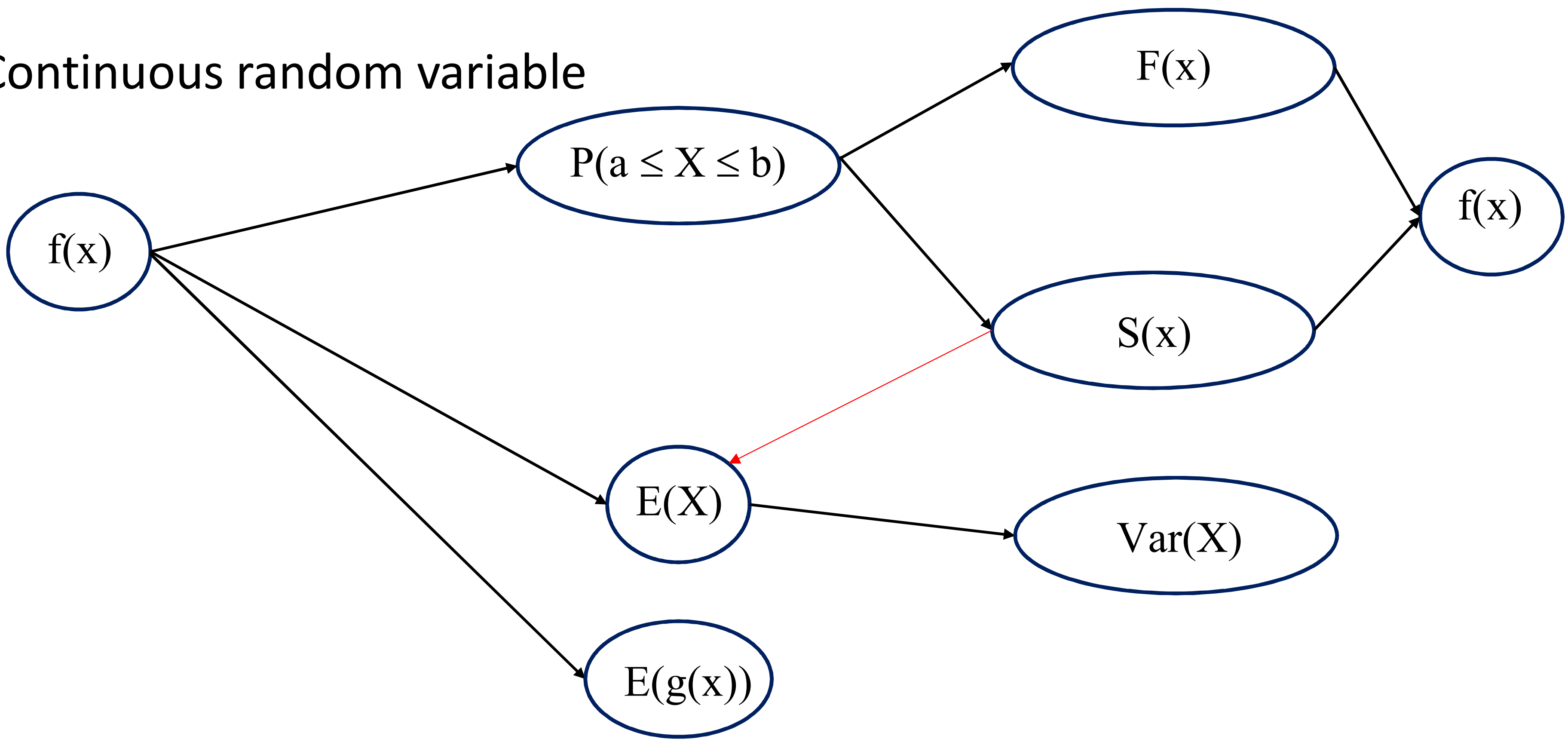
12. A company has five employees on its health insurance plan. Each year, each employee independently has an 80% probability of no hospital admissions. If an employee requires one or more hospital admissions, the number of admissions is modeled by a geometric distribution with a mean of 1.50. The numbers of hospital admissions of different employees are mutually independent.

Each hospital admission costs 20,000.

Calculate the probability that the company's total hospital costs in a year are less than 50,000.

Random variable

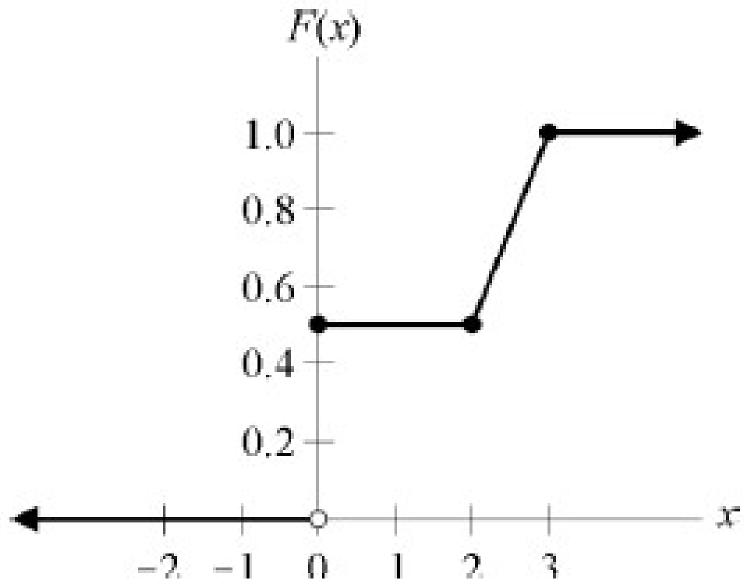
- Continuous random variable



13. The lifetime of a machine part has a continuous distribution on the interval (0, 40) with probability density function $f(x)$, where $f(x)$ is proportional to $(10 + x)^{-2}$ on the interval.

Calculate the probability that the lifetime of the machine part is less than 6.

14. The figure below shows the cumulative distribution function of a random variable, *X*.



Calculate E(X).

15. A random variable X has the cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 1 \\ \dfrac{x^2 - 2x + 2}{2}, & 1 \le x < 2 \\ 1, & x \ge 2. \end{cases}$$

Calculate the variance of X.

16. Let *X* be a continuous random variable with density function

$$f(x) = \begin{cases} \dfrac{|x|}{10}, & -2 \le x \le 4 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the expected value of *X*.

- continuous distribution

Distribution	f(x)	F(x)	S(x)	Expected	Variance
exponential					
gamma					

Integrate Shortcut

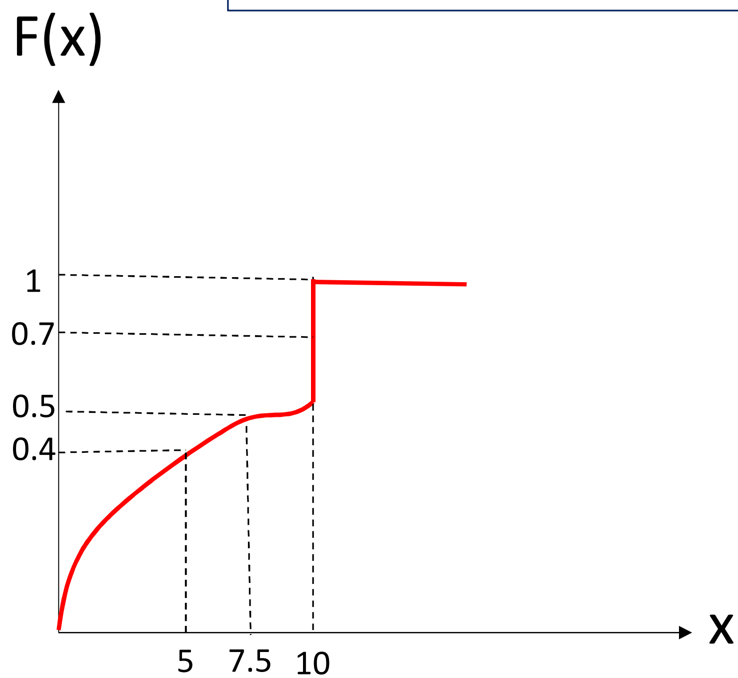
1.

2.

17. A device that continuously measures and records seismic activity is placed in a remote region. The time, T, to failure of this device is exponentially distributed with mean 3 years. Since the device will not be monitored during its first two years of service, the time to discovery of its failure is $X = \max(T, 2)$ Calculate $E(X)$.

Percentile

ถ้า π_r อยู่ตำแหน่ง percentile ที่ r แล้ว $F(\pi_r) = \frac{r}{100}$



40^{th} percentiles = 5

100^{th} percentiles = 10

70^{th} percentiles = 10

Given that x is more than 5.
Find 50^{th} percentiles

18. An insurer's annual weather-related loss, *X*, is a random variable with density function

$$f(x)=\begin{cases}\dfrac{2.5(200)^{2.5}}{x^{3.5}}, & x>200\\ 0, & \text{otherwise.}\end{cases}$$

Calculate the difference between the 30th and 70th percentiles of *X*.

19. An insurance company sells an auto insurance policy that covers losses incurred by a policyholder, subject to a deductible of 100. Losses incurred follow an exponential distribution with mean 300.

Calculate the 95th percentile of losses that exceed the deductible

$$\{\ f(x) = \frac{1}{300} e^{-\frac{x}{300}} \}$$

Maximum / Minimum

- Max

$$F(\max(x_1, x_2)) = F(x_1)F(x_2)$$

- Max

$$S(\min(x_1, x_2)) = S(x_1)S(x_2)$$

20. Claim amounts for wind damage to insured homes are mutually independent random variables with common density function

$$f(x)=\begin{cases}\dfrac{3}{x^4}, & x>1\\ 0, & \text{otherwise,}\end{cases}$$

where *x* is the amount of a claim in thousands.

Suppose 3 such claims will be made.

Calculate the expected value of the largest of the three claims.