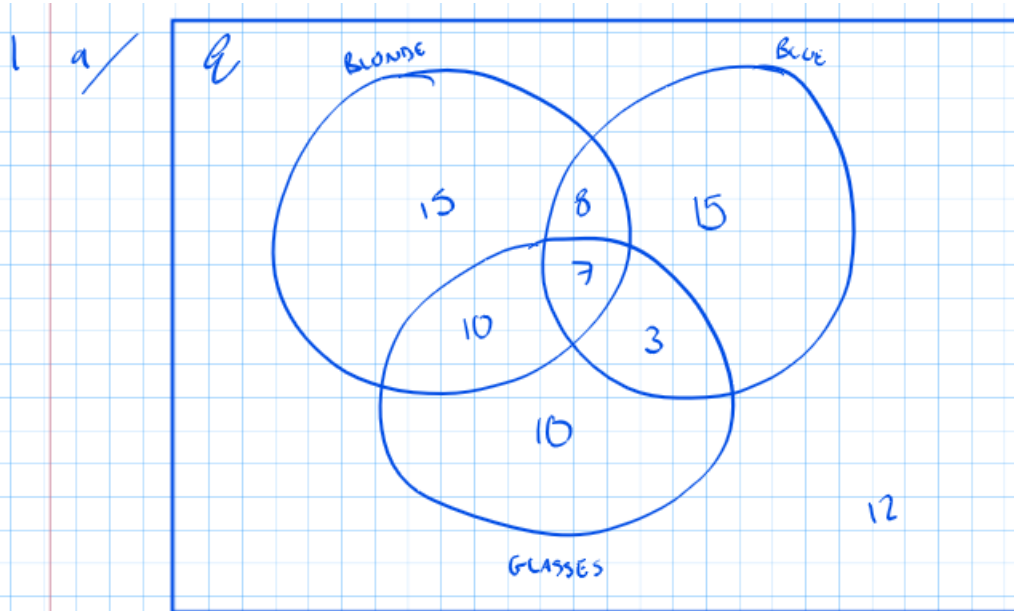Skill: Conditional Probability and Venn Diagrams

Solutions

1 a/

b/ i $\dfrac{7+3}{30} = \dfrac{1}{3}$ ii/ $\dfrac{15+3}{15+3+10+12} = \dfrac{9}{20}$

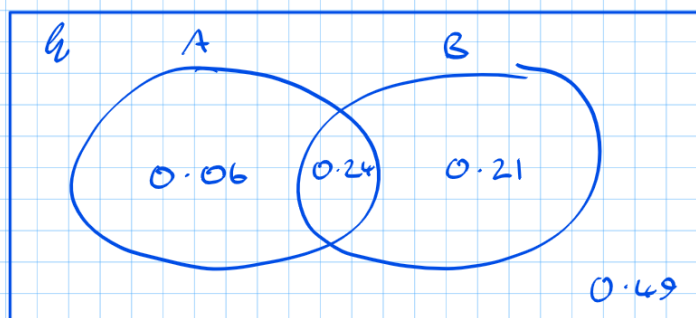iii/ $\dfrac{15+12}{15+10+10+12} = \dfrac{27}{47}$

iv/ $\dfrac{12}{15+3+10+12} = \dfrac{3}{10}$ v/ $\dfrac{7}{7+3} = \dfrac{7}{10}$

vi/ $\dfrac{15}{15+10+8+7} = \dfrac{3}{8}$

2 a/

b' i/ $P(A|B) = \dfrac{0.24}{0.24 + 0.21} = \dfrac{8}{15}$

ii/ $P(A|B') = \dfrac{0.06}{0.06 + 0.49} = \dfrac{6}{55}$

iii/ $P(B'|A) = \dfrac{0.06}{0.06 + 0.24} = \dfrac{1}{5}$

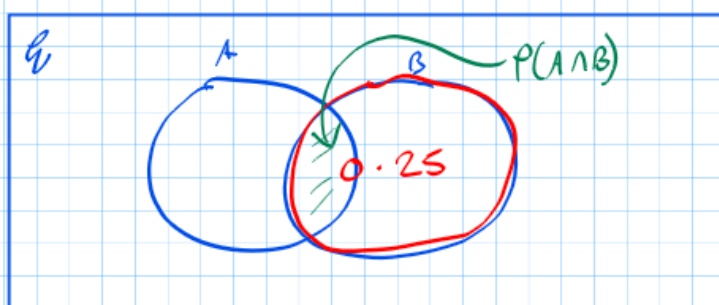iv/ $P(A'|B') = \dfrac{0.49}{0.06 + 0.49} = \dfrac{49}{55}$

c/ $P(A \cap B) = 0.24$

$P(A) \times P(B) = 0.3 \times 0.45 = 0.135 \neq 0.24$

∴ Not independent

3

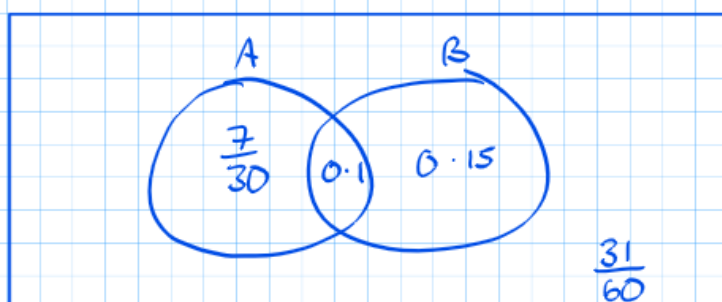ξ A   B   P(A∩B)

0.25

i/ $\dfrac{P(A∩B)}{0.25} = 0.4$

$\Rightarrow P(A∩B) = 0.4 \times 0.25$

$= 0.1$

A   B

$\dfrac{7}{30}$ 0.1   0.15

$\dfrac{31}{60}$

ii/ $P(B|A) = \dfrac{0.1}{\frac{1}{3}}$
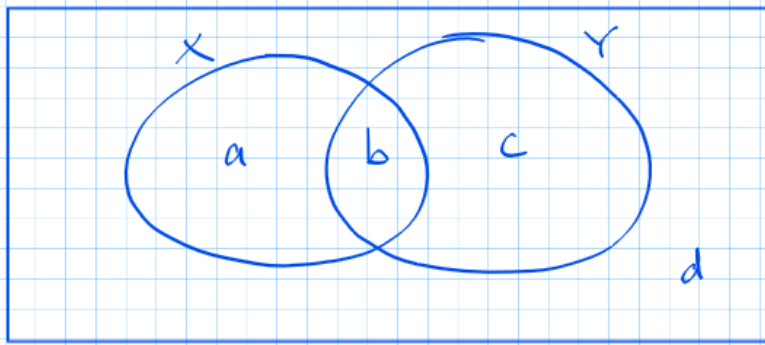
$= 0.3$

iii/ $P(B|A') = \dfrac{0.15}{0.15 + \frac{31}{60}} = \dfrac{9}{40}$

iv/ $\dfrac{0.15}{0.15 + \frac{31}{60}} = \dfrac{9}{40}$

[Bit of a trick Q as since we are given A' occurs, the A∪ is irrelevant]

4

$$P(X|Y) = \frac{b}{b+c} = \frac{1}{2} \implies 2b = b + c$$
$$\implies b = c \quad \text{(A)}$$

$$P(Y|X) = \frac{b}{a+b} = \frac{2}{3} \implies 3b = 2a + 2b$$
$$\implies b = 2a \quad \text{(B)}$$

$$P(X \cup Y) = a + b + c = 0.9 \quad \text{(C)}$$

$$\implies \text{(A)}, \text{(B)} \implies c = 2a$$

$$\therefore \text{(C)} \implies a + 2a + 2a = 0.9$$

$$\implies 5a = 0.9 \implies a = \frac{9}{50}$$

$$\implies b = \frac{9}{25} \implies c = \frac{9}{25} \implies d = 0.1$$
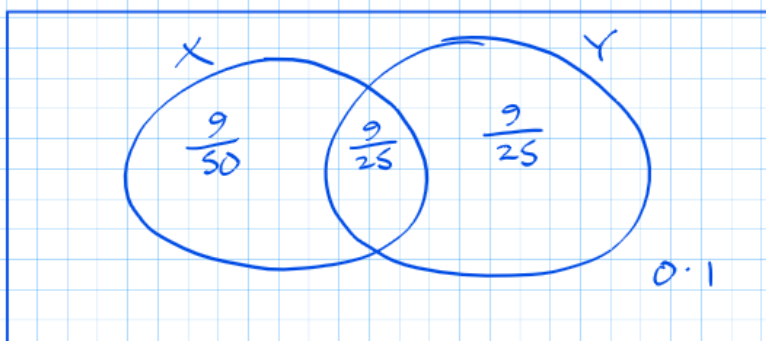
$a^i/\ P(X \cap Y) = \frac{9}{25}$

$ii/\ P(Y) = \frac{9}{25} + \frac{9}{25}$
$$= \frac{18}{25}$$

$$iii/\ P(X'|Y) = \frac{\frac{9}{25}}{\frac{9}{25} + \frac{9}{25}} = \frac{1}{2}$$

iv/ $P(X|Y) = \dfrac{\frac{9}{50}}{\frac{9}{50} + 0.1} = \dfrac{9}{14}$

b/ $P(X \cap Y) = \dfrac{9}{25}$

$P(X) \times P(Y) = \left(\dfrac{9}{50} \times \dfrac{9}{25}\right) \times \dfrac{18}{25} = \dfrac{729}{15625} \neq \dfrac{9}{25}$

$\therefore$ Not independent

c/ $P(X \cap Y) \neq 0$ $\therefore$ not mutually exclusive.