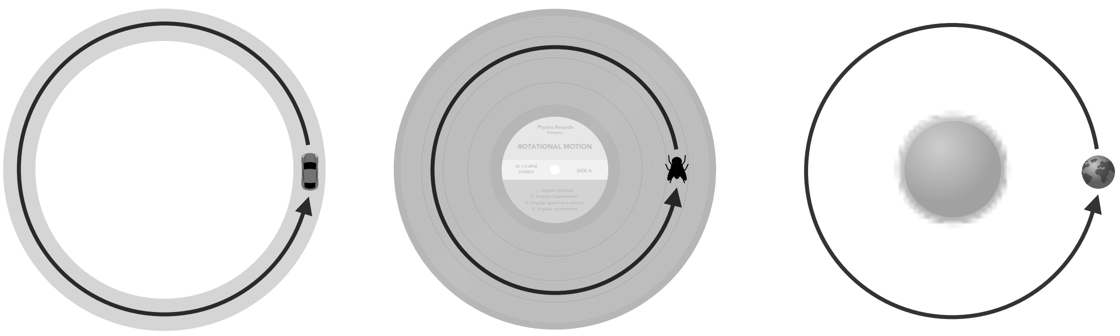
STUDY GUIDE

CIRCULAR & ROTATIONAL MOTION

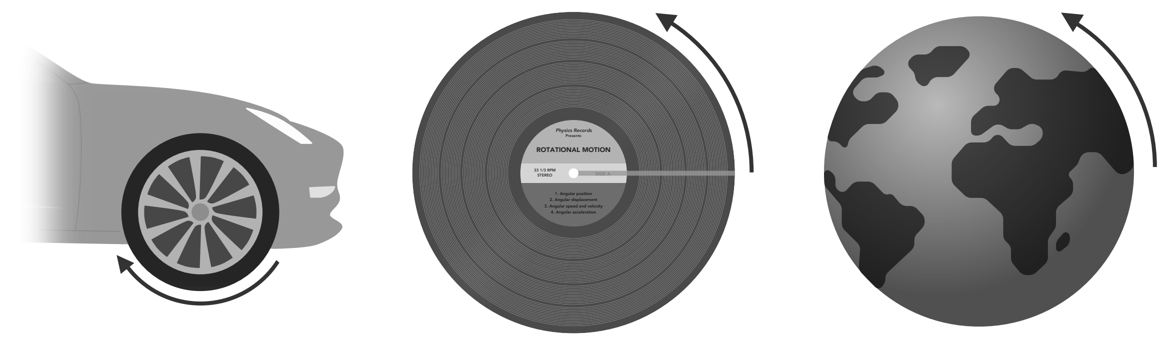
Circular vs Rotational Motion

Circular Motion



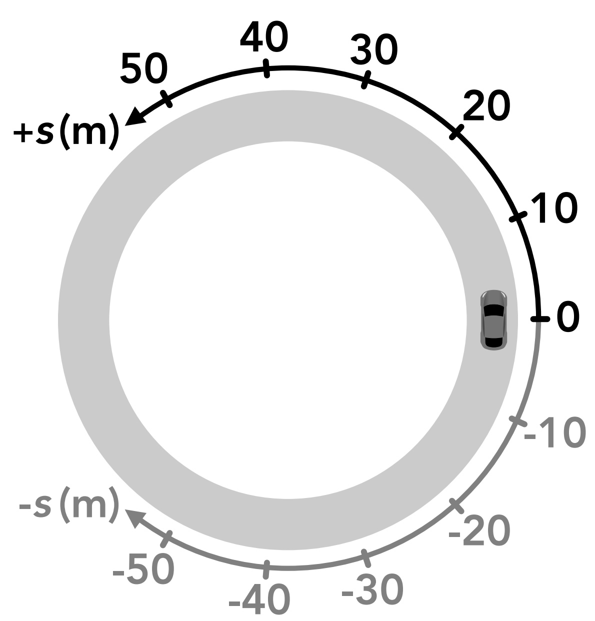
- Object travels along a circular path (circumference of a circle whose center lies outside of the object).
- A point on a rotating object is in circular motion.
- Typically uses the **tangential description of motion**.

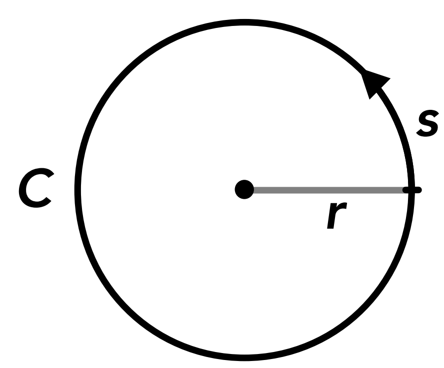
Rotational Motion



- Object rotates about its own center (a point or axis that passes through the object).
- Typically uses the **angular description of motion**.
- All points on the object have the same angular motion.

Circular Motion (Tangential Description)





$$C = 2\pi r$$

Variables		SI Unit
s	tangential position	m
Δs	tangential displacement	m
v_t	tangential velocity	$\frac{m}{s}$
a_t	tangential acceleration	$\frac{m}{s^2}$

$$\Delta s = s_f - s_i$$

Tangential displacement

$$v_t = \frac{\Delta s}{\Delta t}$$

Tangential velocity

$$a_t = \frac{\Delta v_t}{\Delta t}$$

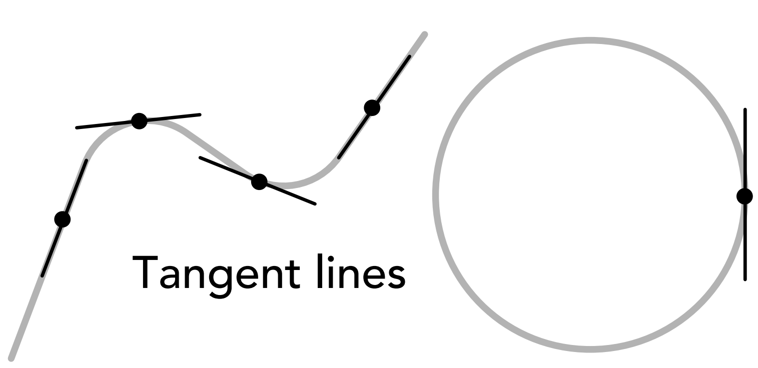
Tangential acceleration

$$s_f = s_i + v_{ti}t + \frac{1}{2}a_t t^2$$

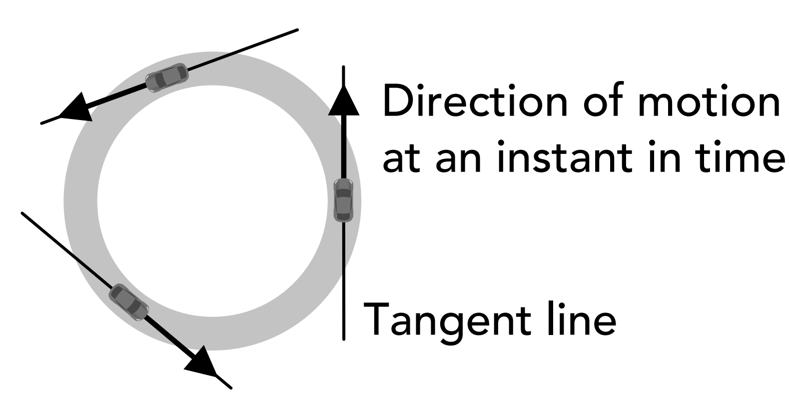
$$v_{tf}^2 = v_{ti}^2 + 2a_t(s_f - s_i)$$

Kinematic equations with constant acceleration

- **Circular motion** typically uses the **tangential** desciption of motion.
- The value of the position will continue to increase past 1 revolution (or decrease in the negative direction).
- Tangential motion is sometimes referred to as the "linear" motion of an object in circular motion because the displacement, velocity and acceleration are directed along a **tangent line**.

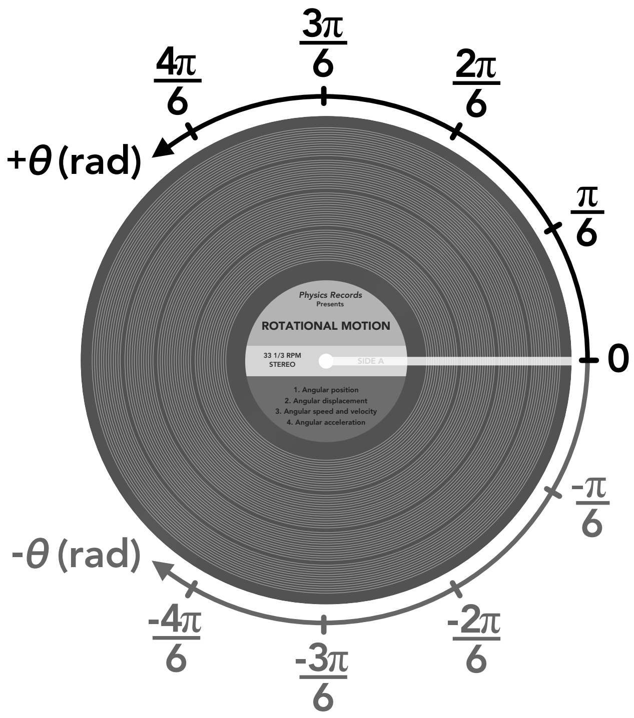


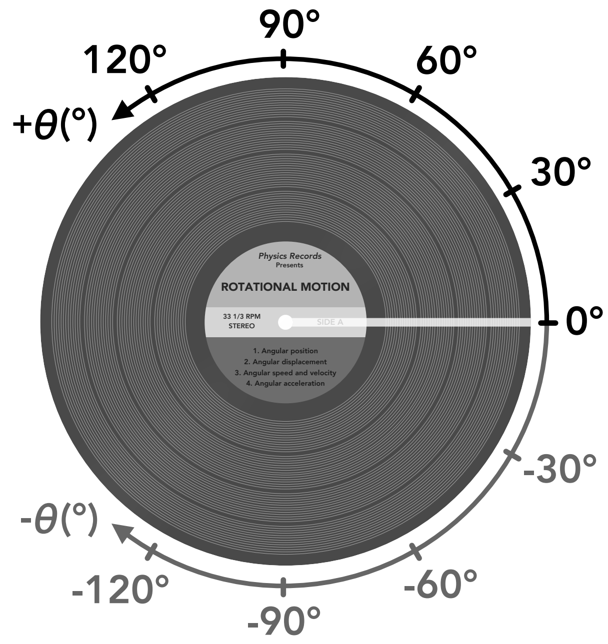
- At a point on a curve, the **tangent line** passing through it matches the curvature or "slope" of the curve.
- For a circle, a tangent line only touches one point.



- For an object in circular motion, the instantaneous direction of the motion is always tangent to the circle.

Rotational Motion (Angular Description)





Variables		SI Unit
θ	angular position	rad
$\Delta\theta$	angular displacement	rad
ω	angular velocity	$\frac{rad}{s}$
α	angular acceleration	$\frac{rad}{s^2}$

θ "theta" ω "omega" α "alpha" RPM: $\frac{\text{revolutions}}{\text{minute}}$

$$\Delta\theta = \theta_f - \theta_i$$

Angular displacement

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Angular velocity

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

Angular acceleration

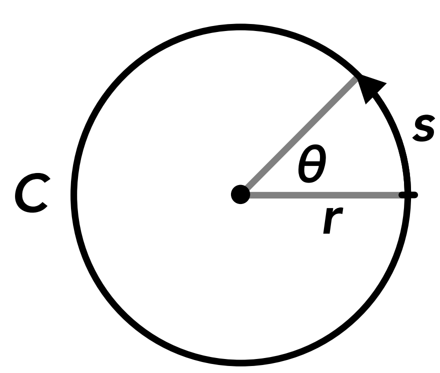
$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$$

Kinematic equations with constant acceleration

- **Rotational motion** typically uses the **angular** description of motion.
- Can also be used to describe the angle that is "swept out" by an object in circular motion.
- All points on a rotating object have the same angular motion because they rotate together (but they may have different tangential motions depending on their distance from the center).
- The value of the position will continue to increase past 1 revolution (or decrease in the negative direction).

Converting Between Tangential & Angular Descriptions



Circumference: $C = 2\pi r$

1 circumference ⟷ 2π radians
1 circumference ⟷ 360°
1 circumference ⟷ 1 revolution
1 circumference ⟷ 1 cycle

Conversion (Angular variable must use *radians)*

	Tangential description	⟷	Angular description
Position:	s m	$s = r\,\theta$	θ rad
Displacement:	$\Delta s = s_f - s_i$ m	$\Delta s = r\,\Delta\theta$	$\Delta\theta = \theta_f - \theta_i$ rad
Velocity:	$v_t = \frac{\Delta s}{\Delta t}$ $\frac{m}{s}$	$v_t = r\,\omega$	$\omega = \frac{\Delta\theta}{\Delta t}$ $\frac{rad}{s}$
Acceleration:	$a_t = \frac{\Delta v_t}{\Delta t}$ $\frac{m}{s^2}$	$a_t = r\,\alpha$	$\alpha = \frac{\Delta\omega}{\Delta t}$ $\frac{rad}{s^2}$

- In some cases, we need to convert from one description to another.
- This conversion is based on the definition of a radian, or the relationship between the circumference and the number of radians in a circle.