Topic: Mixing problems

Question: How much salt is in the tank?

Find the amount of salt in the tank at t minutes.

The tank contains $1{,}000$ L of water and 10 kg of dissolved salt.

Fresh water is entering the tank at 10 L/min (the solution stays perfectly mixed) and the tank drains at a rate of 5 L/min.

Answer choices:

A $y = 5e^{-\frac{1}{100}t}$

B $y = 5e^{\frac{1}{100}t}$

C $y = 10e^{-\frac{1}{200}t}$

D $y = 10e^{\frac{1}{200}t}$

Solution: C

Starting with the mixing problem formula that models the flow of salt in the tank over time, we'll use

$$\frac{dy}{dt} = C_1 r_1 - C_2 r_2$$

In this problem,

$C_1 = 0$ kg/min because no salt is entering the tank with the fresh water

$r_1 = 10$ L/min because water is entering the tank at that rate

$C_2 = \frac{y}{1{,}000}$ kg/L because we're not sure how much salt is leaving the tank, but we know the initial water amount is $1{,}000$ L

$r_2 = 5$ L/min because the solution is leaving the tank at that rate

Plugging these values into the differential equation gives

$$\frac{dy}{dt} = (0)(10) - \left(\frac{y}{1{,}000}\right)(5)$$

$$\frac{dy}{dt} = -\frac{y}{200}$$

This is now a separable differential equation, so we'll split the variables.

$$dy = -\frac{y}{200}\ dt$$

$$\frac{1}{y}\ dy = -\frac{1}{200}\ dt$$

Integrate both sides.

$$\int \frac{1}{y}\ dy = \int -\frac{1}{200}\ dt$$

$$\ln|y| + C_1 = -\frac{1}{200}t + C_2$$

$$\ln|y| = -\frac{1}{200}t + C_2 - C_1$$

$$\ln|y| = -\frac{1}{200}t + C$$

Now we'll solve for y.

$$e^{\ln|y|} = e^{-\frac{1}{200}t+C}$$

$$|y| = e^{-\frac{1}{200}t}e^C$$

Remember that e^C is a constant, so we can replace it just with C.

$$|y| = Ce^{-\frac{1}{200}t}$$

$$y = \pm Ce^{-\frac{1}{200}t}$$

We can absorb the ± into the constant C.

$$y = Ce^{-\frac{1}{200}t}$$

Because we were told that the tank initially contained 10 kg of dissolved salt, we can plug the initial condition $y(0) = 10$ into the equation for y to solve for C.

$$10 = Ce^{-\frac{1}{200}(0)}$$

$$10 = C(1)$$

$$C = 10$$

So we can say that the amount of salt in the tank at any time t can be modeled by

$$y = 10e^{-\frac{1}{200}t}$$

Topic: Mixing problems

Question: How much salt is in the tank?

Find the amount of salt in the tank after 10 minutes.

A tank contains $1,000$ L of water and 15 kg of dissolved salt.

Fresh water is entering the tank at 20 L/min (the solution stays perfectly mixed) and the tank drains at a rate of 10 L/min.

Answer choices:

A 15.4 kg

B 13.6 kg

C 16.0 kg

D 14.0 kg

Solution: B

Starting with the mixing problem formula that models the flow of salt in the tank over time, we'll use

$$\frac{dy}{dt} = C_1 r_1 - C_2 r_2$$

In this problem,

$C_1 = 0$ kg/min because no salt is entering the tank with the fresh water

$r_1 = 20$ L/min because water is entering the tank at that rate

$C_2 = \frac{y}{1{,}000}$ kg/L because we're not sure how much salt is leaving the tank, but we know the initial water amount is $1{,}000$ L

$r_2 = 10$ L/min because the solution is leaving the tank at that rate

Plugging these values into the differential equation gives

$$\frac{dy}{dt} = (0)(20) - \left(\frac{y}{1{,}000}\right)(10)$$

$$\frac{dy}{dt} = -\frac{y}{100}$$

This is now a separable differential equation, so we'll split the variables.

$$dy = -\frac{y}{100}\ dt$$

$$\frac{1}{y}\ dy = -\frac{1}{100}\ dt$$

Integrate both sides.

$$\int \frac{1}{y}\ dy = \int -\frac{1}{100}\ dt$$

$$\ln|y| + C_1 = -\frac{1}{100}t + C_2$$

$$\ln|y| = -\frac{1}{100}t + C_2 - C_1$$

$$\ln|y| = -\frac{1}{100}t + C$$

Now we'll solve for y.

$$e^{\ln|y|} = e^{-\frac{1}{100}t+C}$$

$$|y| = e^{-\frac{1}{100}t}e^{C}$$

Remember that e^C is a constant, so we can replace it just with C.

$$|y| = Ce^{-\frac{1}{100}t}$$

$$y = \pm Ce^{-\frac{1}{100}t}$$

We can absorb the ± into the constant C.

$$y = Ce^{-\frac{1}{100}t}$$

Because we were told that the tank initially contained 15 kg of dissolved salt, we can plug the initial condition $y(0) = 15$ into the equation for y to solve for C.

$$15 = Ce^{-\frac{1}{100}(0)}$$

$$15 = C(1)$$

$$C = 15$$

So we can say that the amount of salt in the tank at any time t can be modeled by

$$y = 15e^{-\frac{1}{100}t}$$

Last, we'll use this equation to say how much salt is in the tank after 10 minutes.

$$y(10) = 15e^{-\frac{1}{100}(10)}$$

$$y(10) = 15e^{-\frac{1}{10}}$$

$$y(10) \approx 13.6$$

After 10 minutes, the tank contains approximately 13.6 kg of salt.