Complex Numbers

► from Maths HL formula booklet - complex numbers ◄

- complex numbers $z = a + ib = r(\cos\theta + \mathrm{i}\sin\theta) = re^{\mathrm{i}\theta} = r\mathrm{cis}\theta$
- De Moivre's theorem $\left[r(\cos\theta + \mathrm{i}\sin\theta)\right]^n = r^n(\cos n\theta + \mathrm{i}\sin n\theta) = r^n e^{\mathrm{i}n\theta} = r^n\mathrm{cis}n\theta$

* answers included

Exercises

1. Find the cube roots of i in the form $a+bi$, where $a, b \in \mathbb{R}$.

2. (a) Find x and y such that $(x+3i)(x+iy) = 1-i$

(b) Find a and b such that $a+bi = \sqrt{5+12i}$

3. Find all solutions to the equation $z^5 - 32 = 0$. Express the solutions exactly.

4. Given that $z_1 = 3+2i$ and $z_2 = 4-3i$

(a) Find $z_1 z_2$ and $\frac{z_1}{z_2}$, and express each in the form $a+bi$ (Cartesian form).

(b) Verify that $|z_1 z_2| = |z_1||z_2|$.

5. The complex number z satisfies the equation $\frac{z}{z+2} = 2-i$.

(a) Find z and express it in the form $a+bi$, where $a, b \in \mathbb{R}$.

(b) Find $|z|$ and $\arg(z)$.

6. (a) Find all solutions to the equation $z^4 + 4 = 0$.

(b) Hence, or otherwise, express $z^4 + 4$ as the product of two quadratic polynomials in z with real coefficients.

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Answers

1. $z_1 = \frac{\sqrt{3}}{2} + \frac{1}{2}i,\ z_2 = -\frac{\sqrt{3}}{2} + \frac{1}{2}i,\ z_3 = -i$

2. (a) $x = \frac{1}{25},\ y = -\frac{7}{25}$ (b) $a = 3, b = 2$ or $a = -3, b = -2$

3. five solutions: $z = 2,\ z = 2\left(\cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}\right),\ z = 2\left(\cos\frac{2\pi}{5} - i\sin\frac{2\pi}{5}\right),$

$z = 2\left(\cos\frac{4\pi}{5} + i\sin\frac{4\pi}{5}\right)$ and $z = 2\left(\cos\frac{4\pi}{5} - i\sin\frac{4\pi}{5}\right)$

4. (a) $z_1 z_2 = 18 - i,\ \frac{z_1}{z_2} = \frac{6}{25} + \frac{17}{25}i$

5. (a) $z = -3 - i$ (b) $|z| = \sqrt{10}$, $\arg(z) \approx -2.82$ radians

6. (a) $z = 1+i,\ 1-i,\ -1+i,\ -1-i$ (b) $z^4 + 4 = (z^2 + 2z + 2)(z^2 - 2z + 2)$