

Chapter 3

Risk Identification and Measurement

Chapter Objectives

- Discuss frameworks for identifying business and individual risk exposures.
- Review concepts from probability and statistics.
- Apply mathematical concepts to understand the frequency and severity of losses.
- Explain the concepts of maximum probable loss and value at risk.

3.1 Risk Identification

As introduced in Chapter 1, the five major steps in the risk management decision-making process are: (1) identify all significant risks that can cause loss; (2) evaluate the potential frequency and severity of losses; (3) develop and select methods for managing risk; (4) implement the risk management methods chosen; and (5) monitor the suitability and performance of the chosen risk management methods and strategies on an ongoing basis. This chapter focuses on the first two steps of this process.

Identifying Business Risk Exposures

The first step in the risk management process is **risk identification**: the identification of loss exposures. Unidentified loss exposures most likely will result in an implicit retention decision, which may not be optimal. There are various methods of identifying exposures. For example, comprehensive checklists of common business exposures can be obtained from risk management consultants and other sources. Loss exposures also can be identified through analysis of the firm's financial statements, discussions with managers throughout the firm, surveys of employees, and discussions with insurance agents and risk management consultants. Regardless of the specific methods used, risk identification requires an overall understanding of the business and the specific economic, legal, and regulatory factors that affect the business.

Table 3.1
Some practical questions in identifying business property and liability loss exposures.

Property Loss Exposures

Some of the major practical questions asked when identifying property loss exposures for businesses are listed in Table 3.1. In addition to identifying what property is exposed to loss and the potential causes of loss, the firm must consider how property should be valued for the purpose of making risk management decisions. Several valuation methods are available. **Book value**—the purchase price minus accounting depreciation—is the method commonly used for financial reporting purposes. However, since book value does not

Table 3.1
Some practical
questions in
identifying
business
property and
liability loss
exposures.

Type of Loss	Property Losses	Liability Losses
Direct Losses	1. What types of property are subject to damage or disappearance? 2. What factors (perils) can lead to loss? 3. What is the value of property exposed to loss? 4. Will the property be replaced if it is lost?	1. What parties might be harmed by the firm (customers, suppliers, and other parties)? 2. How might these parties be harmed? 3. What is the potential magnitude of damages? 4. What is the potential magnitude of defense costs?
Indirect Losses	1. Will the firm have to raise external funds to replace uninsured property? 2. Assuming replacement, will the firm suspend or cut back operations following a direct loss? 3. If the firm suspends or cuts back its operations: (a) What is the potential duration and how much normal profit could be lost? (b) What operating expenses would continue despite the suspension or slowdown? (c) Will revenue losses continue after normal levels of production are resumed, and, if so, what actions might reduce these losses and at what cost? 4. If the firm continues operating at preloss levels: (a) What facilities or resources will be needed? (b) What will be the additional cost from using alternative facilities or resources?	1. Will revenues decline in response to possible damage to the firm's reputation? (a) What is the potential magnitude of this loss? (b) What actions might reduce the resulting indirect losses and at what cost? 2. Will products and services likely be abandoned or products recalled in the event of large uninsured losses? 3. Will the firm have to raise additional capital in the event that cash flows decline? 4. Could large uninsured losses push the firm into financial distress?

necessarily correspond to economic value, it generally is not relevant for risk management purposes (except for the tax reasons discussed in Chapter 21). **Market value** is the value that the next-highest-valued user would pay for the property. **Firm-specific value** is the value of the property to the current owner. If the property does not provide firm-specific benefits, then firm-specific value will equal market value. Otherwise, firm-specific value will exceed market value. **Replacement cost new** is the cost of replacing the damaged property with new property. Due to economic depreciation and improvements in quality, replacement cost new often will exceed the market value of the property.¹

Indirect losses also can arise from damage to property that will be repaired or replaced. For example, if a fire shuts down a plant for four months, the firm not only incurs the cost of replacing the damaged property, it also loses the profits from not being able to produce. In addition, some operating expenses might continue despite the shutdown (e.g., salaries for certain managers and employees and advertising expenses). These exposures are known as **business income exposures** (or, sometimes, business interruption exposures), and they frequently are insured with *business interruption insurance*. Note that business interruption losses also might result from property losses to a firm's major customers or suppliers that prevent them from transacting with the firm. This exposure can be insured with "contingent" business interruption insurance.

Firms also may suffer losses after they resume operations if previous customers that have switched to other sources of supply do not return. In the event that a long-term loss of customers would occur and/or a shutdown temporarily would impose large costs on customers or suppliers, it might be optimal for the firm to keep operating following a loss by arranging for the immediate use of alternative facilities at higher operating costs. The resulting exposure to higher costs is known as the **extra expense exposure**. Insurance purchased to reimburse the firm for these higher costs is known as *extra expense coverage*.

Liability Losses

As we analyze in detail in later chapters, firms face potential legal liability losses as a result of relationships with many parties, including suppliers, customers, employees, shareholders, and members of the public. The settlements, judgments, and legal costs associated with liability suits can impose substantial losses on firms. Lawsuits also may harm firms by damaging their reputation, and they may require expenditures to minimize the costs of this damage. For example, in the case of liability to customers for injuries arising out of the firm's products, the firm might incur product recall expenses and higher marketing costs to rehabilitate a product.

Losses to Human Resources

Losses in firm value due to worker injuries, disabilities, death, retirement, and turnover can be grouped into two categories. First, as a result of contractual commitments and compulsory benefits, firms often compensate employees (or their beneficiaries) for injuries, dis-

¹As noted in Chapter 10 property insurance policies can cover either the replacement cost or the *actual cash value* of the property. Actual cash value commonly is defined as replacement cost new less depreciation. A substantial number of court cases deal with disagreements over what this means. In many cases, actual cash value is treated as equivalent to market value. However, some court decisions might allow a corporation to argue that actual cash value equals firm-specific value if this is greater than the market value.

abilities, death, and retirement. Second, worker injuries, disabilities, death, retirement, and turnover can cause indirect losses when production is interrupted and employees cannot be replaced at zero cost with other employees of the same quality. In some cases, firms purchase life insurance to compensate for the death or disability of important employees. Also, as the discussion of pension benefits in Chapter 18 will show, employment contracts can be designed to reduce employee turnover.

Losses from External Economic Forces

The final category of losses arises from factors that are outside of the firm. Losses can arise because of changes in the prices of inputs and outputs. For example, increases in the price of oil can cause large losses to firms that use oil in the production process. Large changes in the exchange rate between currencies can increase a multinational firm's costs or decrease its revenues. As another example, an important supplier or purchaser can go bankrupt, thus increasing costs or decreasing revenues. We discuss how some of these types of losses can be managed using derivative contracts in later chapters.

Identifying Individual Exposures

One method of identifying individual/family exposures is to analyze the sources and uses of funds in the present and planned for the future. Potential events that cause decreases in the availability of funds or increases in uses of funds represent risk exposures (see Box 3.1). Because both physical and financial assets represent potential future sources of funds, potential losses in asset values also represent risk exposures. Just as business risk management consultants can aid in the identification of business risks, individual/family financial planners can help identify and then manage personal risks.

An important risk for most families is a drop in earnings prior to retirement due to the death or disability of a breadwinner. The magnitude of this risk depends, among other factors, on the number and age of dependents and on alternative sources of income (e.g., a spouse's income or investment income). The losses due to death or disability can be managed with life and disability insurance. The risk of a drop in earnings prior to retirement due to external economic factors is also an important risk facing households. Private methods for dealing with this risk, except for perhaps investments in education, are limited. Some public support often is available in the form of compulsory social insurance and unemployment insurance programs.

One of the most important sources of risk for most individuals and families is from medical expenses. The methods of dealing with this risk vary across countries. Some countries, like the United States, rely largely on the private medical and insurance industry to provide or pay for services and insurance to deal with medical expense risk. Other countries, such as Canada and the United Kingdom, rely more on government provision of medical services and insurance.

Another major source of expense risk is from personal liability exposures. Individuals can be sued and held liable for damages inflicted on others. The main sources of personal liability arise from driving an automobile and owning property with potential hazards. These risks are typically managed by using loss control and purchasing liability insurance.

Retirement often implies a large drop in earnings. To continue to pay living expenses during retirement, an individual needs to have saved substantial funds prior to retirement and/or rely on public programs, such as social security. The risk associated with pre-retirement

Consider some of the risks that you face during a semester as a student. The obvious risks are that you could become ill or injured, you could have an automobile accident, your residence could burn down, your vehicle could be stolen, and so on. A common aspect of these risks is that insurance contracts generally exist to help you manage the risk. In addition, you could reduce your exposure to the risk by taking additional precautions or by avoiding the activity that gives rise to the risk.

Consider some other risks that you face: You could buy food that is contaminated, you could purchase a product that causes an accident, or your bank could fail. A common aspect of these risks is that some type of government or social policy exists to help you deal with the consequences. Notice that the existence of these social policies lessens the extent to which you will deal with them privately, either by purchasing insurance or by taking additional precautions.

You also are exposed to many other risks where neither insurance contracts nor public programs exist to help you. For example, a sibling could die, causing you emotional distress. Your teacher could give a very diffi-

cult exam, or you could forget a fundamental concept—so that in either case you bomb the exam, causing your grade point average to suffer. Alternatively, your best friend could decide to avoid you forever. Generally, the only way to deal with these risks is to engage in some loss control activity (e.g., studying more often) that will reduce either the chance of the loss occurring or the size of the loss if it does occur.

The pervasiveness of risk is apparent. The optimal response to risk from a business's or an individual's perspective is one of the central issues addressed in this book. In addition, we will provide answers to other interesting and important questions, such as: Why do insurance contracts exist for some, but not all risks? Why do we have government programs to lessen some types of risk? What are the effects of these programs on individual behavior? Answers to these questions and many others require a framework in which to analyze risky situations. The framework we use is based on some fundamental concepts from probability and statistics, which are presented in the subsequent sections of this chapter.

savings and thus the risk of not having sufficient assets during retirement to fund expenses depends on how the assets are invested. The choice of assets, (for example, between stocks, bonds, and real estate) is an important risk management decision for all individuals and households. Even after someone has retired with substantial assets, the person faces the risk of living so long all savings are depleted prior to death. This longevity risk can be managed using annuities, including government mandated annuities, such as those provided in the U.S. social security system.

3.2 Basic Concepts from Probability and Statistics

Risk assessment and measurement require a basic understanding of several concepts from probability and statistics. We review these concepts in this section. These concepts also are needed to understand much of the material in subsequent chapters.

Random Variables and Probability Distributions

A **random variable** is a variable whose outcome is uncertain. For example, suppose a coin is to be flipped and the variable X is defined to be equal to \$1 if heads appears and $-\$1$ if tails appears. Then prior to the coin flip, the value of X is unknown; that is, X is a random variable. Once the coin has been flipped and the outcome revealed, the uncertainty about X is resolved, because the value of X is then known.

TABLE 3.2
Probability
distribution for
coin flipping
example.

FIGURE 3.1
Probability
distribution
for coin
flipping
example.

Information about a random variable is summarized by the random variable's probability distribution. In particular, a **probability distribution** identifies all the possible outcomes for the random variable and the probability of the outcomes. For the coin flipping example, Table 3.2 gives the probability distribution for X .

In addition to describing a probability distribution by listing the outcomes and probabilities, we also can describe probability distributions graphically. Figure 3.1 illustrates the probability distribution for the coin flipping example. On the horizontal axis, we graph the possible outcomes. On the vertical axis, we graph the probability of a particular outcome. There are only two possible outcomes in this very simple example: \$1 and -\$1, and the probability of each is 0.5. When discussing random variables, we use the term *actual* or *observed* outcome (or, sometimes *realized* outcome) to refer to the outcome observed (realized) in a particular case, as opposed to the *possible* outcomes that could have occurred. In the coin flipping example, once the coin has been tossed we can observe the actual outcome, which either must be \$1 or -\$1.

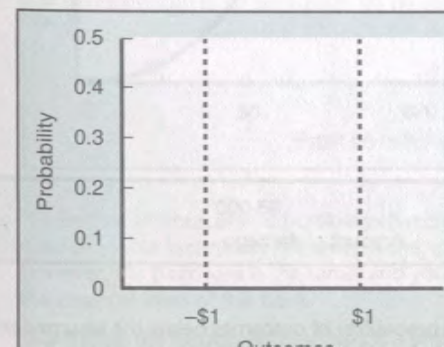
As emphasized in the first two chapters, risk management decisions need to be made prior to knowing what the actual (realized) outcomes of key variables will be. Managers do not know beforehand which outcomes of the random variables affecting the firm's profits will occur. Nevertheless, they must make decisions. Once the outcomes are observed, it usually is easy to say what would have been the best decision. However, we cannot evaluate decisions from this perspective, which is why probability distributions are so important. Probability distributions tell us all of the possible outcomes and the probability of those outcomes. Information about probability distributions is needed to make good risk management decisions.

As a second example of a probability distribution, we can approximate the probability distribution for the dollar amount of damages to your car during the coming year. For simplicity, our approximation will assume only five possible levels of damages: \$0; \$500;

TABLE 3.2
Probability
distribution for
coin flipping
example.

Possible Outcomes for X	Probability
\$1	0.5 or 50%
-\$1	0.5 or 50%

FIGURE 3.1
Probability
distribution
for coin
flipping
example.



\$1,000; \$5,000; and \$10,000. The probabilities of each of these outcomes are listed in Table 3.3. The most likely outcome is zero damages, and the least likely outcome is that damages equal \$10,000. Note that the sum of the probabilities equals 1; this must always be the case. An alternative way of describing the probability distribution is provided by Figure 3.2, where the height of each dotted line gives the probability of each possible outcome.

As a final example, consider an automaker. Two of the many reasons why the automaker's profits are uncertain are steel price changes and labor conditions. In the language just introduced, the automaker's profits are a random variable. There are numerous possible outcomes for the automaker's profits. For example, steel prices could increase so much that profits could be negative. On the other hand, favorable outcomes for steel prices and the economy could cause very high profits.

What is the probability distribution for the automaker's profits? Recall that a probability distribution identifies all of the possible outcomes and associates a probability with each outcome. The coin flipping example had only two possible outcomes and so listing the probabilities was simple. In the automaker example, however, we could spend hours listing all the possible outcomes for profits and still not be finished, due to the large number of possible outcomes. In these situations, it is useful to assume that the possible outcomes can be *any* number between two extremes (the minimum possible outcome and the maximum possible outcome) and that the probability of the outcomes between the extremes is represented by a specific mathematical function.² For example, assume that profits for the automaker

Table 3.3
Probability
distribution for
automobile
damages.

Possible Outcomes for Damages	Probability
\$ 0	0.50
\$ 500	0.30
\$ 1,000	0.10
\$ 5,000	0.06
\$10,000	0.04

FIGURE 3.2 Probability distribution for automobile damages.

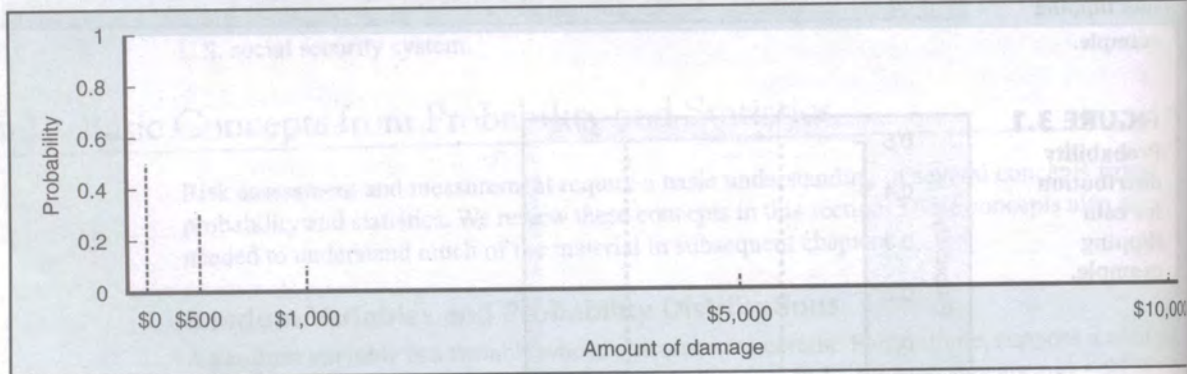


FIGURE 3.3
Probability
distribution
for
automaker's
profits.

²This is equivalent to assuming that the probability of outcomes below the assumed minimum or above the assumed maximum is so small that these outcomes can be ignored.

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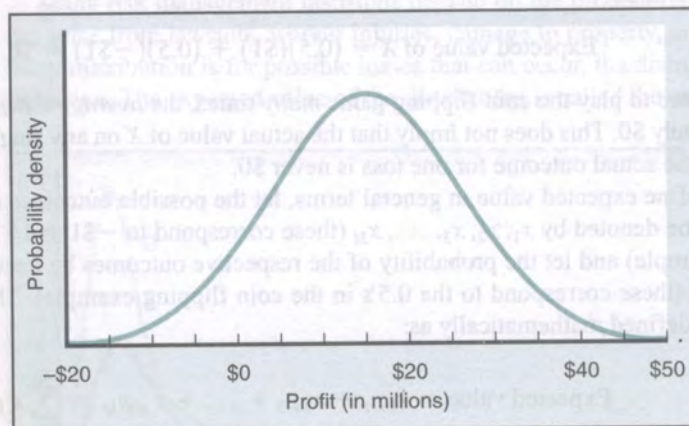
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could be any number between $-\$20$ million and $\$50$ million. Just as with the earlier graphs, we can identify the possible outcomes for profits between these amounts on the horizontal axis of Figure 3.3, which illustrates the probability distribution for the automaker's profits. Analogous to the earlier graphs, the vertical axis will measure the probability of the possible outcomes.³ The probabilities of the outcomes are illustrated in Figure 3.3 by a bell-shaped curve, which might appear familiar to you.

Recall that the sum of the probabilities of all the possible outcomes must equal 1 (some outcome must occur). In the coin flipping example and the automobile damage example, this property is easy to verify because the number of possible outcomes is small. Stating that the probabilities sum to 1 in these examples is equivalent to stating that the heights of the dotted lines in Figures 3.1 and 3.2 sum to 1. This is a useful observation because it helps to illustrate the analogous property in the automaker example, where any outcome between $-\$20$ million and $\$50$ million is possible. You can think of the curve in Figure 3.3 as a curve that connects the tops of many thousands of bars that have very small widths, and the sum of the heights of all these bars is equivalent to the area under the curve.⁴ Thus, stating that the probabilities must sum to 1 is equivalent to stating that the area under the curve must equal 1.

Since the area under the curve in Figure 3.3 equals 1, we can graphically identify the probability that profits are within a certain interval. For example, the probability that profits are greater than $\$40$ million is the area under the curve to the right of $\$40$ million. The probability that profits are less than $\$0$ is the area under the curve to the left of $\$0$. The probability that profits are between $\$10$ and $\$30$ million is the area under the curve between $\$10$ and $\$30$ million. Thus, the bell-shaped curve in Figure 3.3 tells us that for the automaker, there is a relatively high probability that profits will be between $\$10$ and $\$30$ million. In contrast, while very low profits and very high profits are possible, they do not have a high probability of happening.

FIGURE 3.3
Probability
distribution
for
automaker's
profits.



³Given that any outcome is possible between $-\$20$ million and $\$50$ million, the vertical axis measures what technically is known as the "probability density," rather than the probability. However, the basic idea is the same, and you can think of it as the probability in order to understand the essential ideas of this book.

⁴Adding up the heights of these bars is a problem in calculus, which is not needed for understanding the material in this book.

Concept Checks

1. What information is given by a probability distribution? What are the two ways of describing a probability distribution?
2. Earthquakes are rare, but the property damage can be very large when they occur. Illustrate these features by drawing a probability distribution for property losses due to an earthquake for a business that has property valued at \$50 million. Identify on your graph the probability that losses will exceed \$30 million.

Characteristics of Probability Distributions

In many applications, it is necessary to compare probability distributions of different random variables. Indeed, most of the material in this book is concerned with how decisions (e.g., whether to purchase insurance) change probability distributions. Understanding how decisions affect probability distributions will lead to better decisions. The problem is that most probability distributions have many different outcomes and are difficult to compare. It is therefore common to compare certain key characteristics of probability distributions: the expected value, variance or standard deviation, skewness, and correlation.

Expected Value

The **expected value** of a probability distribution provides information about where the outcomes tend to occur, on average. For example, if the expected value of the automaker's profits is \$10 million, then profits should average about \$10 million. Thus, a distribution with a higher expected value will tend to have a higher outcome, on average.

To calculate the expected value, you multiply each possible outcome by its probability and then add up the results. In the coin flipping example there are two possible outcomes for X , either \$1 or -\$1. The probability of each outcome is 0.5. Therefore, the expected value of X is \$0:

$$\text{Expected value of } X = (0.5)(\$1) + (0.5)(-\$1) = \$0$$

If one were to play the coin flipping game *many* times, the *average* outcome would be approximately \$0. This does not imply that the actual value of X on any single toss will be \$0; indeed, the actual outcome for one toss is never \$0.

To define expected value in general terms, let the possible outcomes of a random variable, X , be denoted by $x_1, x_2, x_3, \dots, x_M$ (these correspond to -\$1 and \$1 in the coin flipping example) and let the probability of the respective outcomes be denoted by $p_1, p_2, p_3, \dots, p_M$ (these correspond to the 0.5's in the coin flipping example). Then, the expected value is defined mathematically as:

$$\text{Expected value} = x_1p_1 + x_2p_2 + \dots + x_Mp_M = \sum_{i=1}^M x_i p_i \quad (3.1)$$

If we examine a probability distribution graphically, we often can learn something about the expected value of the distribution. For example, Figure 3.4 illustrates two probability distributions. Since the distribution for A is shifted to the right compared with B, distribution A has a higher expected value than distribution B.

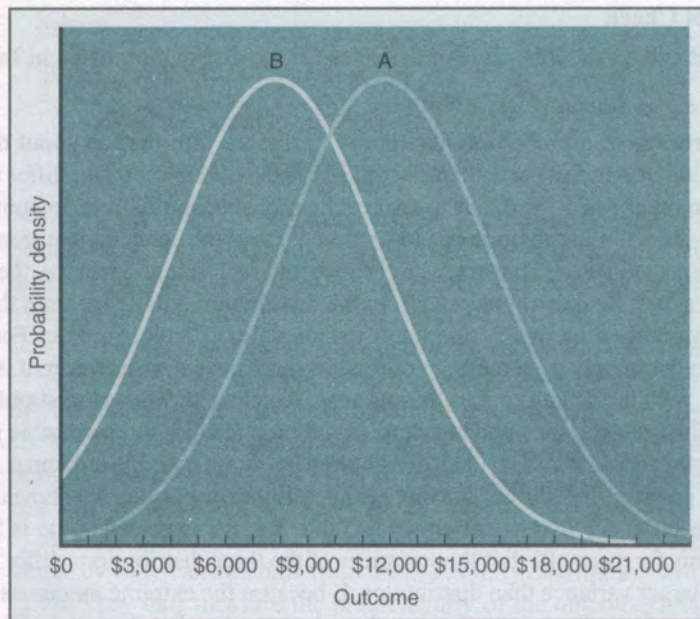
When distributions are symmetric, as in Figure 3.4, identifying the expected value is relatively easy; it is the midpoint in the range of possible outcomes. When the probability distri-

FIGURE 3.4
Comparing the expected values of two distributions (distribution A has a higher expected value than distribution B).

FIGURE 3.5
Comparing expected values of distributions (distribution C has a higher expected value than distribution D).

FIGURE 3.4

Comparing the expected values of two distributions (distribution A has a higher expected value than distribution B).

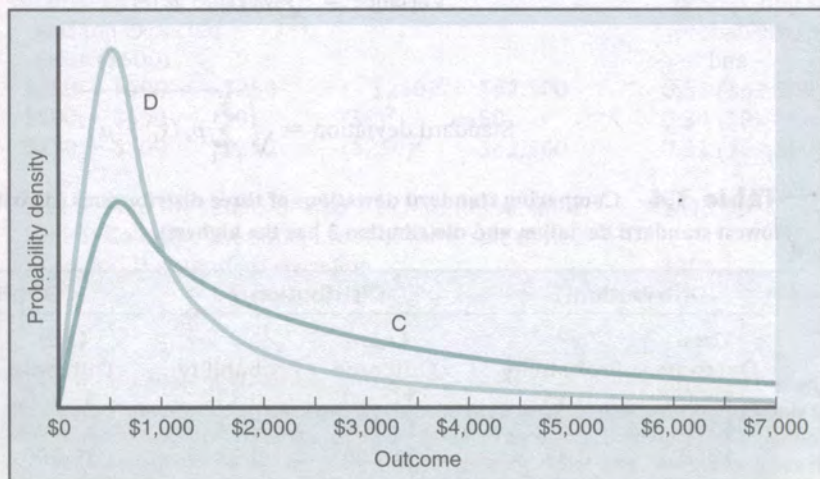


butions are not symmetric, identifying the expected value by examining a diagram sometimes can be difficult. Nevertheless, you often can compare the expected values of different distributions visually. Consider, for example, the two distributions illustrated in Figure 3.5. Distribution C has a higher expected value than distribution D. Intuitively, the high outcomes are more likely with distribution C than with D, and the low outcomes are less likely with C than with D.

Many risk management decisions depend on the probability distribution of losses that can arise from lawsuits, worker injuries, damage to property, and the like. When a probability distribution is for possible losses that can occur, the distribution is called a **loss distribution**. The expected value of the distribution is called the **expected loss**.

FIGURE 3.5

Comparing expected values of distributions (distribution C has a higher expected value than distribution D).



Concept Check

3. What is the expected value of damages for the distribution listed in Table 3.3?

Variance and Standard Deviation

The **variance** of a probability distribution provides information about the likelihood and magnitude by which a particular outcome from the distribution will differ from the expected value. In other words, variance measures the probable variation in outcomes around the expected value. If a distribution has low variance, then the actual outcome is likely to be close to the expected value. Conversely, if the distribution has high variance, then it is more likely that the actual (realized) outcome from the distribution will be far from the expected value. A high variance therefore implies that outcomes are difficult to predict. For this reason, variance is a commonly used measure of risk. In some instances, however, it is more convenient to work with the square root of the variance, which is known as the **standard deviation**.

To illustrate variance and standard deviation, consider three possible probability distributions for accident losses. Each distribution has three possible outcomes, but the outcomes and the probabilities differ. The three probability distributions are shown in Table 3.4.

For each of the loss distributions in Table 3.4, the expected value is \$500 (you should verify this for yourself), but the variances of the three distributions differ. Loss distribution 2 has a larger variance than distribution 1, because the extreme outcomes for distribution 2 are farther from the expected value than they are for distribution 1. Distribution 3 has a larger variance than distribution 2, because even though the outcomes are the same for distributions 2 and 3, the extreme outcomes are more likely with distribution 3 than with distribution 2. That is, the probability of having a loss far from the expected value (\$500) is greater with distribution 3 than with distribution 2. The comparison of distributions 2 and 3 illustrates that the variance depends not only on the dispersion of the possible outcomes but also on the probability of the possible outcomes.

The mathematical definitions of variance and standard deviation show precisely how the probabilities of the different outcomes and the deviation of each outcome from the expected value affect these measures of risk. The definitions are:

$$\text{Variance} = \sum_{i=1}^N p_i (x_i - \mu)^2 \quad (3.2)$$

and

$$\text{Standard deviation} = \sqrt{\sum_{i=1}^N p_i (x_i - \mu)^2} \quad (3.3)$$

Table 3.4 Comparing standard deviations of three distributions (distribution 1 has the lowest standard deviation and distribution 3 has the highest).

Distribution 1		Distribution 2		Distribution 3	
Loss Outcome	Probability	Loss Outcome	Probability	Loss Outcome	Probability
\$250	0.33	\$ 0	0.33	\$ 0	0.4
\$500	0.34	\$ 500	0.34	\$ 500	0.2
\$750	0.33	\$1,000	0.33	\$1,000	0.4

Table 3.5 Calculating variance and standard deviation for distribution 1 from Table 3.4.

where

$\mu(\text{mu})$ = the expected value;

x_i = the possible outcome; and

p_i = the probability of the outcome.

Notice that the quantity in parentheses measures the deviation of each outcome from the expected value. This difference is squared so that positive differences do not offset negative differences. Each squared difference is then multiplied by the probability of the particular outcome so those outcomes that are more likely to occur receive greater weight in the final sum than those outcomes that have a low probability of occurrence.

Additional insights about these measures of risk can be gained by going step-by-step through the calculations for distribution 1 introduced above. Table 3.5 provides this analysis. It indicates that distribution 1 has a standard deviation equal to \$204. Similar calculations for distributions 2 and 3 (not shown) indicate that their standard deviations equal \$408 and \$447, respectively.

As noted earlier, variance and standard deviation measure the likelihood that and magnitude by which an outcome from the probability distribution will deviate from the expected value. They thus measure the predictability of the outcomes. As a consequence, when referring to risk as variability around the expected value, *we generally will measure risk using variance or standard deviation.*⁵

Like expected values, standard deviations of distributions often can be compared by visually inspecting the probability distributions. For example, Figure 3.6 illustrates two distributions for accident losses. Both have an expected value of \$1,000, but they differ in their standard deviations. There is a greater chance that an outcome from distribution A will be close to the expected value of \$1,000 than with distribution B.

Table 3.5

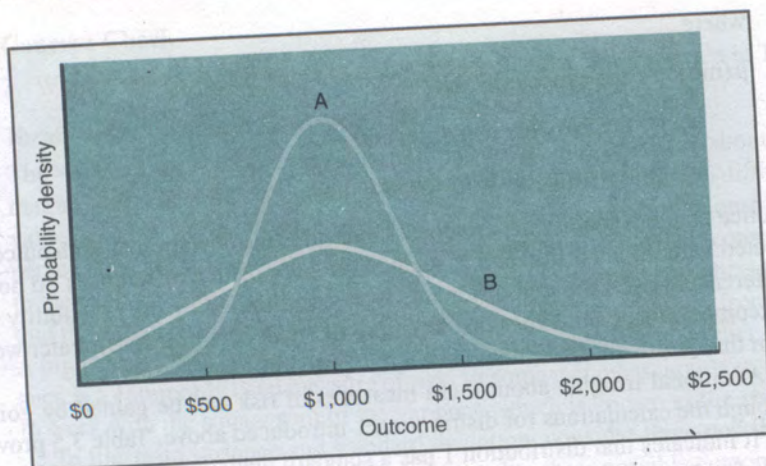
Calculating variance and standard deviation for distribution 1 from Table 3.4.

Step 1: Take difference between each outcome and the expected value (\$500).	Step 2: Square the results of step 1.	Step 3: Multiply the results of step 2 by the respective probabilities.
$\$250 - \$500 = -\$250$	$(-\$250)^2 = \$62,500$	$0.33 (\$62,500) = \$20,833$
$\$500 - \$500 = \$0$	$(\$0)^2 = \$0$	$0.34 (\$0) = \$0$
$\$750 - \$500 = \$250$	$(\$250)^2 = \$62,500$	$0.33 (\$62,500) = \$20,833$
Step 4: Sum the results of step 3 to find the variance.		\$41,666
Step 5: Calculate the square root of the result of step 4 to find the standard deviation.		\$204

⁵Other measures of risk sometimes are used. For example, in some situations it is useful to measure risk as the probability of an extreme outcome (e.g., a large loss). Another commonly used measure of risk is the maximum probable loss or value at risk, both of which identify the loss amount that will not be exceeded with some confidence, say 95 percent of the time. We define these risk concepts later in this chapter.

FIGURE 3.6

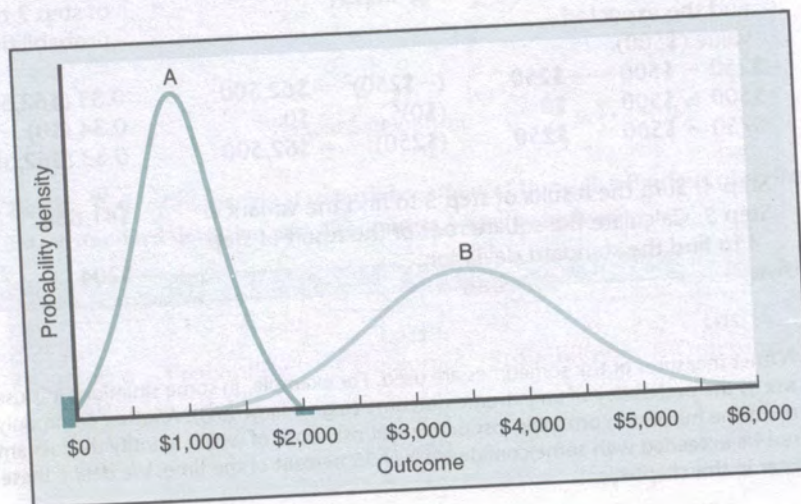
Comparing the standard deviations of two distributions (distribution B has a larger standard deviation).

**Concept Checks**

4. Explain why variance and standard deviation are useful measures of risk.
5. Without doing any calculations, can you compare the standard deviations of the following distributions?

Distribution 1		Distribution 2		Distribution 3	
Loss Outcome	Probability	Loss Outcome	Probability	Loss Outcome	Probability
\$ 5,000	0.33	\$ 5,000	0.00	\$ 0	0.2
\$10,000	0.34	\$10,000	1.00	\$10,000	0.6
\$15,000	0.33	\$15,000	0.00	\$20,000	0.2

6. Compare the expected values and standard deviations of distributions A and B illustrated in the following figure:



Sample Mean and Sample Standard Deviation

Sometimes the expected value is called the *mean of the distribution*. We avoid using this term because it leads to confusion with another concept: the average value from a sample of outcomes from a distribution, which also is known as the **sample mean**. A simple illustration will help you understand the difference between the average outcome from a sample (the sample mean) and the expected value of the probability distribution. Assume that there is a 0.5 probability that the fertilization of an egg will produce a female, and there is a 0.5 probability that the fertilization will produce a male.⁶ The group of babies born this month in the town where you live can be viewed as a sample from this distribution. The sample mean proportion of females is the number of females in the sample divided by the total number of newborns in the sample. The sample mean proportion generally will differ from the expected value of 0.5 due to random fluctuations (unless there are lots and lots of babies in the sample). Similarly, if the expected loss from accidents for a large group of people is \$500, the sample mean loss or **average loss** during a given time period for a sample of these people will differ from the expected value due to random fluctuations.

The **sample standard deviation** (or, similarly, the sample variance) reflects the variation in outcomes of a particular sample from a distribution. It is calculated with the same formula that we used above for the standard deviation but with three differences. First, only the outcomes that occur in the sample are used. Second, the sample mean is used instead of the expected value, which usually is not known. Third, the squared deviations between the outcomes and the sample mean are multiplied by the proportion of times that the particular outcome actually occurs in the sample—rather than by the proportion of times that the outcome is likely to occur, according to the probability distribution.⁷

It is useful to introduce the sample mean and sample standard deviation at this point for several reasons. First, the probability distributions for random variables that concern managers generally are not known. The sample mean and sample standard deviation sometimes can be used to estimate the unknown expected value and standard deviation of a probability distribution. Thus, estimation of the expected value and standard deviation of losses is often very important in risk management. In addition, the concept of the average loss for a group of people that pools its risk (i.e., the sample mean loss for the group) and the standard deviation of the average loss for the group (i.e., the sample standard deviation) are used in Chapter 4 to explain how pooling can reduce risk. Finally, you will no doubt calculate sample means and sample standard deviations if you take a statistics course. We don't want you to confuse the expected value and standard deviation of the underlying probability distribution with the sample mean and sample standard deviation for a particular sample.

Concept Check

- Recall the coin flipping game discussed earlier in the chapter where you win \$1 if heads appears and lose \$1 if tails appears. What is the expected value of the outcome from the

⁶Actually, evidence suggests that the probability that a female will be conceived is very slightly greater than 0.5.

⁷This calculation is equivalent to adding the squared deviations, dividing by the sample size, and then taking the square root. (In many cases, statisticians divide by the sample size minus one instead. This adjustment causes the sample standard deviation to be a better [unbiased] estimator of the true standard deviation.)

game if it is played only one time? Calculate the sample mean and sample standard deviation if the game is played five times with the following results: T, T, H, T, H.

Skewness

Another statistical concept that is important in the practice of risk management is the **skewness** of a probability distribution. Skewness measures the symmetry of the distribution. If the distribution is symmetric, it has no skewness. For example, consider the two distributions for accident losses illustrated in Figure 3.7. The distribution at the top of Figure 3.7 is symmetric; it has zero skewness. However, the distribution at the bottom is not symmetric; it has positive skewness. Many of the loss distributions that are relevant to risk management are skewed.

Note how the skewed distribution has a higher probability of very low losses and a higher probability of very high losses when compared to the symmetric distribution. Recognizing this characteristic of skewed distributions is important when assessing the likelihood of large losses. If you incorrectly assume that the loss distribution is symmetric (you

FIGURE 3.7

Skewness in probability distributions (top distribution is symmetric; bottom distribution is skewed).

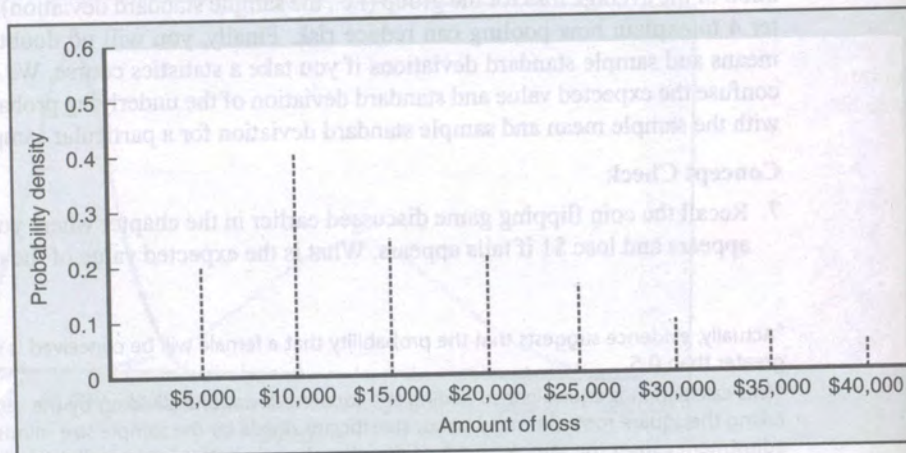
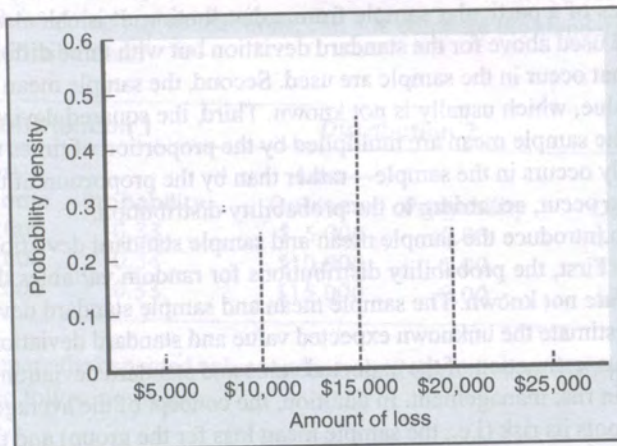
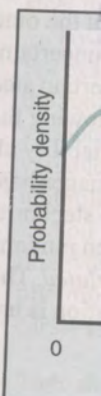


FIGURE 3.8

Maximum probable loss.



ample standard deviation, H, T, H, T, H.

ement is the skewness of the distribution. If you compare the two distributions in Figure 3.7, the distribution on the left is not symmetric; it is skewed to the right. This is a risk management

by low losses and a high probability of large losses. Recalculating the likelihood of a loss is symmetric (you

think that losses have distribution 1 when they really have distribution 2 in Figure 3.7), you will underestimate the likelihood of very large losses. As you will see in later chapters, large losses usually are the most harmful.

Concept Check

8. Draw a distribution that might describe your automobile liability losses for the coming year (i.e., the losses that you could cause to other people for which you could be sued and held liable).

Maximum Probable Loss and Value-at-Risk

A frequently used measure of risk is maximum probable loss or value-at-risk. Although used in different contexts, these terms essentially mean the same thing. **Maximum probable loss** usually describes a loss distribution, whereas **value-at-risk** describes the probability distribution for the value of a portfolio or the value of a firm subject to loss. These concepts are easily illustrated with simple examples.

Suppose that the probability distribution for annual liability losses is described by the probability density function in Figure 3.8. Since the random variable being described is losses, high values are bad and low values are good. If \$20 million is the maximum probable loss (MPL) at the 5 percent level, the probability that losses will be greater than \$20 million is 5 percent. (That is, the area under the probability density function to the right of \$20 million is 0.05.) If \$30 million is the MPL at the 1 percent level, the probability that losses will be greater than \$30 million is 0.01.

To illustrate value-at-risk, consider the probability distribution for the change in the value of an investment portfolio over a month depicted in Figure 3.9. Since the random variable being described is portfolio value changes, high values are good and low values are bad. If \$5 million is the monthly value-at-risk for this portfolio at the 5 percent level, the probability that the portfolio will lose more than \$5 million over the month is 5 percent. (The area under the density function to the left of $-\$5$ million is 0.05.) If \$7.5 million is the monthly value-at-risk at the 1 percent level, the probability that the portfolio will lose more than \$7.5 million over the month is 0.01.

FIGURE 3.8
Maximum probable loss.

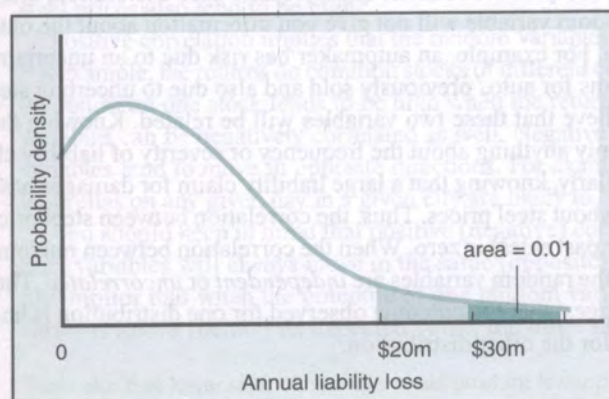
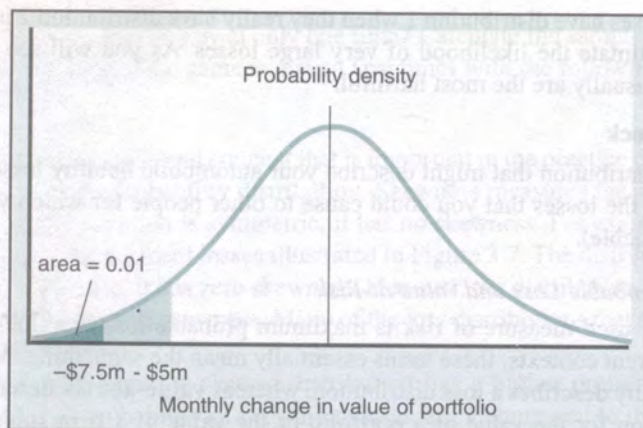


FIGURE 3.9
Value-at-risk.



Many large corporations estimate maximum probable losses from different exposures to evaluate risk. Most large financial institutions calculate a daily measure of value-at-risk.⁸ To illustrate this concept, suppose that Mr. David, the risk manager at First Babbel Corp., receives a report that the firm's daily value-at-risk at the 5 percent level is \$50 million. This number tells Mr. David that the firm has a 5 percent chance of losing more than \$50 million over the coming day. If Mr. David determines that the firm should not take this much risk, he might take actions to reduce the firm's value-at-risk, such as hedging or selling some risky assets. After taking these risk management actions, presumably the firm's value at risk would drop to an acceptable level. See Box 3.2.

Correlation

To this point, we have limited our discussion to probability distributions of a single random variable. Because businesses and individuals are exposed to many types of risk, it is important to identify the relationships among random variables. The **correlation** between random variables measures how random variables are related.

If the correlation between two random variables is zero, then the random variables are not related. Intuitively, if two random variables have zero correlation, then knowing the outcome of one random variable will not give you information about the outcome of the other random variable. For example, an automaker has risk due to an uncertain number of product liability claims for autos previously sold and also due to uncertain steel prices. There is no reason to believe that these two variables will be related. Knowing that steel prices are high will not imply anything about the frequency or severity of liability claims for autos already sold. Similarly, knowing that a large liability claim for damages has occurred will not imply anything about steel prices. Thus, the correlation between steel prices and product liability costs (for past sales) is zero. When the correlation between random variables is zero, we will say that the random variables are *independent* or *uncorrelated*. These terms are used because they suggest that the outcome observed for one distribution is unrelated to the outcome observed for the other distribution.

⁸We illustrate some of the tools for estimating maximum probable loss and value-at-risk, such as Monte Carlo simulation, later in the book.

Character

One of the most frequent functions of the normal distribution is the normal distribution is a symmetric, bell-shaped curve. The normal distribution is used to model returns on financial assets. In many chapters, it is used to model many individual, uncorrelated random variables.

The following properties are useful for calculating the probability of a variable loss) if changes in value are normally distributed. If the variable has an expected value of μ , then

Probability ($X > \mu + z\sigma$)

Probability ($X < \mu - z\sigma$)

Probability ($X > \mu + z\sigma$)

Probability ($X < \mu - z\sigma$)

Figure 3.10 illustrates the changes in a portfolio.

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Characteristics of the Normal Distribution

3.2

One of the most frequently used probability distributions is the normal distribution. The probability density function of the normal distribution is the familiar symmetric, bell-shaped curve illustrated in Figure 3.10. The normal distribution is frequently used to describe the returns on financial assets and, as you will see in later chapters, it is used to describe the average loss from many individual, uncorrelated exposures.

The following properties of the normal distribution are useful for calculating value-at-risk (maximum probable loss) if changes in value (losses) are assumed to be normally distributed. If X is normally distributed with an expected value of μ and standard deviation of σ , then

$$\text{Probability } (X > \mu + 2.33\sigma) = 0.01 \\ \text{and}$$

$$\text{Probability } (X < \mu - 2.33\sigma) = 0.01$$

$$\text{Probability } (X > \mu + 1.645\sigma) = 0.05 \\ \text{and}$$

$$\text{Probability } (X < \mu - 1.645\sigma) = 0.05$$

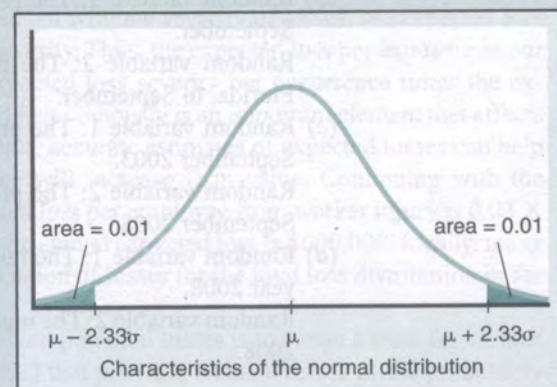
Figure 3.10 illustrates the relationship. If, for example, the changes in a portfolio's value over the coming

day are normally distributed with mean of \$0 and standard deviation of \$10 million, then,

$$\text{Probability [change in value} < \$0 - 2.33 (\$10 \text{ million})] = 0.01.$$

That is, the probability that the portfolio will drop in value by more than \$23.3 million is 1 percent.

FIGURE 3.10 Characteristics of the normal distribution.



In many cases random variables will be correlated. For example, a recession may decrease the demand for new cars and also decrease steel prices. Thus, the demand for new cars and steel prices both are affected by general economic conditions, and as a result, the demand for new cars and steel prices are correlated. When demand for new cars is high, steel prices also tend to be high.⁹

Positive correlation implies that the random variables tend to move in the same direction. For example, the returns on common stocks of different companies are positively correlated—the return on one stock tends to be high when the returns on other stocks are high. Random variables can be negatively correlated as well. Negative correlation implies that the random variables tend to move in opposite directions. For example, sales of sunglasses and sales of umbrellas on any given day in a given city are likely to be negatively correlated.

You should keep in mind that positive (negative) correlation does not imply that the random variables will always move in the same (opposite) direction. Positive correlation simply implies that when the outcome of one random variable—for example, the demand for cars—is above (below) its expected value, the other random variable—for example, steel

⁹Note also that lower sales of new cars could produce fewer product liability claims in the future. Thus, while steel prices and the number of liability claims for autos previously sold will likely be uncorrelated, liability claims arising from new sales and steel prices will likely be correlated.

costs—tends to be above (below) its expected value. Similarly, negative correlation implies that when one random variable—for example, sales of sunglasses—is above (below) its expected value, the other random variable—for example, umbrella sales—tends to be below (above) its expected value.

Concept Check

9. For each scenario below, explain whether the correlation between random variable 1 and random variable 2 is likely to be zero (the random variables are uncorrelated), positive, or negative.
 - (a) Random variable 1: Your automobile accident costs for the coming year.
Random variable 2: The automobile accident costs of a student in another country for the coming year.
 - (b) Random variable 1: The property damage due to hurricanes in Miami, Florida, in September.
Random variable 2: The property damage due to hurricanes in Ft. Lauderdale, Florida, in September.
 - (c) Random variable 1: The property damage due to hurricanes in Miami, Florida, in September 2003.
Random variable 2: The property damage due to hurricanes in Miami, Florida, in September 2008.
 - (d) Random variable 1: The number of people in New York who die from AIDS in the year 2008.
Random variable 2: The number of people in London who die from AIDS in the year 2008.

3.3 Evaluating the Frequency and Severity of Losses

After identifying loss exposures, a risk manager ideally would obtain information about the entire probability distribution of losses and how different risk management methods affect this distribution. We illustrate how larger firms might estimate the relevant loss distributions in Chapter 26. Frequently, risk managers use summary measures of probability distributions, such as frequency and severity measures, as well as expected losses and the standard deviation of losses during a given period. These measures help a risk manager assess the costs and benefits of loss control and retention versus insurance. We therefore illustrate how these summary measures can be obtained in practice.

Frequency

The **frequency** of loss measures the number of losses in a given period of time. If historical data exist on a large number of exposures, then the probability of a loss per exposure (or the expected frequency per exposure) can be estimated by the number of losses divided by the number of exposures. For example, if Sharon Steel Corp. had 10,000 employees in each of the past five years and over the five-year period there were 1,500 workers injured, then an estimate of the probability of a particular worker becoming injured would be 0.03 per year (1,500 injuries/50,000 employee-years). When historical data do not exist for a firm, frequency of losses can be difficult to quantify. In this case, industry data might be used, or an informed judgment would need to be made about the frequency of losses.

Table 3.6
Categorization
of Sharon
Steel's
property losses.

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Severity

The **severity** of loss measures the magnitude of loss per occurrence. One way to estimate expected severity is to use the average severity of loss per occurrence during a historical period. If the 1,500 worker injuries for Sharon Steel cost \$3 million in total (adjusted for inflation), then the expected severity of worker injuries would be estimated at \$2,000 ($\$3,000,000/1,500$). That is, on average, each worker injury imposed a \$2,000 loss on the firm. Again due to the lack of historical data and the infrequency of losses, adequate data may not be available to estimate precisely the expected severity per occurrence. With a little effort, however, risk managers can estimate the range of possible loss severity (minimum and maximum loss) for a given exposure.

Expected Loss and Standard Deviation

When the frequency of losses is uncorrelated with the severity of losses, the expected loss is simply the product of frequency and severity. Thus, the expected loss per exposure in our example can be estimated by taking expected loss severity per occurrence times the expected frequency per exposure. Expected loss obviously is an important element that affects business value and insurance pricing. Thus, accurate estimates of expected losses can help a manager determine whether insurance will increase firm value. Continuing with the Sharon Steel example, the annual expected loss per employee from worker injury is $0.03 \times \$2,000 = \$60$. With 10,000 employees, the annual expected loss is \$600,000. Ideally, many firms also will estimate the standard deviation of losses for the total loss distribution or for losses in different size ranges.

One way to summarize information about potential losses is to create a table for various types of exposures (property, liability, etc.) that provides characteristics of the probability distribution of losses for the particular type of exposure. An example for Sharon Steel's property exposures is provided in Table 3.6.

To create an accurate categorization of a firm's loss exposures (like Table 3.6), considerable information, time, and expertise are needed. For most companies, especially smaller ones and new ones, detailed data on loss exposures do not exist. Nevertheless, the framework of Table 3.6 still can be used. For example, each type of exposure can be classified as having low, medium, or high frequency and severity. Table 3.7 provides an example for Penn Steel Corp., a firm that is engaged in the same activities and is of the same size as Sharon Steel Corp.

Tables 3.6 and 3.7 both show that the standard deviation of losses for high frequency, low severity losses is low, while the standard deviation is high for low frequency losses with

Table 3.6
Categorization
of Sharon
Steel's
property losses.

Property Exposures	Frequency of Losses per Year	Severity Range	Average Severity	Expected Loss	Standard Deviation
Damage to automobiles	100	\$0–\$20,000	\$5,000	\$500,000	\$100,000
Stolen property	200	0–2,000	500	100,000	20,000
Small fires	1	100,000–500,000	125,000	125,000	400,000
Major fires	.05	500,000–10,000,000	2,000,000	100,000	800,000

Table 3.7
Categorization
of Penn Steel
Corp.'s
property losses.

Property Exposures	Frequency	Severity Range	Average Severity	Expected Loss	Standard Deviation
Damage to automobiles	Medium	\$0–\$20,000	Low	Medium	Medium
Stolen property	High	0–2,000	Low	Low	Low
Small fires	Low	100,000–500,000	Medium	Low	High
Major fires	Low	500,000–10,000,000	High	Low	High

high potential severity. This relationship is fairly general: Infrequent but potentially large losses are less predictable and pose greater risk than more frequent, smaller losses. Using the type of information illustrated in these tables, firms pay particular attention to exposures that can produce potentially large, disruptive losses, either from a single event or from the accumulation of a number of smaller but still significant losses during a given period.

3.4 Summary

- The risk management process begins with risk identification.
- Businesses typically identify their major property risks, liability risks, human resource risks, and risks arising from external economic events.
- Individuals typically identify their major earnings risks, expense risks, asset risks, and longevity risks.
- A probability distribution describes the possible outcomes and the probabilities of those outcomes for a random variable.
- The expected value of a probability distribution is the weighted average of the possible outcomes, where the weights are the probabilities.
- Standard deviation or variance is a measure of probable variation around the expected value of a probability distribution for a random variable and, thus, of the risk (unpredictability) of the variable.
- Skewness measures symmetry of a distribution. Many loss exposures have skewed probability distributions.

Key Terms

risk identification 30	probability distribution 35	sample standard deviation 43
book value 31	expected value 38	skewness 44
market value 32	loss distribution 39	maximum probable loss 45
firm-specific value 32	expected loss 39	value-at-risk 45
replacement cost new 32	variance 40	correlation 46
business income exposures 32	standard deviation 40	frequency 48
extra expense exposure 32	sample mean 43	severity 49
random variable 34	average loss 43	

Questions and

1. Suppose that property loss has the following

\$90,000 v

$L = \$10,000$ v

\$ 0 v

What is the losses (i.e., th

2. Suppose that to profits from beach and th bility distribu

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3. Assume that p Brewery have t

\$3,000,000 v

\$1,500,000 v

Loss = \$ 800,000 v

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4. Assume that Bu that its liability distribution:

Answers to Conc

1. A probability dis possible outcome those outcomes

ected ss	Standard Deviation
lium	Medium
	Low
	High
	High

Questions and Problems

- Suppose that L is a random variable equal to property losses from a hurricane and that L has the following probability distribution:

\$90,000 with probability 0.02

$L = \$10,000$ with probability 0.06

\$ 0 with probability 0.92

What is the expected value of hurricane losses (i.e., the expected loss)?

- Suppose that P is a random variable equal to profits from an ice cream stand at the beach and that P has the following probability distribution:

\$70,000 with probability 0.05

\$50,000 with probability 0.25

$P = \$30,000$ with probability 0.35

\$10,000 with probability 0.20

-\$10,000 with probability 0.15

What is the expected value of profits?

- Assume that property losses for Buckeye Brewery have the following distribution:

\$3,000,000 with probability 0.004

\$1,500,000 with probability 0.010

Loss = \$ 800,000 with probability 0.026

\$ 0 with probability 0.96

What is the expected value of property losses (i.e., the expected loss)?

- Assume that Buckeye Brewery determines that its liability losses have the following distribution:

\$5,000,000 with probability 0.004

\$1,500,000 with probability 0.025

Loss = \$ 500,000 with probability 0.030

\$ 0 with probability 0.941

What is the expected value of liability losses?

- Do you think that Buckeye Brewery's property losses are independent, positively correlated, or negatively correlated with its liability losses?
- Company Blue is located in Toronto and has property valued at \$5 million. Sketch a reasonable probability distribution of Company Blue's property losses.
- Company Red is located in Cincinnati, Ohio, and has property valued at \$5 million. Sketch a reasonable probability distribution for Company Red's property losses.
- Suppose that Company Blue buys Company Red and the new firm is called Big Red (not to be confused with Big Blue). Sketch a reasonable probability distribution for Big Red's property losses.
- Bell Curve, Inc., estimates the expected value and standard deviation of its total liability losses for the forthcoming year as \$10 million and \$3 million, respectively. If Bell Curve assumes that total losses have the normal distribution, what is the predicted maximum probable loss at the 95 percent level? At the 99 percent level?

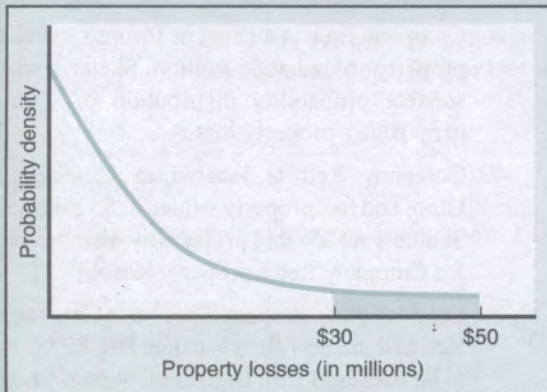
Answers to Concept Checks

- A probability distribution identifies all the possible outcomes and the probabilities of those outcomes for a particular random

variable. Simple probability distributions can be described by listing the possible outcomes and the corresponding probabilities.

Probability distributions also can be described graphically, with the possible outcomes listed on the horizontal axis and the probabilities of these outcomes measured on the vertical axis.

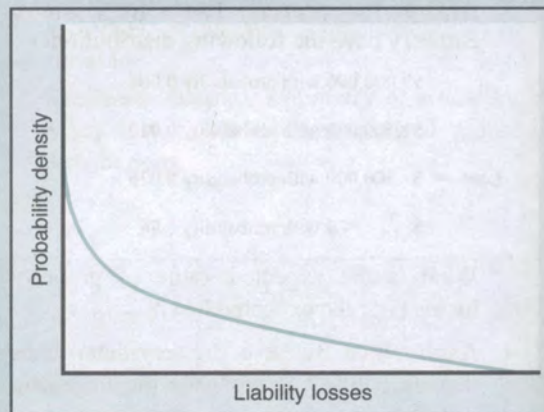
2. The following probability distribution indicates that the probability of low losses is relatively high, but that the probability of very high losses is relatively low. The maximum loss (ignoring indirect losses) is \$50 million. The shaded area is the probability that losses exceed \$30 million.



3. Expected loss = $(\$0 \times 0.5) + (500 \times 0.3) + (\$1,000 \times 0.1) + (\$5,000 \times 0.06) + (\$10,000 \times 0.04) = \$0 + \$150 + \$100 + \$300 + \$400 = \$950$
4. Variance (standard deviation) is a measure of risk, because variance measures the predictability of outcomes. The greater the variance, the more likely it is that a realization from the distribution will deviate materially from the expected value.
5. First, note that the expected value of each distribution is \$10,000. Distribution 2 has zero standard deviation; the \$10,000 outcome occurs all the time. Thus, there is no variation around the expected value. The standard deviation of distribution 1 is difficult to compare to that of distribution 3 without doing the calculations, because the

probability of an outcome other than \$10,000 is greater with distribution 1, but the deviation of the outcomes from the expected value is greater with distribution 3.

6. Distribution A has a lower expected value and a lower standard deviation than distribution B.
7. The expected value of the game is \$0, as our earlier calculation demonstrated $((0.5)(\$1) + (0.5)(-\$1) = \$0)$. The sample mean equals $(2/5)(\$1) + (3/5)(-\$1) = -\$0.20$. The point is that the sample mean can and usually will differ from the expected value. The sample standard deviation equals $[(2/5)(\$1 + 0.20)^2 + (3/5)(-\$1 + 0.20)^2]^{1/2} = [(2/5)(6/5)^2 + (-4/5)^2]^{1/2} = [(72 + 48)/125]^{1/2} = (120/125)^{1/2} = (24/25)^{1/2} = \$0.98$.
8. The distribution would be expected to be highly skewed (i.e., you most likely would have a relatively high probability of no liability losses and a very low probability of extremely high liability losses). The following distribution is consistent with this description.



9. a. Uncorrelated.
- b. Given the proximity of Miami and Ft. Lauderdale, if Miami experiences hurricane losses greater than the expected value, then Ft. Lauderdale also is likely to experience hurricane losses greater than

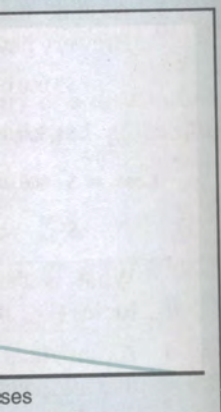
the expected variables.
c. Given the likelihood of the hurricane in another

come other than distribution 1, but comes from the ex- with distribution 3.

ver expected value viation than distri-

game is \$0, as our strated $((0.5) (\$1) +$ sample mean equals $= -\$0.20$. The point can and usually will value. The sample $[(2/5) (\$1 + 0.20)^2 + (4/5) (\$0.20)^2]^{1/2} = [(72/25)^2]^{1/2} = (24/25)^{1/2}$

be expected to be most likely would bability of no liabil- w probability of ex- (esses). The following with this description.



y of Miami and Ft. ni experiences hurri- r than the expected erdale also is likely to e losses greater than

- the expected value. Thus, these random variables would be positively correlated.
- c. Given that the weather in one year is likely to be independent of the weather in the subsequent year, hurricane losses in one year are likely to be independent of the hurricane losses in the same location in another year.

- d. Since the number of people who die from AIDS in a given year (regardless of where they live) will be affected by the development of drugs to treat or possibly cure AIDS, these random variables would likely be positively correlated.

Chapter 26

Analysis Tools Used in Corporate Risk Management

Chapter Objectives

- Show how to use historical loss data to calculate the characteristics of a probability distribution of future losses.
- Illustrate how to use regression analysis in risk management problems and how to interpret regression results.
- Show how to calculate and use correlation coefficients.
- Illustrate how to use Monte Carlo simulation to assess the desirability of alternative risk management policies.
- Show how to calculate incremental cash flows and their present values.

26.1 Risk Management Tools

In Chapter 3, you learned about probability distributions and the characteristics of probability distributions (expected values, standard deviations, correlations). In this chapter, we extend the analysis of Chapter 3 by presenting some tools used by corporate risk managers to analyze their risks and improve decision making. We note at the outset that the tools discussed in this chapter are primarily used by large firms. Generally it is not worthwhile for small businesses (and individuals) to spend resources conducting the analysis described in this chapter. The information generated from the analysis would have little impact on determining how much risk they can reasonably retain. For small businesses and individuals, risk management decisions depend largely on judgment, and commonly they are influenced by the advice of insurance agents and brokers. For large firms, the value of information provided by statistical analysis is much more likely to exceed the cost of undertaking the analysis.

26.2 Calculating Risk from Historical Data

TABLE 26.1
Data on loss payments for Klare Corporation for 2002.

Accident Date	Loss Payment (\$)
01/01	0
02/01	0
03/01	0
04/01	0
05/01	0
06/01	0
07/01	0
08/01	0
09/01	0
10/01	0
11/01	0
12/01	0
12/02	0

Total loss
Frequency
Average s

sis (the cost of data collection, analysis, and interpretation). The information provided is likely to be more meaningful and reliable, and large firms can spread the fixed costs of analysis over a larger number of exposures.

26.2 Calculating Frequency and Severity of Losses from Historical Data

Table 26.1 presents data on medical payments resulting from workplace accidents at two restaurants owned by Klare Corporation. The restaurants are of similar size and design, employ the same number of people, conduct similar amounts of business (revenue), and are located in areas with similar health care costs. The table provides the date of the accident and the amount of the loss for the year 2002. In this particular case, the loss payments are matched to the date of the accident, not when the payments were actually made. Sandeep, the risk manager for Klare Corporation, would like to examine the frequency and severity

TABLE 26.1

Data on loss
payments for
Klare
Corporation
for 2002.

Restaurant A		Restaurant B	
Accident Date	Medical Payments	Accident Date	Medical Payments
01/24	\$ 3,000	01/25	\$ 2,000
02/08	12,000	02/12	92,000
02/11	800	02/22	1,800
03/23	7,500	02/23	1,500
04/01	4,000	04/28	14,000
04/07	15,000	04/29	2,000
05/16	105,000	05/01	12,000
06/07	1,800	05/08	500
06/21	500	06/23	7,500
07/21	9,000	06/30	4,000
08/01	45,000	07/03	12,000
08/16	12,000	07/15	77,000
09/26	500	07/21	900
10/15	5,500	08/23	10,500
10/25	8,000	09/08	34,000
11/24	1,000	10/24	42,000
11/30	10,000	10/28	12,000
12/07	400	11/01	800
12/23	7,000	11/13	18,500
12/28	2,000	11/28	25,000
12/29	2,000	12/01	22,000
		12/06	6,000
		12/18	2,000
		12/27	32,000
		12/29	18,000
Total losses	\$252,000		\$450,000
Frequency of accidents	21		25
Average severity	\$ 12,000		\$ 18,000

of losses during the year at the two restaurants. Using the data in the top panel, total loss payments for the year, the frequency (number) of losses per year, and the average severity (the total loss payments divided by the frequency) are calculated and reported in the bottom panel. The data indicate that restaurant B had greater frequency and severity of losses in 2002 than restaurant A.

Because one year of data is insufficient to conclude that restaurant B has a worse safety record, Sandeep collects loss data for the two restaurants for the past eight years. Before comparing these data, he adjusts the historical data to take into account health care cost inflation. More specifically, he multiplies each loss payment by one plus the rate of inflation from the year of the loss to 2002. For example, if health care costs increased by 4 percent from 2000 to 2001 and by 5 percent from 2001 to 2002, then Sandeep multiplies the 2001 losses by 1.05 to put them in 2002 dollars. He multiplies the 2000 losses by $(1.04)(1.05) = 1.092$ to put them in 2002 dollars.

The results of Sandeep's analysis are reported in Table 26.2. On average, restaurant A has about 20 losses per year compared to about 22 for restaurant B. This is a relatively large difference.¹ In contrast to the 2002 results, the average severity of accidents at restaurant B are slightly lower than those at A. The difference in average severity, however, is not very large. Thus, Sandeep concludes that restaurant B appears to have an accident frequency problem relative to A.

After examining the frequency numbers in more detail, Sandeep suspects that restaurant B's frequency of loss has shown an increasing trend over time. To visually inspect the data, he plots the loss frequency in each year for both restaurants as in Figure 26.1. The figure strongly suggests an increasing trend in loss frequency for restaurant B. Sandeep could use this information to justify expending some resources on trying to identify the reasons for the increasing frequency and thereby identify some potential ways of reducing loss frequency.

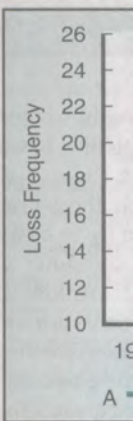
Now suppose that Sandeep would like to forecast expected losses for the year 2003 at each restaurant. If Sandeep makes the reasonable assumption that the probability distributions for loss frequency and loss severity have been and will continue to be stable over time for restaurant A, then he can use the historical averages to calculate expected frequency and expected severity for 2003. That is, he can expect about 19.6 accidents in 2003 with an av-

TABLE 26.2
Frequency and
severity data
(adjusted for
inflation).

Year	Restaurant A		Restaurant B	
	Frequency	Average Severity	Frequency	Average Severity
1995	20	\$13,000	18	\$11,000
1996	18	11,000	20	14,000
1997	18	9,000	21	8,000
1998	21	10,500	21	11,500
1999	19	19,000	23	13,000
2000	21	18,500	25	12,500
2001	19	13,000	24	12,000
2002	21	12,000	25	18,000
Average	19.6	\$13,250	22.1	\$12,500

¹A t-test indicates that the difference in means is statistically significant at the 0.03 level using a two-tailed test.

FIGURE 26.1
Loss frequency for Klare Corporation over time.



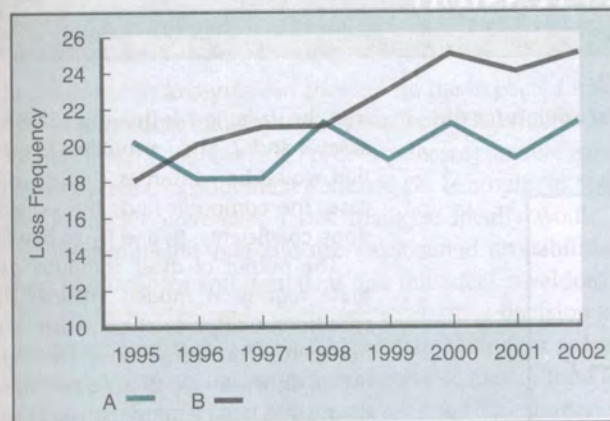
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average loss of about \$13,250 per accident. Multiplying expected frequency and severity, he finds the expected total loss for restaurant A to be about \$260,000 (in 2002 dollars).

The same type of analysis, however, is problematic for restaurant B, because the assumption that the frequency distribution is stable over time for restaurant B is not likely to be appropriate. The data suggest an upward trend in accident frequency for restaurant B; this means the historical average accident frequency will underestimate the true expected frequency in 2003. Essentially, Sandeep must find a way to extrapolate from the trend the expected frequency in 2003. The easiest way to do this is to estimate a linear regression line for restaurant B's frequency, where the explanatory variable is the number of years since 1994, denoted as TIME TREND. While the details of regression analysis are beyond the scope of this book, essential ideas of linear regression are conveyed in Box 26.1. In this example, Sandeep is using the variable TIME TREND (equal to the number of years since 1994) to explain restaurant B's accident frequency.

Sandeep's estimated regression line for the loss frequency and year data for restaurant B is

$$\text{FREQUENCY} = 17.7 + 0.99 (\text{TIME TREND})$$

$$(28.3) \quad (7.9)$$

The equation indicates that accident frequency increases by about 1 (0.99) each additional year. In Figure 26.2 we plot the actual frequency data for restaurant B versus YEAR, as well as the estimated regression line. As you can see, the regression line fits the data well.

Returning now to the problem of estimating expected losses for restaurant B for 2003, Sandeep can use the estimated regression line to forecast accident frequency (assuming no changes are made at the restaurant). By plugging 9 for the variable TIME TREND variable (year equals 2003) into the estimated regression line equation, frequency is forecasted to be 26.6 ($17.7 + 0.99 \times 9$). Assuming that the probability distribution for severity is stable over time, Sandeep can use the historical average of \$12,500 as the expected severity of losses in 2003. Multiplying expected frequency by expected severity gives \$332,500 as an estimate of expected losses for restaurant B in 2003. Note that if Sandeep had simply used the historical average frequency (22.1), he would have predicted a much lower level of expected losses.

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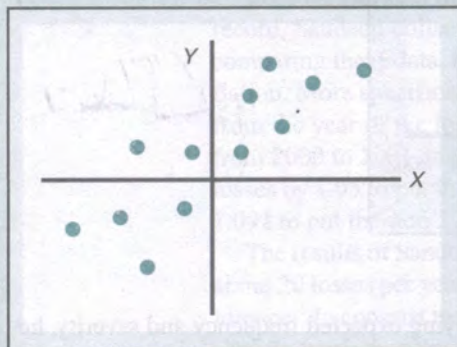
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Restaurant B	
Frequency	Average Severity
1	\$11,000
2	14,000
3	8,000
4	11,500
5	13,000
6	12,500
7	12,000
8	18,000
9	\$12,500

nt at the 0.03 level using a two-

Consider the scatter plot depicting data points for the variables X and Y .



A simple **linear regression line** is the equation of a line that best approximates the actual data for X and Y . Recall from your high school algebra classes that the equation of a line can be written as $Y = \alpha + \beta X$, where α (alpha) is the y-intercept and β (beta) is the slope. When a computer estimates a linear regression line, it finds the intercept and the slope that best approximates the actual data. In essence, the computer chooses α and β so that the errors between the actual data and the line are minimized.*

A multiple linear regression equation extends the idea of a simple regression line by including more than one variable on the right side of the equation. Suppose

that the variable Y is thought to be related to the variables X and Z . Then a multiple linear regression equation would be written as $Y = \alpha + \beta_1 X + \beta_2 Z$. In this case, the computer finds the y -intercept, α , and the slope coefficients, β_1 and β_2 , to best fit the actual data.

The output of most computer packages that estimate regression models includes statistical tests for whether the slope coefficients are significantly different from zero. The most common method of testing for statistical significance is to use a *t*-statistic. A coefficient estimate that has a *t*-statistic greater than two is generally considered to be reliably different from zero. For example, suppose that we obtain the following estimated multiple regression equation, where the *t*-statistics are reported in parentheses below the estimated slope coefficients.

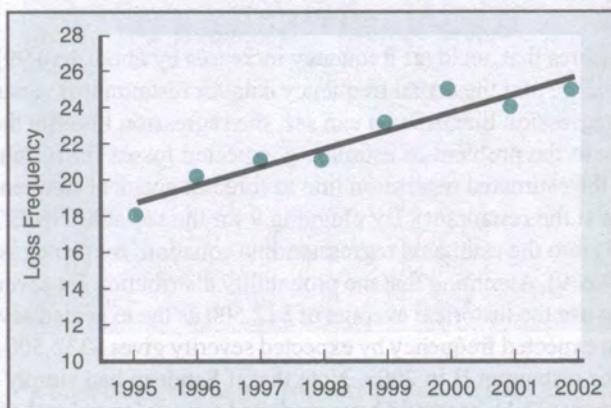
$$Y = 2.5 + 1.2X + 0.6Z$$

$$(3.5) \quad (2.7) \quad (1.1)$$

In this case, the slope coefficient for X , 1.2, is reliably different from zero, because the t -statistic is 2.7. Thus, there is a statistically significant relationship between Y and X . However, the slope coefficient for Z , 0.6, is not significantly different from zero, indicating that there is not a significant linear relationship between Y and Z .

* More formally, regression analysis finds the intercept and slope that minimizes the sum of squared deviations from each point and the regression line.

FIGURE 26.2
Frequency data for restaurant B and a linear regression line.



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A nice feature of the standard deviation is that you can find the probability of a loss is the same as the probability of a gain exceeding the expected value. In the example, the expected total loss distribution is normal with a mean of \$100 and a standard deviation of \$100. The standard deviation is 10 times the standard deviation of the individual losses.

Illustration

To illustrate the distribution, consider a loss of \$100 (the loss of a day's wages) to current workers (the loss of profits to the firm). The same loss to the firm (workers) is \$100 (the loss of a day's wages). The firm alone has 10% of the loss, which will be normal. The standard deviation

26.1

Using Entire Probability Distributions

In the previous analysis, we focused on the expected values of the frequency and severity of losses. In some situations, analysts would like to know the entire probability distribution for important variables (e.g., property losses, taxable earnings, after-tax cash flows) under alternative risk management policies. For example, to determine the optimal liability property insurance coverage, a risk manager ideally would like to know each possible loss amount during the year and their associated probabilities under alternative policy provisions (deductibles and limits). While this ideal is seldom achieved in practice, a variety of approaches can be used as a means to improve decision making. These approaches usually involve approximating the total loss distribution, or at least estimating the expected value, standard deviation, and maximum probable loss of total losses or costs.

The remainder of this section focuses on two common approaches to estimating loss distributions or key summary measures of these distributions: (1) relying on large sample probability distribution theory (i.e., assuming that the loss distribution can be approximated by the normal distribution), and (2) using computer simulation.

Approximating Loss Distributions with the Normal Distribution

You learned in Chapters 3 and 4 that if losses are independent across exposures, then the probability distribution of the average loss per exposure approaches the normal probability distribution, with its familiar bell shape, as the number of exposures increases. This result is known as the *central limit theorem*. Thus, if a firm has a large number of exposures with zero or very low correlation in losses across exposures, the distribution of its average loss will be approximately normal. The distribution of the total loss for all exposures also will be approximately normal.

A nice feature of the normal distribution is that knowledge of the expected value and standard deviation is sufficient to obtain the entire probability distribution. Consequently, if we can find the expected value and standard deviation of total losses, then we know the entire probability distribution for total losses (provided the total loss distribution is normal, which is the same as assuming a large number of uncorrelated exposure units). Fortunately, the expected value and standard deviation of total losses can be calculated given information about the expected value and standard deviation of loss *per exposure*. The expected value of the total loss distribution will equal the number of exposures times the expected loss per exposure. The standard deviation of total losses will equal the square root of the number of exposures times the standard deviation of loss for each exposure (see the appendix to Chapter 3).

Illustration

To illustrate how the firm might rely on the assumption that losses have the normal distribution, consider the case of Stallone Steel. Stallone has obtained data on workers' compensation losses for each of its workers for the last five years and has adjusted the historical data to current values based on inflation (and possible changes in workers' compensation benefits). The sample mean loss per worker (total losses during the sample period/number of workers) is \$300 and the sample standard deviation (see Chapter 3) is \$20,000. Because Stallone has 10,000 workers, it is willing to assume that the total losses for the upcoming year will be normally distributed with expected total loss of \$3,000,000 ($10,000 \times \$300$) and a standard deviation of total losses of \$2,000,000 (100 [the square root of 10,000] $\times \$20,000$).

Using the properties of the normal distribution, Stallone can estimate both the probability that losses are less than any given amount during the year and the maximum probable loss at any desired level of confidence. For example, there is a 0.95 probability that any normally distributed random variable is less than its expected value plus 1.645 standard deviations. Thus, Stallone's maximum probable loss at the 95 percent level is approximately \$6,300,000 ($\$3,000,000 + 1.645 \times \$2,000,000$). Similarly, there is a 0.99 probability that any normally distributed random variable is less than its expected value plus 2.33 standard deviations, which implies that Stallone's maximum probable loss at the 99 percent level is \$7,660,000 ($\$3,000,000 + 2.33 \times \$2,000,000$).

While the formulas are too complicated for us to cover here, the expected value and standard deviation of a normally distributed random variable that is "truncated" at particular points also can be readily calculated (using the right formula). This means, for example, that if Stallone decided to purchase excess insurance with a \$5 million annual aggregate retention, it could calculate the expected value and standard deviation of retained losses (losses less than the \$5 million retention limit).

Problems and Limitations

The principal limitation of assuming that losses are normally distributed is that the assumption often will be inappropriate in practice. First, the assumption is based on the central limit theorem, which simply describes a tendency as the number of exposures becomes infinitely large. Unless firms have very large numbers of exposures, the true distribution of total losses will likely exhibit positive skewness, and the difference from the normal distribution could be substantial. Compared to the true, positively skewed distribution, the normal distribution will tend to understate the probability of large losses. (We illustrate this result in the next section.) As a consequence, the firm will underestimate the likelihood of large, potentially disruptive losses.² This is not the type of mistake that firms wish to make.

Second, in many cases losses will not be independent across exposures. In the case of worker injuries, for example, many workers might be hurt in a single accident. Positive correlation in losses across exposures increases the standard deviation of total losses. Again, the result is that the procedure illustrated above will tend to underestimate the probability of large losses. In addition, the central limit theorem assumes independence so that assuming the normal distribution may produce significant errors even if a firm is very large.

A third limitation of the procedure illustrated above is that it does not facilitate analysis of per occurrence deductibles and policy limits. The reason is that the procedure provides estimates of the aggregate loss distribution only; it does not provide evidence about the loss distribution for individual losses (occurrences). As a consequence, while the method might help some firms make decisions about aggregate annual retention limits without per occurrence deductibles, it is not useful in the many cases where firms wish to consider limiting their exposure to losses from a single occurrence.

Computer Simulation of Loss Distributions

In many cases, realistic probability distributions for loss frequency and severity cannot be combined mathematically to derive the distribution of total losses. If, however, a firm is

²Actuaries and statisticians have developed several approximation methods to allow estimation of the maximum probable loss at specified confidence levels that incorporate information about skewness in the distribution of losses.

TABLE 26.3
Get-Well's
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willing to estimate or make assumptions concerning the shape of the loss frequency and severity distributions, **computer simulation of loss distributions** using one of several software packages can be employed to estimate the probability distribution of total losses, as well as the effects of per occurrence deductibles and policy limits. Computer simulation also can be used to obtain this type of information on loss frequency and severity and expected losses. In general, computer simulation of loss distributions often provides a versatile and valuable method of providing insight into the distribution of losses and the effects of alternative risk management strategies. It does not rely on the assumption that total losses are normally distributed. While more difficult to do accurately, it also is possible to incorporate in the simulation assumptions concerning possible correlation in losses across exposures.

Illustration

We illustrate the method of computer simulation for the product liability exposure of Get-Well Pharmaceutical Company. Based on an analysis of its own data as well as industry data on product liability claims, Get-Well assumes that the frequency of lawsuits by consumers claiming damages for adverse reactions to drugs that it has manufactured can be reasonably approximated by a **Poisson distribution**. This probability distribution often is used to describe accident probabilities. The Poisson distribution assumes that the probability that an event will occur (such as an accident) is constant in any small time period. However, you do not need to worry about the technical details of this distribution. All that you need to recognize is that this distribution often provides a reasonable description of reality and that it gives the probability that there will be zero claims, one claim, two claims, and so on during a given year (or other specified time period).

Get-Well believes that the expected number of lawsuits during the year is 30, but it also wishes to allow for parameter uncertainty (see Chapter 10) that arises due, among other factors, to uncertainty associated with legal standards that will be applied to product-related injuries. Get-Well decides to allow for this uncertainty by making the assumptions shown in Table 26.3. In this way, Get-Well builds assumptions about uncertainty in the legal system into its analysis. In effect, it allows for increased risk due to dependence in claim frequency across exposures that can arise from changes in legal standards.

Based on analysis of historical data and published research on distributions for the severity of the costs of liability claims, Get-Well assumes that the loss severity will have the **log-normal distribution**, a distribution that is commonly used to describe the severity of property and liability losses.³ For a random variable with a lognormal distribution, the natural logarithm of the variable's value has the normal distribution—thus the name *lognormal*. The lognormal distribution is characterized by positive skewness, as is true for most

TABLE 26.3
Get-Well's
assumptions
about expected
claim
frequency.

Legal Environment	Probability of Environment	Expected Annual Claim Frequency
Current standards applied	1/3	30
Stricter standards applied	1/3	20
More lenient standards applied	1/3	40

³For simplicity we assume that claim costs include defense costs and that all costs are discounted to present value.

severity distributions. Get-Well estimates that the expected value of the severity distribution is \$100,000 and that the standard deviation is \$300,000. If desirable, Get-Well also could allow for uncertainty in these parameters, but we assume that you will get the main idea without us going to any more trouble.

The loss frequency and severity distributions for Get-Well are illustrated in Figure 26.3. The first panel shows the claim frequency distribution assuming a Poisson distribution with an expected value of 30. Note that the distribution is bell shaped for an expected number of losses this large. For lower expected numbers of claims per period, such as 0.1, which might describe the frequency of your having an auto accident, the distribution would be highly skewed (large probability of no loss, increasingly small probabilities for one, two, three losses, etc.). The second panel shows the sample frequency distribution obtained by randomly drawing 1,000 outcomes (i.e., from 1,000 "trials" of a simulation) from a Poisson distribution where the expected value varies according to the assumptions shown in Table 26.3. As can be seen, allowing for uncertainty in the expected value increases the variability of claim frequency compared to the first panel. The third panel shows the sample distribution of 1,000 trials from a lognormal distribution with expected value equal to \$100,000 and standard deviation equal to \$300,000. Note that most claims are less than \$600,000, but some are much larger.

Given these assumed distributions, Get-Well can use computer simulation to generate many years of hypothetical losses in order to estimate its probability distribution of losses and the effects of different insurance arrangements. The simulation proceeds as follows for each trial (year):

1. In the first step, an expected claim frequency is randomly sampled from the distribution shown in Table 26.3. The outcome of this step is either 20, 30, or 40.
2. In the second step, the number of claims is randomly sampled from a Poisson distribution with expected value determined by the first step. That is, if the first step produces an expected value of 20, then the sampled frequency distribution has an expected value of 20, and so on. The specific value drawn might be 4, 9, 19, 32, and so on.
3. In the third step, a claim severity amount for each claim is randomly sampled from a lognormal distribution with expected value of \$100,000 and standard deviation of \$300,000. For example, if the second step produces a claim frequency of 35, then 35 claim amounts are sampled in this step. The specific severities drawn might range from a very small loss to a loss in the millions of dollars.
4. Summing the claim severities sampled in step 3 yields the total loss for the given trial (year).
5. The effects of alternative insurance arrangements on the distribution of retained losses can be estimated using the output of the simulation by applying coverage limits and deductibles to the loss amounts generated in steps 3 and/or 4. For example, with a \$500,000 per occurrence retention, Get-Well will retain the amount of each loss sampled in step 3 up to a maximum of \$500,000.
6. Repeating steps 1 through 5 over many trials provides an estimate of the probability distribution of annual total losses and the probability distribution of annual retained losses for any arrangements considered in step 5.

We summarize the results of a 1,000 trial simulation of Get-Well's loss distribution and its retained losses for the three alternative insurance policies summarized in Table 26.4. The

FIGURE 26.3

Loss frequency and severity distributions for Get-Well.

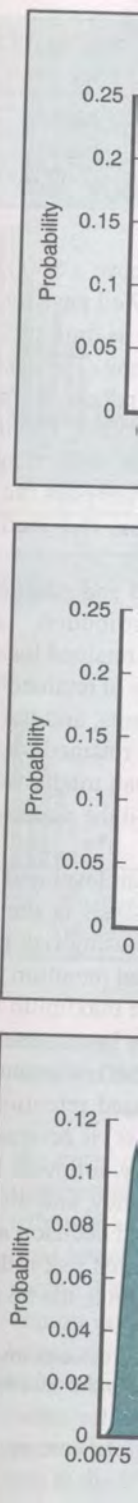
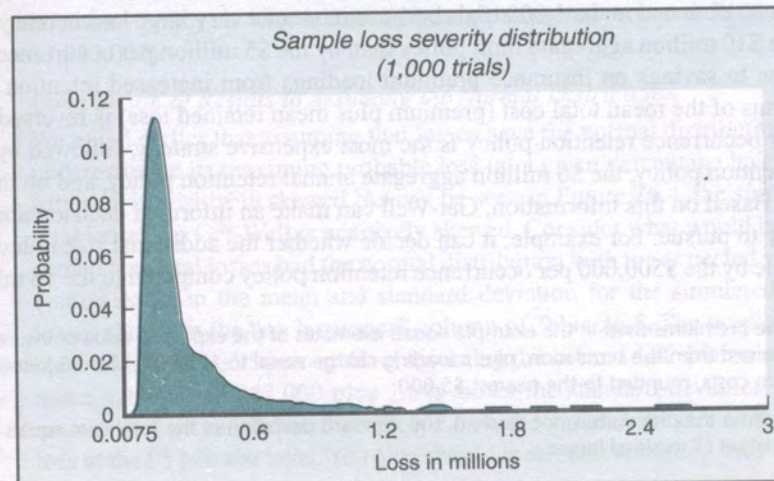
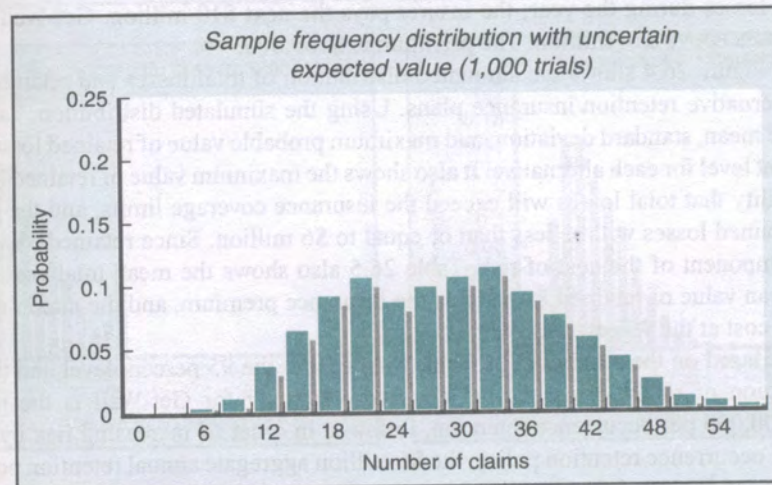
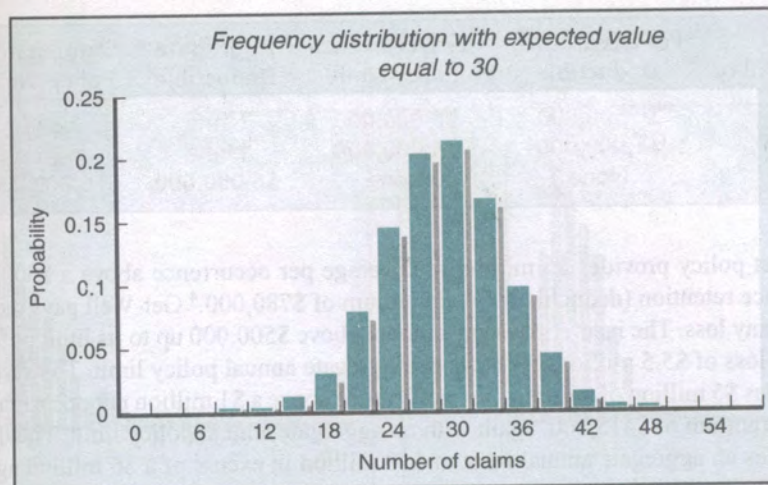


FIGURE 26.3
Frequency and severity distributions for Get-Well.



of the severity distribution of desirable, Get-Well also that you will get the main

illustrated in Figure 26.3. Using a Poisson distribution shaped for an expected number of claims per period, such as 0.1, percent, the distribution would have small probabilities for one, two, or three claims. The frequency distribution obtained from a simulation (see Figure 26.3) is very different from the assumptions shown. As the expected value increases, the distribution becomes more spread out. The third panel shows the same distribution with an expected value equal to 30. In most cases, the number of claims are less than

the simulation to generate a probability distribution of losses. The simulation proceeds as follows for

randomly sampled from the distribution, 30, or 40.

sampled from a Poisson distribution, if the first step produces a number with an expected value of 30 and so on.

randomly sampled from a normal distribution with a mean and standard deviation of 35, then 35 claim numbers might range from a very

total loss for the given trial

the distribution of retained losses. Applying coverage limits and, for example, with a \$500,000 limit, the loss sampled in step 3 up

estimate of the probability of annual retained losses

Get-Well's loss distribution and summarized in Table 26.4. The

TABLE 26.4

Alternative insurance policies for Get-Well.

Policy	Per Occurrence Deductible	Per Occurrence Policy Limit	Aggregate Deductible	Aggregate Policy Limit	Premium
1	\$ 500,000	\$5,000,000	None	None	\$780,000
2	\$1,000,000	\$5,000,000	None	None	\$415,000
3	None	None	\$6,000,000	\$10,000,000	\$165,000

first policy provides \$5 million of coverage per occurrence above a \$500,000 per occurrence retention (deductible) for a premium of \$780,000.⁴ Get-Well pays the first \$500,000 of any loss. The insurer pays any amount above \$500,000 up to its limit of \$5 million (a total loss of \$5.5 million). There is no aggregate annual policy limit. The second policy provides \$5 million of coverage per occurrence above a \$1 million per occurrence retention for a premium of \$415,000, again with no aggregate annual policy limit. The third policy provides an aggregate annual limit of \$10 million in excess of a \$6 million aggregate annual retention. There is no per occurrence limit or deductible. Get-Well pays the first \$6 million of losses during the year; the insurer pays the next \$10 million. Get-Well has to pay any losses above \$16 million. The premium is \$165,000.

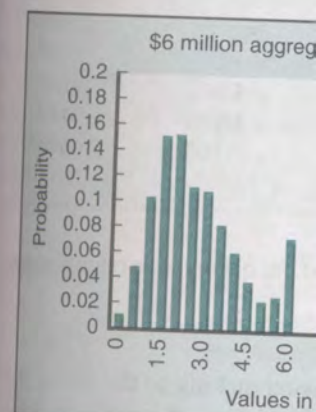
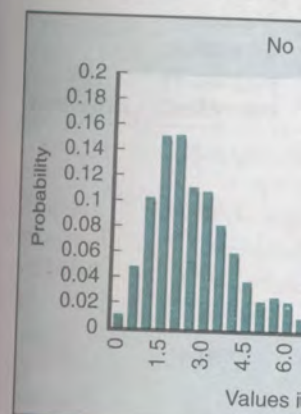
Figure 26.4 shows the simulated distribution of total losses and retained losses for the alternative retention/insurance plans. Using the simulated distribution, Table 26.5 shows the mean, standard deviation, and maximum probable value of retained losses at the 95 percent level for each alternative. It also shows the maximum value of retained losses, the probability that total losses will exceed the insurance coverage limits, and the probability that retained losses will be less than or equal to \$6 million. Since retained losses are only one component of the cost of risk, Table 26.5 also shows the mean total cost, defined as the mean value of retained losses plus the insurance premium, and the maximum probable total cost at the 95 percent level.

Based on the maximum probable total cost at the 95 percent level and the standard deviation of retained losses, the least risky strategy for Get-Well is the policy with the \$500,000 per occurrence retention, followed in order of increasing risk by the \$1 million per occurrence retention policy, the \$6 million aggregate annual retention policy, and no insurance.⁵ (Note, however, that this order does not hold for the maximum value of retained losses obtained in the 1,000 trials because more of a very large loss in one year was paid by the \$10 million aggregate limit policy than by the \$5 million per occurrence limit policies.) Due to savings on insurance premium loadings from increased retention, the ranking in terms of the mean total cost (premium plus mean retained loss) is reversed. The \$500,000 per occurrence retention policy is the most expensive strategy, followed by the \$1 million retention policy, the \$6 million aggregate annual retention policy, and no insurance.

Based on this information, Get-Well can make an informed decision about which strategy to pursue. For example, it can decide whether the additional risk reduction made possible by the \$500,000 per occurrence retention policy compared to the \$6 million aggregate

⁴ The premiums used in the example equals estimates of the expected value of insured claim costs obtained from the simulation, plus a loading charge equal to \$25,000, plus 20 percent of expected claim costs, rounded to the nearest \$5,000.

⁵ Because the premium is not random, the standard deviation of the total cost equals the standard deviation of retained losses.

FIGURE 26.4 Simulated distribution of total losses and retained losses (1,000 trials).

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viation equa
losses shown
ble total loss
mean value o
amount is 6 p
loss at the 95

Aggregate Policy Limit	Premium
None	\$780,000
None	\$415,000
\$10,000,000	\$165,000

ove a \$500,000 per occurrence. Well pays the first \$500,000 of its limit of \$5 million (a total). The second policy provides occurrence retention for its limit. The third policy provides a \$6 million aggregate annual limit. The first \$6 million of losses is paid by Get-Well. Get-Well has to pay any

and retained losses for the retention. Table 26.5 shows the simulated retained losses at the 95 percent level. The probability of retained losses, the probability that retained losses are only one percent of the total cost, defined as the maximum probable to-

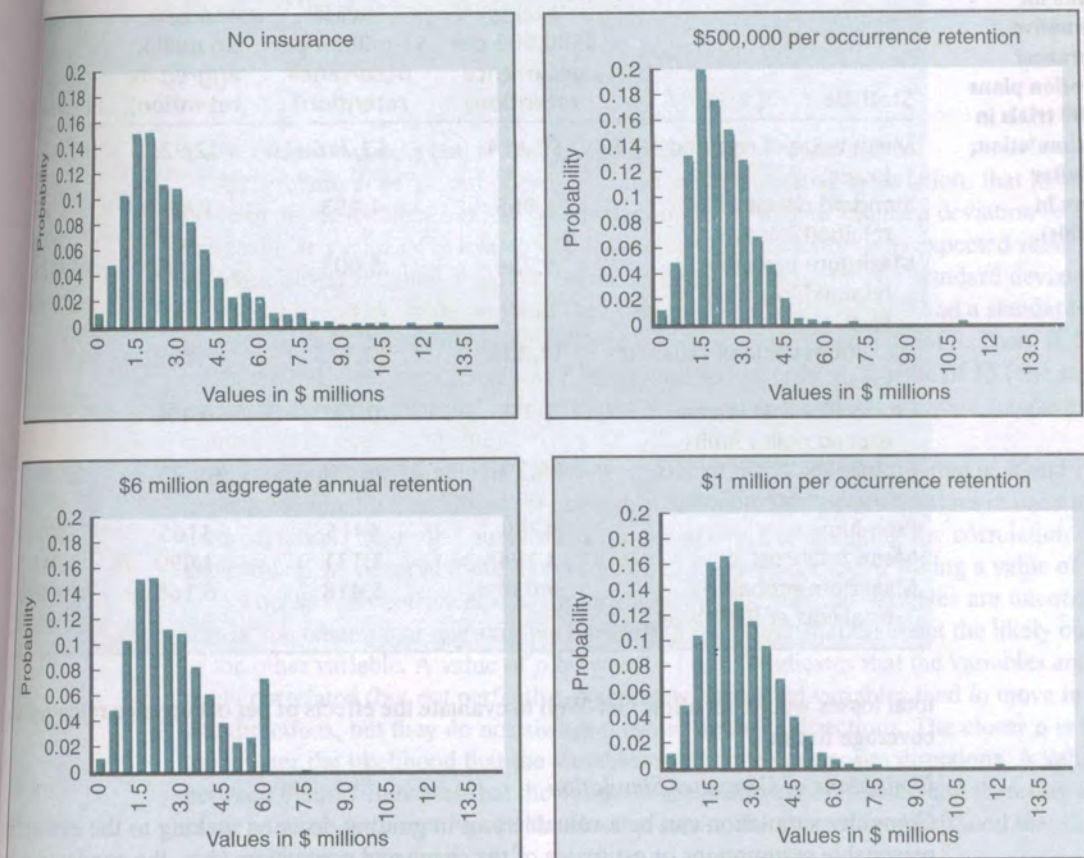
el and the standard deviation of the policy with the risk by the \$1 million retention policy, and no insurance value of retained losses in one year was paid by occurrence limit policies.) retention, the ranking is reversed. The \$500,000 retained by the \$1 million no insurance.

n about which strategy reduction made possible \$6 million aggregate

insured claim costs percent of expected

equals the standard

FIGURE 26.4 Simulated distributions of retained losses for alternative retention arrangements (10,000 trials).



retention policy is worth the additional expected cost of \$104,000 (\$3,194,000 - \$3,090,000).

Comparison of Results to Assuming the Normal Distribution

We noted earlier that assuming that losses have the normal distribution can cause a firm to underestimate its maximum probable loss (at a given percentage level) if the true loss distribution is positively skewed. As can be seen in Figure 26.4, the simulated distribution of total losses for Get-Well is positively skewed. Consider what would happen if Get-Well assumed that total losses had the normal distribution with the expected value and standard deviation equal to the mean and standard deviation for the simulated distribution of total losses shown in the "no insurance" column of Table 26.5. The predicted maximum probable total loss at the 95 percent level assuming the normal distribution equals \$6,067,000 (the mean value of \$3,042,000 plus 1.645 times the standard deviation of \$1,839,000). This amount is 6 percent lower than \$6,462,000, the simulated value of the maximum probable loss at the 95 percent level. You also should recall that assuming the normal distribution for

TABLE 26.5

Simulation results for alternative insurance/retention plans (1,000 trials in the simulation; all dollar values in \$1,000s).

Statistic	Policy 1 (\$5 million excess \$500,000 per occurrence retention)	Policy 2 (\$5 million excess \$1 million per occurrence retention)	Policy 3 (\$10 million excess \$6 million aggregate retention)	No Insurance
Mean value of retained losses	\$2,414	\$2,716	\$2,925	\$3,042
Standard deviation of retained losses	1,065	1,293	1,494	1,839
Maximum probable retained loss at 95% level	4,254	5,003	6,000	6,462
Maximum value of retained losses	11,325	12,125	7,899	18,898
Probability that losses exceed policy limits	1.1%	0.7%	0.1%	n.a.
Probability that retained losses $\leq$ \$6 million	99.7%	98.7%	99.9%	92.7%
Premium	\$780	\$415	\$165	\$0
Mean total cost	3,194	3,131	3,090	3,042
Maximum probable total cost at 95% level	5,034	5,418	6,165	6,462

total losses would not allow Get-Well to evaluate the effects of per occurrence retention and coverage limits.

Limitations of Computer Simulation

Computer simulation can be a valuable tool in guiding decision making to the extent that reasonable assumptions or estimates of the shape and parameters (e.g., the expected value and standard deviation) of loss frequency and severity distributions can be obtained. The principal limitation of the method arises from parameter uncertainty: At best, the inputs to the simulation are only estimates of the true parameters and distributions. It might be especially difficult to incorporate accurate information about dependence in losses across exposures. Note, however, that parameter uncertainty can be incorporated in the analysis, as was illustrated for Get-Well in the case of claim frequency. In addition, different simulations can be run to examine the sensitivity of the results to different assumptions concerning the type and parameters of the assumed frequency and severity distributions. For example, Get-Well might examine the results assuming that claim frequency is described by one of the commonly used alternatives to the Poisson distribution.

26.4 Correlation Analysis

In Chapter 3, we discussed correlation between two random variables. Recall that two random variables can be positively correlated, negatively correlated, or uncorrelated. In this section, we describe how to calculate correlation coefficients and present an application.

Policy 3 (\$10 million excess \$6 million aggregate retention)	No Insurance
\$2,925	\$3,042
1,494	1,839
6,000	6,462
7,899	18,898
0.1%	n.a.
99.9%	92.7%
\$165	\$0
3,090	3,042
6,165	6,462

per occurrence retention and

ion making to the extent that
eters (e.g., the expected value
butions can be obtained. The
ertainty: At best, the inputs to
distributions. It might be es-
pendence in losses across ex-
corporated in the analysis, as
In addition, different simula-
fferent assumptions concern-
d severity distributions. For
claim frequency is described
tribution.

variables. Recall that two ran-
lated, or uncorrelated. In this
s and present an application.

The **correlation coefficient**, which is typically denoted by the Greek letter rho (ρ), between two random variables, X and Y , is defined as the covariance between X and Y divided by the product of the standard deviations of X and Y :

$$\rho = \frac{\text{Cov}(X, Y)}{\text{Std}(X) \text{Std}(Y)}$$

The correlation coefficient is always within the range -1 through 1 .

A correlation coefficient of -1 indicates perfect negative correlation; that is, X and Y move in opposite directions and by equal amounts in units of standard deviation relative to their expected values. For example, if the realization of X is above its expected value by one standard deviation, then Y will be below its expected value by one standard deviation. To give a numerical example, suppose that X has an expected value of 5 and a standard deviation of 10, and Y has an expected value of 6 and a standard deviation of 3. Then, if the correlation coefficient between X and Y is -1 , and we observe an X value of 15 (one standard deviation above its expected value), then Y will take on the value of 3 (one standard deviation below its expected value).

A correlation coefficient of 1 indicates perfect positive correlation, that is, X and Y move in the same direction and by equal amounts relative to their expected values in units of standard deviation. Using the numerical example above, but changing the correlation coefficient to 1, if we observe X taking a value of 15, then we observe Y taking a value of 9.

A correlation coefficient of zero indicates that the random variables are uncorrelated; that is, the outcome of one variable does not provide information about the likely outcome of the other variable. A value of ρ between -1 and 0 indicates that the variables are negatively correlated (but not perfectly). Negatively correlated variables tend to move in opposite directions, but they do not always move in opposite directions. The closer ρ is to -1 , the greater the likelihood that the variables will move in opposite directions. A value of ρ between 0 and 1 indicates that the variables are positively correlated and therefore tend to move in same direction. Again, the closer ρ is to 1, the greater the likelihood that the variables will move together.

The typical method of calculating the correlation coefficient between two variables is to gather historical data on each variable and then calculate the sample covariance between the two variables and divide by the sample standard deviations. The sample covariance can be calculated as follows:

$$\text{sample covariance between } X \text{ and } Y = \frac{\sum_{i=1}^T (x_i - \bar{x})(y_i - \bar{y})}{T - 1}$$

where T is the number of observations, x_i and y_i are observations for X and Y during period i , $\bar{x}$ and $\bar{y}$ are the sample means for X and Y . The sample standard deviation is given by the formula:

$$\text{sample standard deviation for } X = \sqrt{\frac{\sum_{i=1}^T (x_i - \bar{x})^2}{T - 1}}$$

TABLE 26.6

Correlation between McMinn's fuel costs and the price of oil.

Month	Barrels of Fuel Purchased	Fuel Cost per Barrel	Total Fuel Costs	Price of Oil
1	9,915	\$23.26	\$230,646	\$24.80
2	10,372	28.51	295,751	28.95
3	9,737	29.58	288,018	29.29
4	9,354	25.69	240,351	27.49
5	9,041	28.95	261,729	28.13
6	9,738	23.07	224,692	22.90
7	12,309	26.48	325,962	27.24
8	10,618	26.45	280,857	26.85
9	9,337	22.34	208,585	21.41
10	8,313	27.30	226,941	28.47
11	9,961	17.89	178,179	18.55
12	9,278	24.47	227,045	25.16
13	11,650	24.96	290,768	25.14
14	10,223	20.37	208,251	22.75
15	10,084	28.17	284,020	27.59
16	9,975	25.51	254,451	27.01
17	10,532	25.52	268,733	25.02
18	11,337	23.74	269,158	24.40
19	11,738	19.66	230,752	20.25
20	10,609	27.78	294,699	28.71
21	8,427	23.92	201,610	24.92
22	10,258	29.55	303,126	27.84
23	10,063	26.16	263,265	26.17
24	9,807	20.35	199,570	21.96
Correlation Coefficient with Price of Oil		0.95	0.73	1.00

One application of correlation coefficients is for assessing basis risk associated with a hedging instrument (Chapter 25). Table 26.6 gives data for fuel purchased (in barrels), fuel cost per barrel, and total fuel costs for McMinn Trucking Company during a 24-month period, as well as the price of oil on which a futures contract is based. McMinn would like to know whether the futures contract is likely to provide a good hedge for its fuel costs. McMinn therefore calculates the correlation coefficient between its per barrel fuel costs and the price of oil and the correlation coefficient between its total fuel costs and the price of oil. As indicated at the bottom of the table, the correlation between the per barrel fuel cost is 0.95, indicating that the price of oil and McMinn fuel costs per barrel are highly correlated. However, the correlation between McMinn's *total* fuel costs and the price of oil is only 0.73. The reason for the lower correlation with total fuel costs is because of the variation over time in the amount of fuel purchased by McMinn. Thus, the futures contract would have relatively little basis risk with McMinn's per unit fuel costs, but considerable basis risk with McMinn's total fuel costs.

26.5 Use of Disco

The Net

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Calculat

When usir cash flows flows that expected c around its exist for th line deprec and, theref Security ex

Table 2 fence proj years 1 thr security c (\$150,000 of \$20,000

5 Use of Discounted Cash Flow Analysis

Total Fuel Costs	Price of Oil
\$230,646	\$24.80
295,751	28.95
288,018	29.29
240,351	27.49
261,729	28.13
224,692	22.90
325,962	27.24
280,857	26.85
208,585	21.41
226,941	28.47
178,179	18.55
227,045	25.16
290,768	25.14
208,251	22.75
284,020	27.59
254,451	27.01
268,733	25.02
269,158	24.40
230,752	20.25
294,699	28.71
201,610	24.92
303,126	27.84
263,265	26.17
199,570	21.96
0.73	1.00

The Net Present Value Criterion

To analyze the effects of risk management decisions that involve cash flows over multiple periods, it is necessary to discount the expected net cash flows to present value. Discounting cash flows was employed to determine the fair price of insurance in Chapter 8, to illustrate the effects of risk management on firm value in Chapter 20, and to illustrate the tax consequences of insurance versus retention in Chapter 21. In this section we review the key concept and illustrate its application.

Assuming that cash flows occur initially (beginning of year 1) and then at the end of each year for n years, the discounted value of expected net cash flows equals:

$$\sum_{t=0}^n \frac{E(NCF_t)}{(1+r)^t} \quad (26.1)$$

where $E(NCF_t)$ is the expected value of the net cash flow in year t and r is the opportunity cost of capital. It is important that the numerators in equation 26.1 reflect cash flows, not accounting earnings.

Many risk management decisions involve a comparatively large up-front investment of funds that produces positive expected net cash flows in future years. For example, the earlier example of the decision to install sprinklers was of this type. In these cases, the expression for the discounted value of expected net cash flows is known as the **net present value** of the investment. The net present value often is written in a slightly different way to emphasize the initial cash outflow for the investment (which we denote as I_0):

$$\sum_{t=0}^{t=n} \frac{E(NCF_t)}{(1+r)^t} - I_0 \quad (26.2)$$

Calculating Incremental Expected Cash Flows

When using discounted cash flow analysis, it is important to properly calculate expected cash flows. Always remember that only incremental cash flows are relevant; that is, the cash flows that change as a result of the decision being considered. To illustrate how to calculate expected cash flows, suppose that Zoe Building Supply is considering building a fence around its premises to reduce theft losses. For simplicity, we assume that the firm will only exist for three years, the fence will cost \$150,000, and it will be depreciated using straight-line depreciation over three years. The fence will reduce theft losses by \$55,000 each year and, therefore, increase net sales revenue (sales revenue less theft losses) by that amount. Security expenses also will be reduced each year by \$15,000.

Table 26.6 shows how Zoe could calculate the expected incremental cash flows from the fence project. The only cash flow immediately is the capital investment of \$150,000. For years 1 through 3, the reduction in theft losses increases expected cash flows by \$55,000, security costs decrease by \$15,000, and depreciation expense increases by \$50,000 (\$150,000 divided by three), resulting in an increase in earnings before interest and taxes of \$20,000. Using a 34 percent tax rate, taxes increase by \$6,800, yielding an increase in

TABLE 26.6 Incremental expected cash flows from Zoe's fence project.

	This Year	Year 1	Year 2	Year 3
Capital investment	-\$150,000			
Reduction in theft losses		\$55,000	\$55,000	\$55,000
Reduction in security		15,000	15,000	15,000
Depreciation		50,000	50,000	50,000
Earnings before interest and taxes		20,000	20,000	20,000
Taxes (34%)		6,800	6,800	6,800
After-tax earnings		13,200	13,200	13,200
Cash flow adjustments				
Add depreciation		50,000	50,000	50,000
Expected net cash flow		63,200	63,200	63,200
PV of cash flow				
(10% discount rate)	-\$150,000	57,454	52,231	47,483
NPV =	\$ 7,169			

after-tax earnings of \$13,200. Earnings, however, are not cash flows. Depreciation expenses need to be added back because depreciation is not a cash flow.⁶ After making the cash flow adjustments to after-tax earnings, the expected cash flows from the project can be calculated: \$63,200 in years one through three. Assuming that the appropriate discount rate is 10 percent, the present value of the expected cash flows can be calculated. Summing the present values of the individual expected cash flows, we find that the net present value of the project is \$7,169. Assuming Zoe is confident in all of the assumptions, the fence project should be undertaken.

Example: Forming a Captive Insurer

To provide another illustration of the use of discounted cash flow analysis, consider the case of Long-Haul Trucking. Long-Haul currently has an annual self-insured retention of \$500,000 per occurrence for both workers' compensation and auto liability insurance. The combined premium for its excess workers' compensation insurance, which pays losses without a per occurrence limit, and for excess auto liability coverage, with a \$3 million per occurrence limit, is \$2.15 million. Long-Haul is considering establishing a captive in Vermont to fund its retained losses. If it forms the captive, it will incur start-up costs in the amount of \$140,000 for licensing the captive and one-time administrative costs. This expenditure can be regarded as the initial investment required to establish the captive.

If Long-Haul forms the captive, it will be able to obtain comparable excess coverage in the reinsurance market for an annual premium of \$2 million, for a savings of \$150,000. (Thus, in this example, other cash flows also occur at the time that the investment is made.) Long-Haul also plans to use a captive management firm to administer the captive on an ongoing basis. The fees paid to this firm will increase its total administrative costs by \$30,000 per year. If it establishes the captive, Long-Haul also will use a fronting arrangement with

⁶Similarly, any change in working capital (inventory, accounts receivable, accounts payable) resulting from the project would be cash flows.

TABLE 26.7
Change in
Long-Haul's
expected net
cash flows from
establishing a
captive.

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The Appropriate Cost of Capital

Basic financial theory suggests that the cost of capital used in discounting the expected net cash flows for a particular investment (or cost) should reflect the nondiversifiable risk associated with that particular investment. The key idea is that the rate of return required to compensate shareholders for risk will depend on the riskiness of the cash flows of the particular project from the shareholders' perspective. Since shareholders generally are assumed to be diversified, the appropriate way to measure the riskiness of the cash flows is by the amount of nondiversifiable risk.

In practice, risk managers sometimes must choose between two alternative (mutually exclusive) methods of financing losses that have different patterns of expenditures but do not materially affect the variability of *any of the firm's cash flows* (e.g., insurance premiums, losses, or other cash flows). In these cases, the appropriate cost of capital for discounting these expenditures is the rate that the firm would use to discount riskless cash flows. For example, Long-Haul's decision to form a captive versus paying losses from internal funds does not change the expected value or the risk associated with retained losses. Consequently, the decision to form a captive depends critically on the present value of the costs of forming a captive (the fronting fees, administrative costs, etc.). Since these cash flows generally are certain or their risk is diversifiable, the cash flows associated with captive formation (or other retention methods that do not affect the level of retained losses) should be treated as riskless.

26.6 Summary

- Larger firms often conduct a detailed statistical analysis to estimate the probability distribution of retained losses/costs for different types of losses and loss financing arrangements, or to estimate key features of these loss distributions, such as the expected value, standard deviation, and maximum probable loss.
- Historical loss data can be used to identify areas within a firm that might have unusually high frequency or severity of losses.
- Regression analysis can be used to forecast loss distribution characteristics.
- While loss distributions sometimes can be approximated with the normal distribution, prob-

lems and limitations of assuming the normal distribution often favor more elaborate and accurate estimation methods. In particular, computer simulation can be used in many cases to obtain more accurate estimates of losses and costs for different loss financing arrangements.

- Discounted cash flow analysis often is used by firms to compare the net present value of various loss control and loss financing decisions. This procedure requires an estimation of the expected value and timing of cash flows for each alternative and the choice of an appropriate discount rate in view of the risk of the cash flows.

Key Terms

linear regression line 574
computer simulation of loss distributions 577
Poisson distribution 577

lognormal distribution 577
correlation coefficient 583
net present value 585

Questions and Problems

1. Use the following data to estimate the total losses for 2000. Assume a normal probability distribution. What are the probabilities that have been and will be?

Year	Frequency of Accidents
1996	33
1997	25
1998	35
1999	31
2000	29
2001	27
2002	34

2. What is the correlation coefficient between Irwin's sales and GNP growth?

Year	GNP growth
1990	2.0%
1991	1.5
1992	-2.0
1993	-1.0
1994	0.5
1995	-1.0
1996	1.0
1997	2.0
1998	2.5
1999	0.0
2000	2.0
2001	1.5
2002	3.0

3. Describe the major components of a computer simulation. How would you describe the advantages and disadvantages of a simulation (compared to a normal distribution)? How would you have the normal distribution? How would you have the normal distribution of computer simulation?
4. Wood Warehouse is considering a replacement of its automatic sprinkler system. The initial cost of \$100,000 is expected to last five years (the life expectancy in real life). Annual

Questions and Problems

Use the following data to calculate expected total losses for 2003 assuming that the probability distributions of frequency and severity have been and will continue to be stable.

	Frequency of Accidents	Average Accident Severity	Inflation Rate during Year
26	33	\$1,800	2.0%
27	25	2,100	1.5
28	35	3,000	2.5
29	31	2,800	1.5
30	29	3,100	2.0
31	27	3,200	1.8
32	34	3,500	1.5

3. What is the correlation coefficient between Irwin's sales and costs and between sales and GNP growth?

	GNP growth	Sales	Costs
33	2.0%	\$105,400	\$71,000
34	1.5	101,900	72,200
35	-2.0	100,400	72,300
36	-1.0	102,200	72,500
37	0.5	104,000	72,000
38	-1.0	99,950	71,300
39	1.0	103,300	72,300
40	2.0	102,800	72,100
41	2.5	104,900	72,300
42	0.0	100,100	72,500
43	2.0	104,600	71,200
44	1.5	102,000	71,730
45	3.0	103,300	70,600

4. Describe the major steps involved in a computer simulation of loss distributions. Also describe the advantages of computer simulation (compared to assuming that losses have the normal distribution) and the limitations of computer simulation.
5. Wood Warehousing has property insurance on a replacement cost basis with a \$5,000 per occurrence deductible. It can install automatic sprinklers in its warehouse for an initial cost of \$10,000. The sprinklers will last five years (they would last much longer in real life). Annual upkeep of the sprinklers

will be \$1,000, payable at the end of each of the first four years from the time of installation. If Wood installs the sprinklers, it will deduct one-fifth of the installation price at the end of each year for five years as depreciation (a noncash expense) for tax purposes. Without the sprinklers, the firm believes that its expected cash outflow for losses less than the deductible will be \$1,000 per year and that its insurance premium will be \$10,000 per year. With the sprinklers, it believes that its expected cash outflow for losses less than the deductible will be \$500 per year and that its insurance premium will be \$7,000 per year. Wood's opportunity cost of capital for this decision is 10 percent. Retained losses are paid and deductible for tax purposes at the end of each year. Insurance premiums are paid at the beginning of the year and are deductible for tax purposes when paid. The tax rate is 34 percent. Should Wood install the sprinklers? Support your answer by calculating the net present value of installing the sprinklers.

6. Seward Go-Carts has annual sales of \$150 million with an expected profit of \$5 million. It currently has a retention limit for annual liability losses equal to \$5 million. The premium for excess insurance coverage above the \$5 million retention is \$50,000. With the \$5 million retention, its expected retained losses (including claims settlement costs) for accidents during the year equal \$1 million. Half of all retained losses for accidents during the year will be paid at the end of the year; the remaining half will be paid at the end of the next year. If Seward changes its retention level to \$2 million, the premium for excess coverage will be \$170,000. Its expected retained losses for accidents during the year will drop to \$900,000. Again, half of retained losses for accidents during the year will be paid at the end of the year; the remaining half will be

discounting the expected net the nondiversifiable risk as the rate of return required to of the cash flows of the shareholders generally are as-riskiness of the cash flows is

two alternative (mutually ex-ns of expenditures but do not s (e.g., insurance premiums, ost of capital for discounting ount riskless cash flows. For ng losses from internal funds with retained losses. Conse-e present value of the costs etc.). Since these cash flows s associated with captive for-of retained losses) should be

of assuming the normal dis-more elaborate and accurate In particular, computer sim-in many cases to obtain more of losses and costs for differ-rangements. ow analysis often is used by e net present value of various ss financing decisions. This an estimation of the expected cash flows for each alterna-f an appropriate discount rate of the cash flows.

normal distribution 577
relation coefficient 583
present value 585

paid at the end of the second year. Premiums are paid at the beginning of the year. Seward's opportunity cost of capital for this decision is 10 percent. The tax rate is 34 percent. The decision horizon is one year of coverage because the firm can change its retention level again the following year. (Note, however, that cash flows associated with the decision occur over two years.)

- (a) Calculate the present value of the after-tax change in expected net cash flows from reducing Seward's retention level from \$5 million to \$2 million.

- (b) Calculate the present value of the after-tax change in net cash flows from reducing Seward's retention level *assuming that actual retained losses equal the retention limits* rather than the expected values.
- (c) Which approach do you think the firm should take? Why? What factors (other than the present values that you calculated) would be likely to influence its decision?

References

Holler, Keith D. "The Confidence Game: Using Confidence Levels to Estimate Losses." *Risk Management*, August 1995, pp. 53–57.

(Practitioner-oriented introduction to estimation of losses, including the use of computer simulation.)

Chapter 2

Enterprise Risk Management

Chapter 27

- Description of enterprise risk management
- Illustration of enterprise risk management

27.1 Enterprise Risk Management

For many years, risk management has been a topic of interest for executives. In the 1990s, risk management became a topic of interest for the general public. This was due to a number of factors, including the increasing complexity of the business environment, the increasing awareness of the potential consequences of risk, and the increasing demand for risk management by investors and other stakeholders. This chapter discusses the importance of risk management and provides a framework for understanding the various risks that a firm faces. It also discusses the various tools and techniques that can be used to manage risk.

¹Much of the risk management literature is based on the work of the National Commission on the Causes and Prevention of the Collapse of Enron.

²Example of a risk management framework is provided in the next chapter.