Test 2 – Polynomial Functions, Equations & Inequalities

Syllabus content on test: ▪ polynomial functions ▪ quadratic formula & discriminant ▪ factor & remainder theorems ▪ polynomial division ▪ sum and product of roots of a polynomial equation ▪ rational functions ▪ solving inequalities

total marks on test: **60**

Part I: **No** calculator – questions 1-6 **[34 marks]**

1. When the polynomial $2x^3+ax^2+b$ is divided by $(x-2)$, the remainder is 2, and when divided by $(x+1)$, the remainder is -1. Find the value of *a* and the value of *b*. [5 marks]

2. Find a cubic polynomial with integer coefficients that has zeros of $x=2$ and $x=1+3\text{i}$. [4 marks]

3. Given that $m>0$, find the value(s) of *m* that solve the inequality $mx^2+mx+3>0$. [5 marks]

4. If α and β are the roots of the quadratic equation $2x^2-6x+1=0$, find a quadratic equation whose roots are:

(a) $2\alpha,\ 2\beta$ (b) $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ [8 marks]

5. (x^2-1) is a factor of the cubic polynomial x^3+px^2+qx+r, and the polynomial leaves a remainder of 12 when divided by $(x-2)$. Find the value of *p*, the value of *q* and the value of *r*. [6 marks]

6. Consider the quartic equation $2x^4-11x^3+20x^2-7x-10=0$. Given that one of the zeros of the equation is $r_1=2-\text{i}$, find the other three zeros r_2, r_3 and r_4. [6 marks]

Part II: calculator allowed – questions 7-11 **[26 marks]**

7. Sketch the graph of $y=\frac{x-10}{5x-2}$. Clearly label any *x*- or *y*-intercepts and any asymptotes. [5 marks]

8. The cubic polynomial x^3+mx^2+n has a double root of $x=c$ and a single root of $x=2$. Given that $n\neq 0$, find the value of *c*. [6 marks]

9. Solve for *x*: $\frac{3x-3}{4-x}\leq 3$ [4 marks]

10. Find the range of values of *k* such that the equation $kx^2-2x+k-1=0$ has no real solutions. Express your answer **exactly**. [6 marks]

11. Consider the rational function $g(x)=\frac{x+a}{bx+c}$, $x\neq -\frac{c}{b}$. The graph of *g* has asymptotes $x=-6$ and $y=3$, and the point $\left(6,\frac{5}{2}\right)$ lies on the graph. Find the values of *a*, *b* and *c*. [5 marks]