Complex Numbers

IB Math Minds

▶ from Maths HL formula booklet - complex numbers ◀

- complex numbers $z = a + ib = r(\cos\theta + \mathrm{i}\sin\theta) = re^{\mathrm{i}\theta} = r\mathrm{cis}\theta$
- De Moivre's theorem $[r(\cos\theta + \mathrm{i}\sin\theta)]^n = r^n(\cos n\theta + \mathrm{i}\sin n\theta) = r^n e^{\mathrm{i}n\theta} = r^n \mathrm{cis} n\theta$

Problem Set 3

1. Find the cube roots of i in the form $a + bi$, where $a, b \in \mathbb{R}$.

2. (a) Find x and y such that $(x + 3i)(x + iy) = 1 - i$

(b) Find a and b such that $a + bi = \sqrt{5 + 12i}$

3. Find all solutions to the equation $z^5 - 32 = 0$. Express the solutions exactly.

4. Given that $z_1 = 3 + 2i$ and $z_2 = 4 - 3i$

(a) Find $z_1 z_2$ and $\frac{z_1}{z_2}$, and express each in the form $a + bi$ (Cartesian form).

(b) Verify that $|z_1 z_2| = |z_1||z_2|$.

5. The complex number z satisfies the equation $\frac{z}{z+2} = 2 - i$.

(a) Find z and express it in the form $a + bi$, where $a, b \in \mathbb{R}$.

(b) Find $|z|$ and $\arg(z)$.

6. (a) Find all solutions to the equation $z^4 + 4 = 0$.

(b) Hence, or otherwise, express $z^4 + 4$ as the product of two quadratic polynomials in z with real coefficients.