STUDY GUIDE

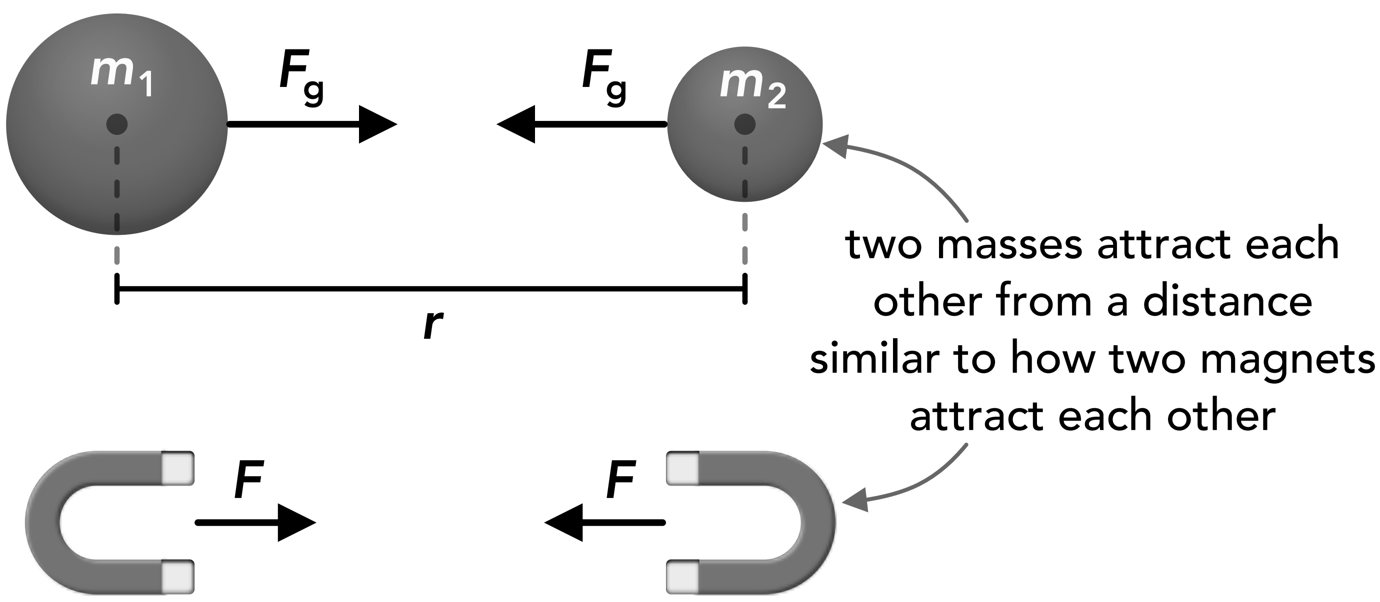
GRAVITY & WEIGHT

Newton's Law of Universal Gravitation

Constants		Unit	Name
G	6.67×10^{-11}	$\frac{m^3}{kg\cdot s^2}$	gravitational constant

Newton's Law of Universal Gravitation (gravitational force)

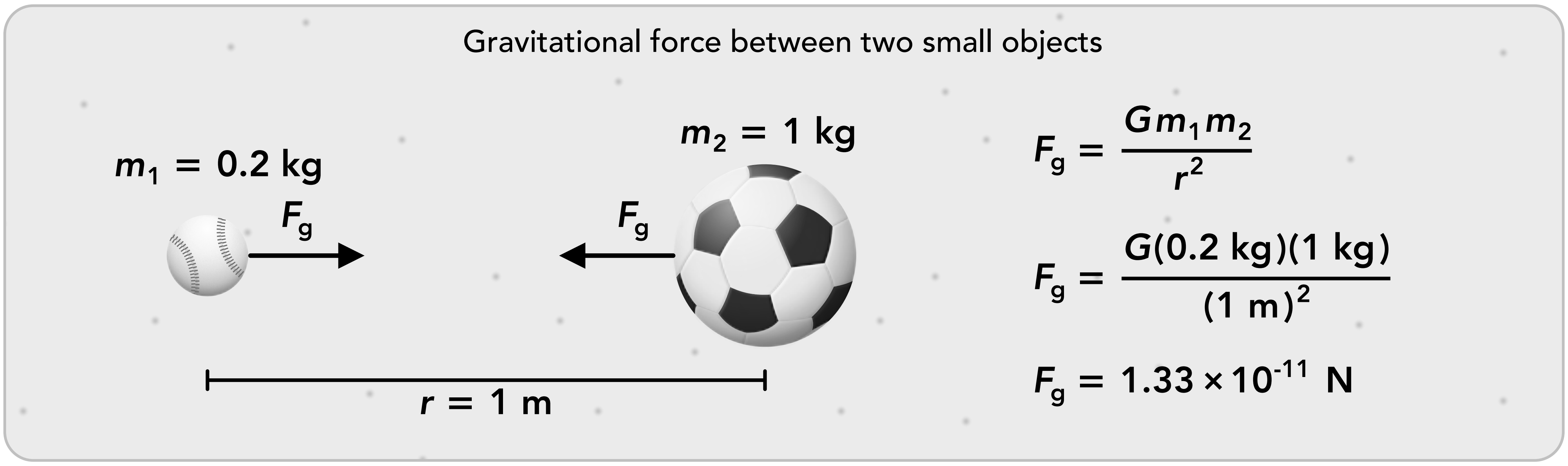
$$F_g = \frac{Gm_1m_2}{r^2} = F_{1\text{ on }2} = F_{2\text{ on }1}$$

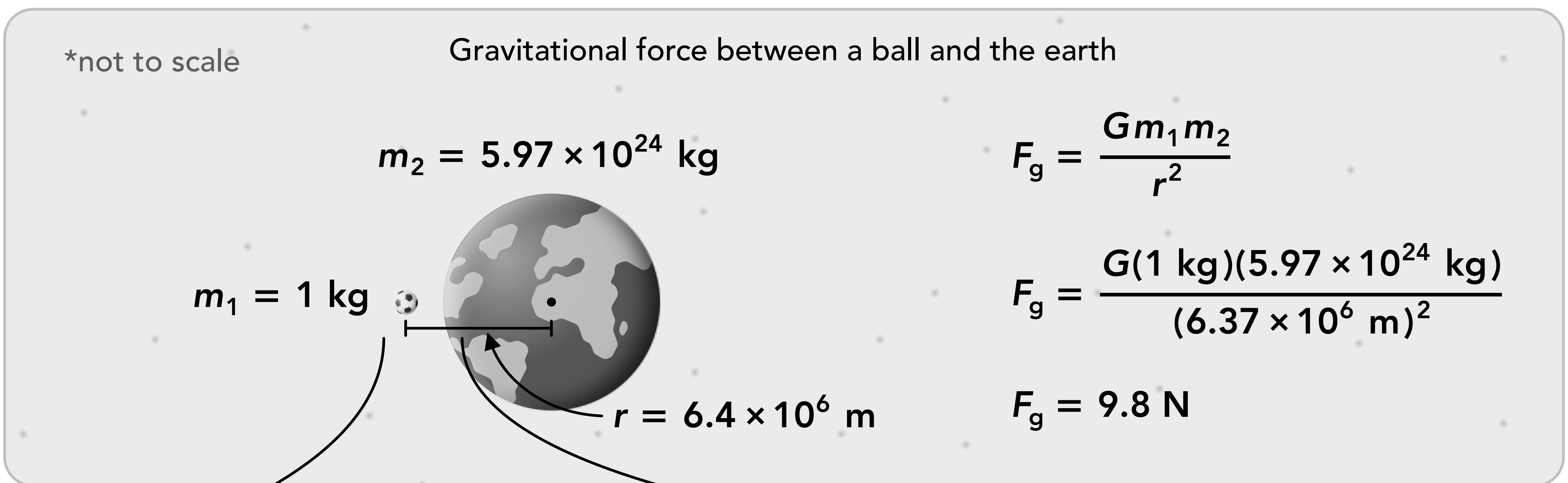


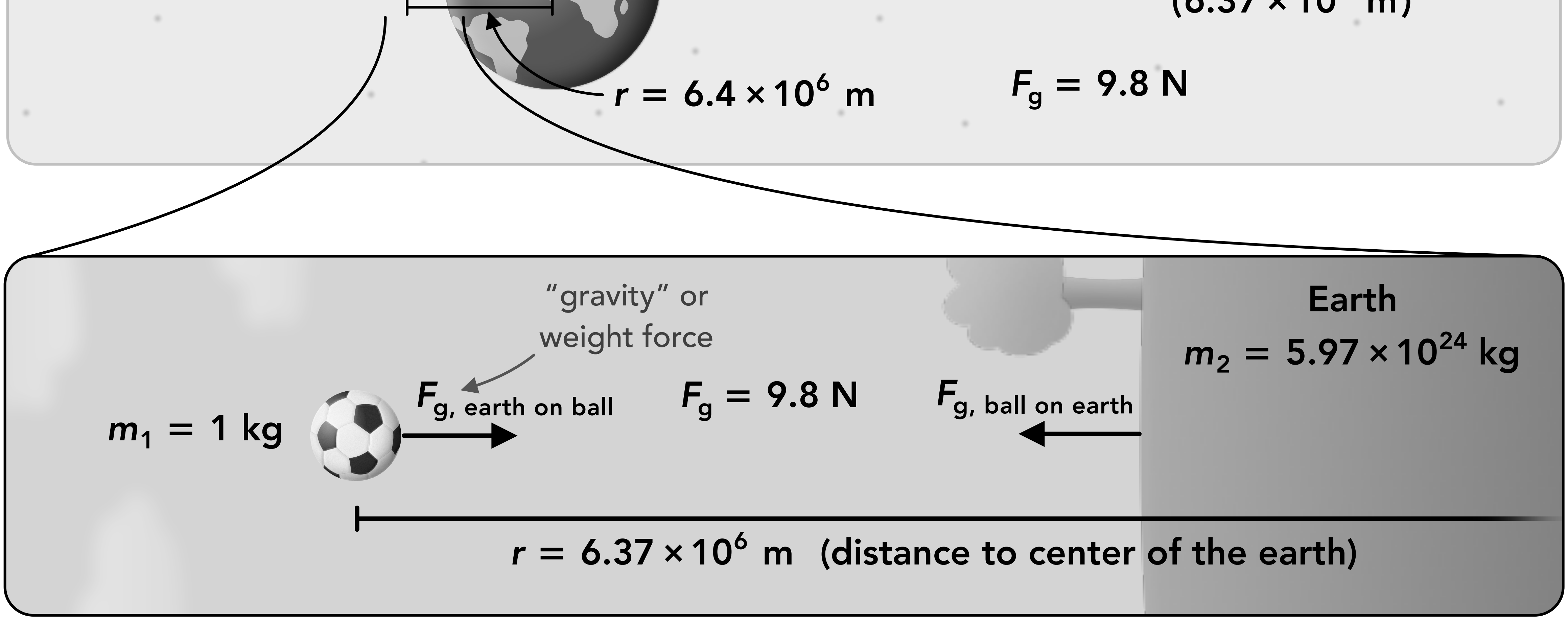
Variables		SI Unit
F_g	gravitational force	N
w	weight force	N
m	mass	kg
M	mass producing a field	kg
r	distance between centers	m
g	gravitational acceleration	$\frac{m}{s^2}$

- **Newton's law of universal gravitation:** Every object in the universe attracts every other object in the universe with a gravitational force that depends on their masses and the distance between their centers.
- This law treats objects as **point masses** which means the gravitational force behaves as if each object's mass is concentrated at a single point (its center of mass, which depends on the object's shape).
- Remember that r is the distance between the centers of the two objects, not between their surfaces.
- A gravitational force is always an **attractive** force that acts towards the center of the other object.
- Based on this equation, the greater the mass of either object the greater the gravitational force. The farther apart the two objects are the smaller the gravitational force.
- The constant G in the equation is the universal gravitational constant whose value is given above.
- It doesn't matter which mass is m_1 and m_2 , and one mass does not need to be larger than the other. It's not the case that only large masses pull on small masses, any two masses pull on each other with the equal force.
- Gravitational forces come in pairs as described in Newton's 3rd law of motion. The gravitational force exerted on mass 1 by mass 2 is equal in magnitude and opposite in direction to the gravitational force exerted on mass 2 by mass 1. Each mass pulls on the other with the same amount of force.

- This gravitational force is what we experience as gravity on earth. However, notice that Newton's law of universal gravitation does not describe gravity using the words "earth", "falling", "down", etc. A gravitational force acts between every two objects in the universe: the earth and the moon attract each other, the earth and a book attract each other, and a book and a cup attract each other because they all have mass.







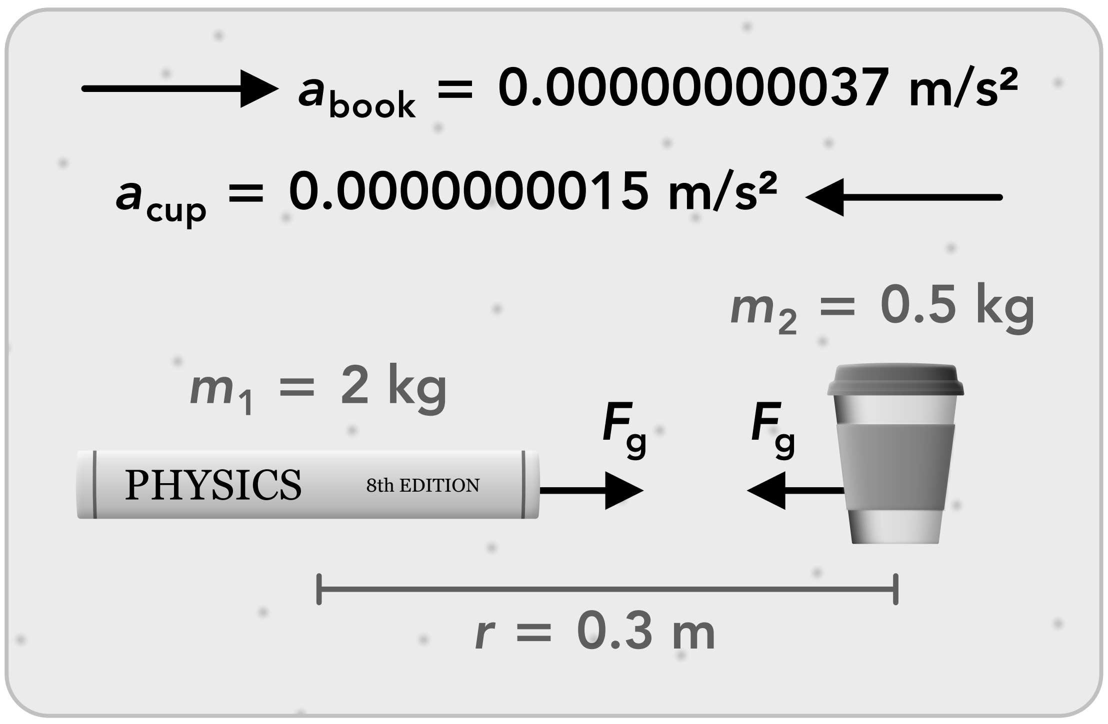
- We call the gravitational force between the earth and an object near the surface of the earth the **weight force** acting on the object. Although we're used to saying objects are pulled "down" by gravity, it's more accurate to say that objects are pulled towards the center of the earth.

- If all objects are attracted to each other by a gravitational force, why don't we experience this? For example, if a book and a cup are sitting next to each other on a table, why don't they move towards each other? There must be one or more forces acting in the opposite direction as the gravitational force so that the net force on each object is zero. In most cases that force is **friction**, but electrostatic forces or other forces may also be involved.

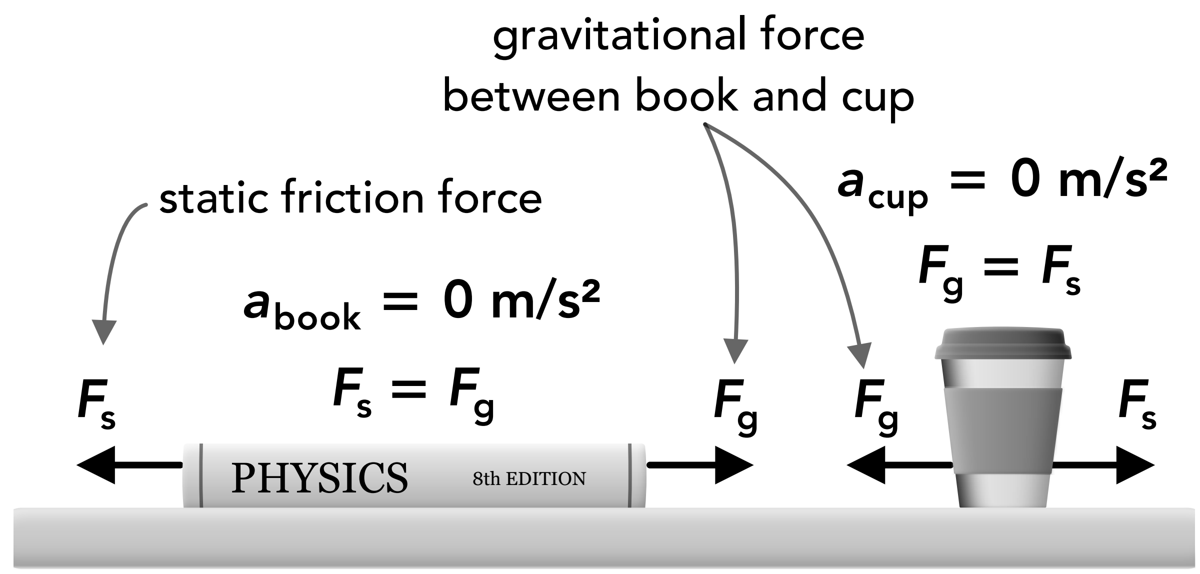
Gravitational force between a book and a cup

$$F_g = \frac{G(2\text{ kg})(0.5\text{ kg})}{(0.3\text{ m})^2} = 7.4 \times 10^{-10}\text{ N} = 0.00000000074\text{ N}$$

If there's no friction, the book and the cup will slowly accelerate towards each other



Friction forces prevent the book and the cup from accelerating towards each other



- If the book and the cup were floating in space with no other forces acting on them besides the gravitational force between them, they would slowly accelerate towards each other. In the example above, it would take a few hours for the book and the cup to hit each other (starting from rest).
- In most scenarios there is a static friction force acting between the objects and a surface that opposes the gravitational force and prevents the objects from accelerating towards each other. For a 2 kg book the maximum static friction force could be around 4 N, which means you could push against a resting book with up to 4 N of force before you overcome friction and it begins to slide. That's much more than the gravitational force.
- The gravitational force is very weak compared to the other fundamental forces and the forces we normally experience. Gravity is often associated with the earth and other planets because planets have such a large mass that the gravitational force is significant compared to other forces.

Gravitational Field and Weight

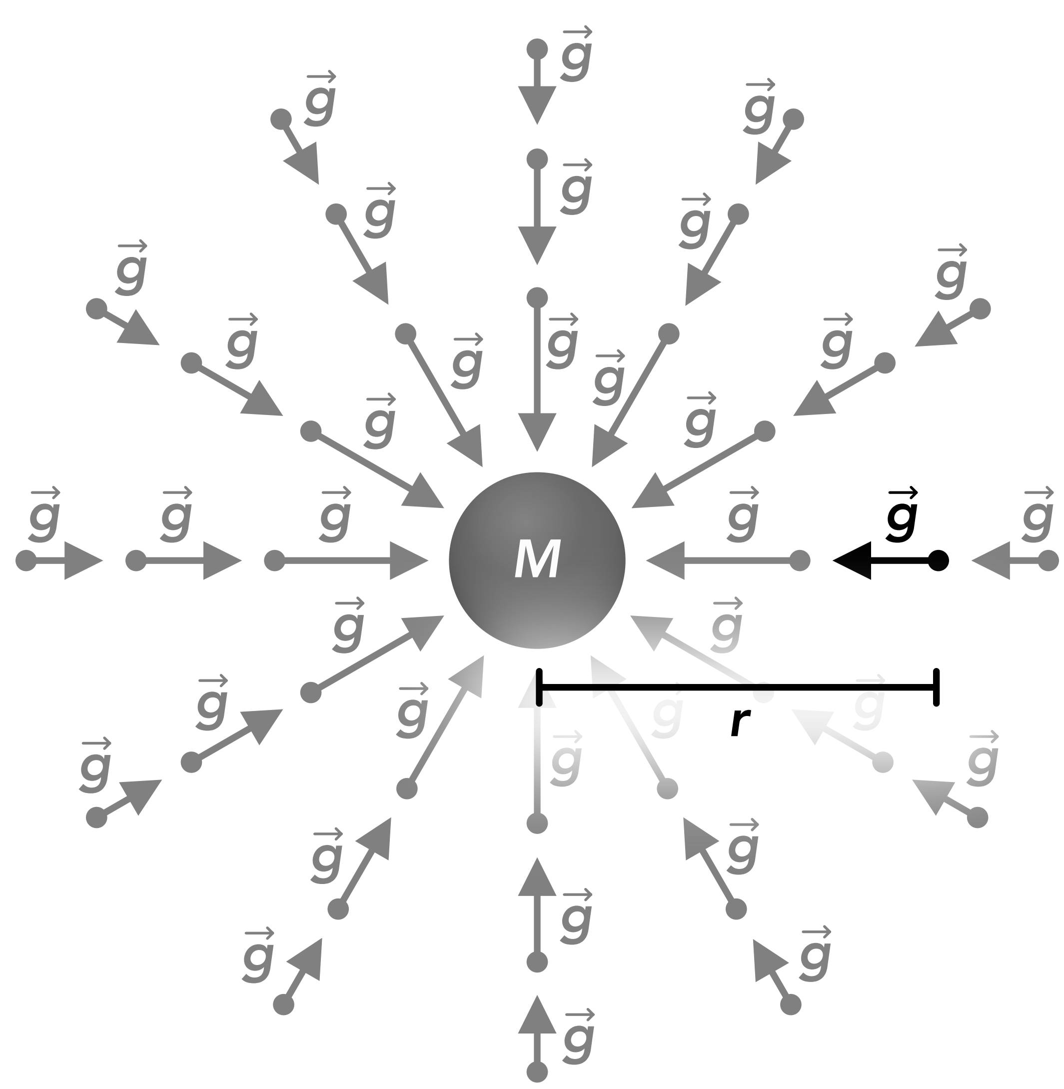
- While Newton's law of universal gravitation treats gravity as a force that exists between two point masses, there is another way to think about gravity: the interaction between a mass and a gravitational field.
- A **gravitational field** exists around every object due to its mass. The field is not visible on its own and is more like a mathematical representation of how a second mass would interact with the mass creating the field at any position in space. A gravitational field is a vector field and is sometimes referred to as a "gravitational acceleration field" because it consists of a vector at every point in space which shows the direction and magnitude of the gravitational acceleration vector at that point.
- The direction of the gravitational field is always **towards the center of the mass producing the field**.
- Note that **a gravitational field is produced by a single mass**. If a second mass is placed in that gravitational field it will experience a gravitational force towards the first mass due to the field. (The second mass also produces its own field which causes the first mass to also experience a gravitational force towards the second mass).
- A mass does not experience a force from its own gravitational field, only the field from another mass.
- In the equations below the variable ***M*** represents the mass producing the field and the variable ***m*** represents a a second mass in that field, experiencing a force. These two masses can be any size, ***M*** does not have to be larger than ***m***, but this is often applied to a planet and a small mass where the planet mass is ***M***.

A gravitational field exists around every mass

g is the **gravitational field strength** and the **acceleration due to gravity** at every point in space

$$g = \frac{GM}{r^2}$$

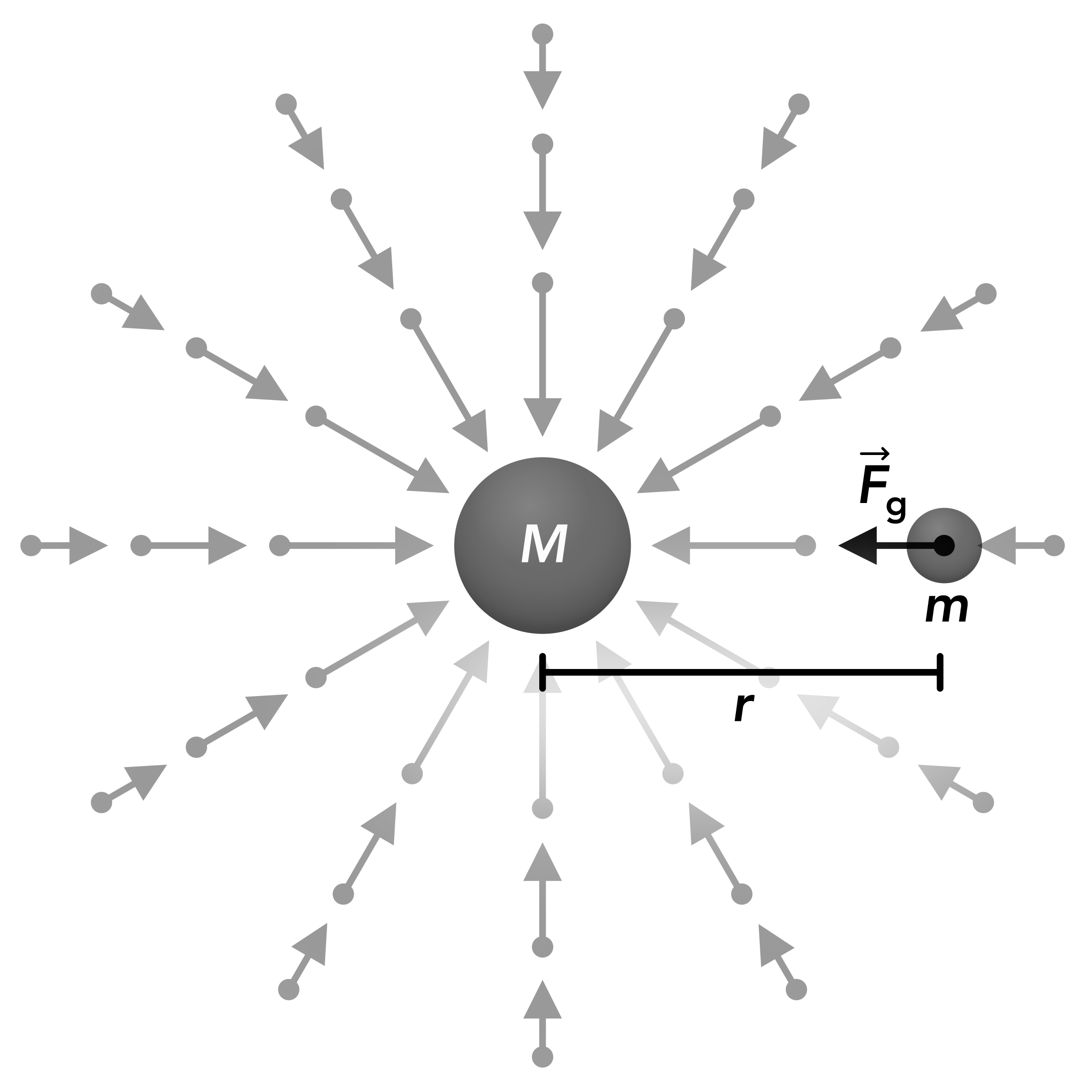
Units: $\frac{\text{N}}{\text{kg}} = \frac{\text{m}}{\text{s}^2}$



A second mass placed in that gravitational field will experience a gravitational force towards the mass that is creating the field

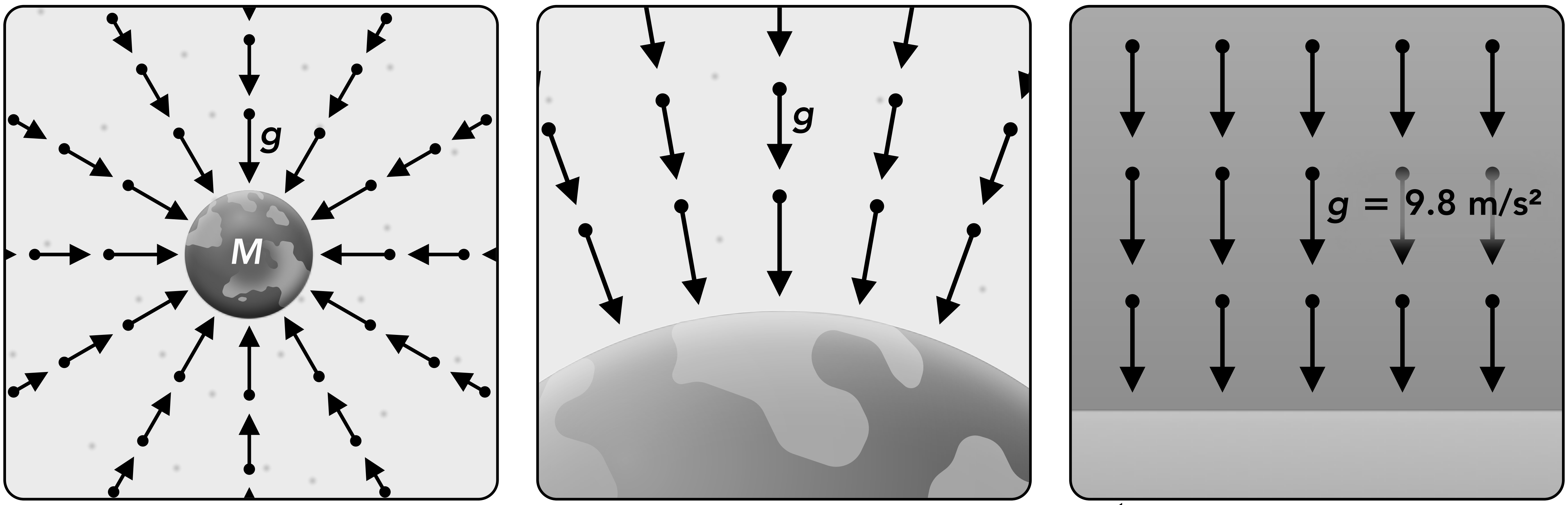
Gravitational force on mass in gravitational field

$$F_g = mg \quad = \quad F_g = \frac{GMm}{r^2}$$



- If the mass producing the gravitational field is the earth, we can look at the strength of the field at different distances from the center of the earth. As we zoom in, the gravitational field lines appear to be more parallel (the earth doesn't appear as curved) and the field strength doesn't vary as much within the smaller window height.
- Near the surface of the earth (where the value of r is equal to the radius of the earth) the value of g is 9.8 N/kg or 9.8 m/s². That's the strength of the gravitational field and the acceleration due to gravity for any object.

The value of g depends on the distance r from the center of the earth



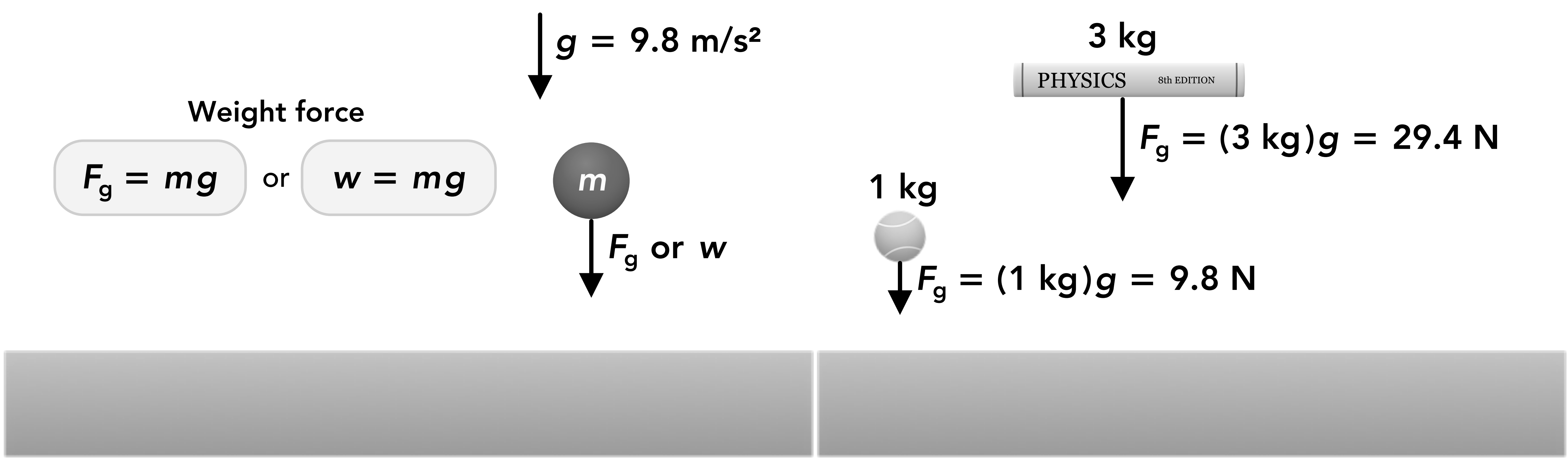
$$g = \frac{G(5.97 \times 10^{24}\ \text{kg})}{(6.37 \times 10^{6}\ \text{m})^2} = 9.8\ \text{m/s}^2$$

Near the surface of the earth the value of g is about 9.8 m/s²

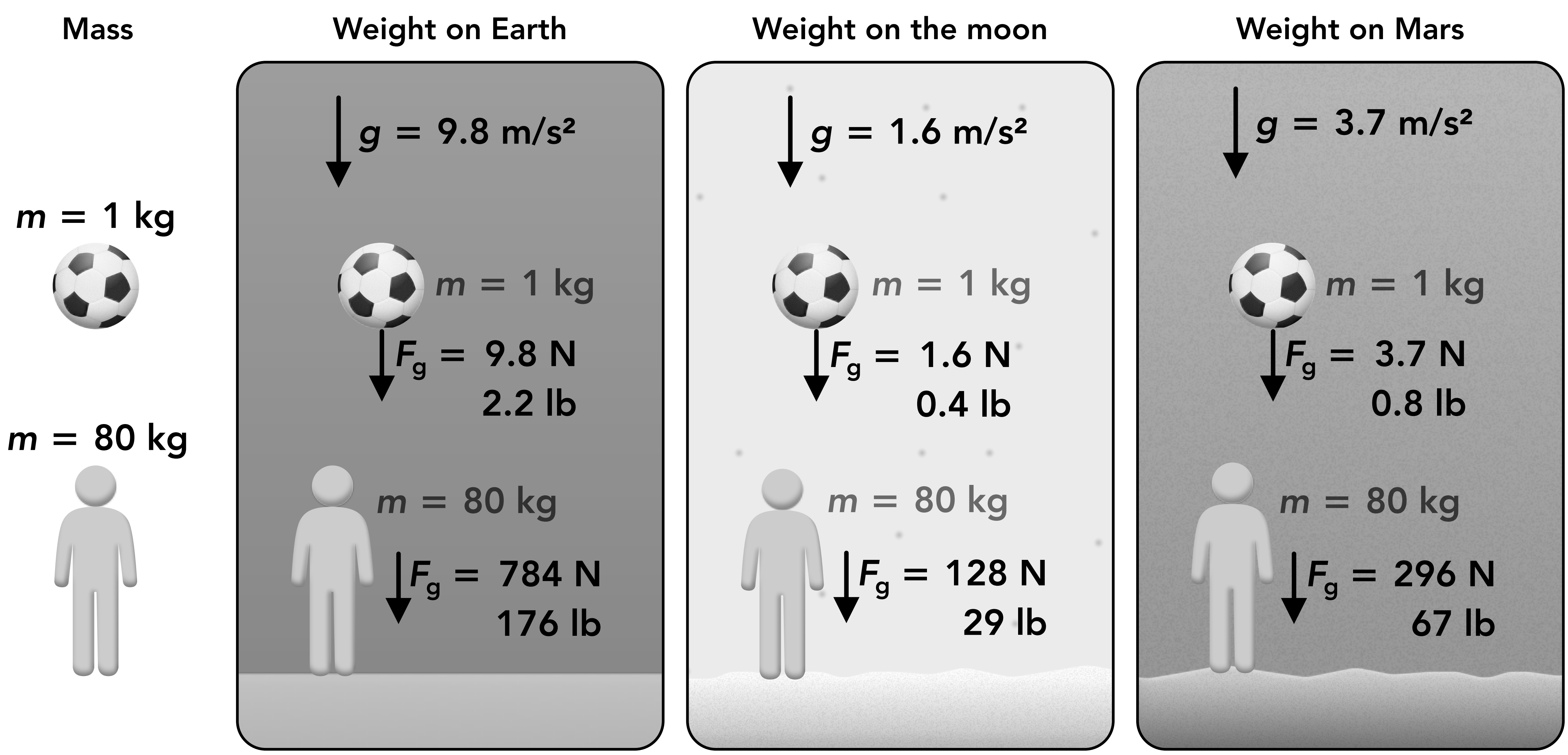
- The **weight force** acting on an object (sometimes referred to as "the weight of an object") is just the gravitational force acting on that object when it's near the earth (or any large body like the moon or another planet).
- Unless a different value is given, **assume the value of g is 9.8 m/s²** when finding the weight of an object on earth.

Weight force

$F_g = mg$ or $w = mg$



- A common confusion is the difference between mass (m) and weight (F_g) because they are sometimes used interchangeably outside of physics.
- An object's mass is its **inertia**, which is how much it resists a change to its state of motion as described in Newton's 1st law of motion. An object's mass doesn't change no matter where it is in the universe.
- An object's weight is the **gravitational force** acting on the object when it's in the gravitational field of another mass (usually the earth, the moon or another planet).
- An object's weight is proportional to its mass so we often measure an object's weight instead of its mass because the value of g is relatively constant near the surface of earth.
- The strength of gravity is different near the surface of the moon and other planets because of the difference in the the planet's mass and radius. Even though the mass of an object is the same everywhere, its weight will change.



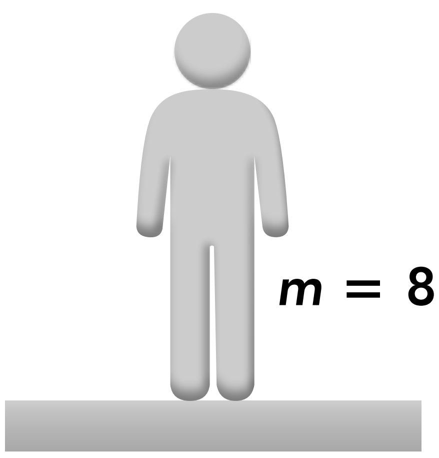
Apparent Weight

- The weight of an object is the gravitational force pulling the object down (towards the center of the planet). The weight does not change due to the motion of the object or other forces acting on it.
- The **apparent weight** of an object is the **normal force** between the object and the surface below it, or the **tension force** in the rope that the object is hanging from.
- Think about how and why you feel your own weight when standing or sitting in a chair. It may seem like you're feeling the force of gravity, but you're actually feeling the **contact forces** that are supporting you from below (the normal force acting upwards on your body from the floor or the chair).
- Remember that a scale placed between two objects (or an object and a surface) measures the normal force between the two objects, so **a scale measures your apparent weight, not your actual weight**.

- **Weightlessness** is a term used to describe when an object has **zero apparent weight**. This does not mean the object has zero weight. If there is a gravitational force acting on the object then it still has weight.
- In the fifth elevator example, the elevator is accelerating downwards at 9.8 m/s² and the person's apparent weight is zero. There is still a weight force pulling them down, causing them to accelerate downwards at g, 9.8 m/s². In this example the elevator, the person and the scale are all in **free fall**. The person still has weight but they are experiencing weightlessness, just like if they were falling through the air without an elevator.

- **If you're not accelerating up or down then your weight and apparent weight are equal**. The net force in the vertical direction is zero, and your weight force is equal in magnitude to the normal force acting upwards. This is the case in many "normal" scenarios (like standing on the ground, sitting in a chair) because we're usually not accelerating up or down.

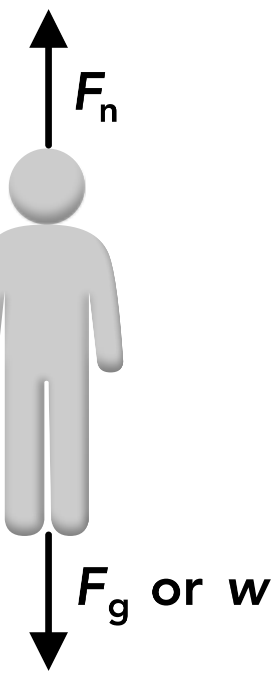
The apparent weight (normal force) is equal to the actual weight if the acceleration is zero



$m = 80\ \text{kg}$

$F_g = mg = 784\ \text{N}$

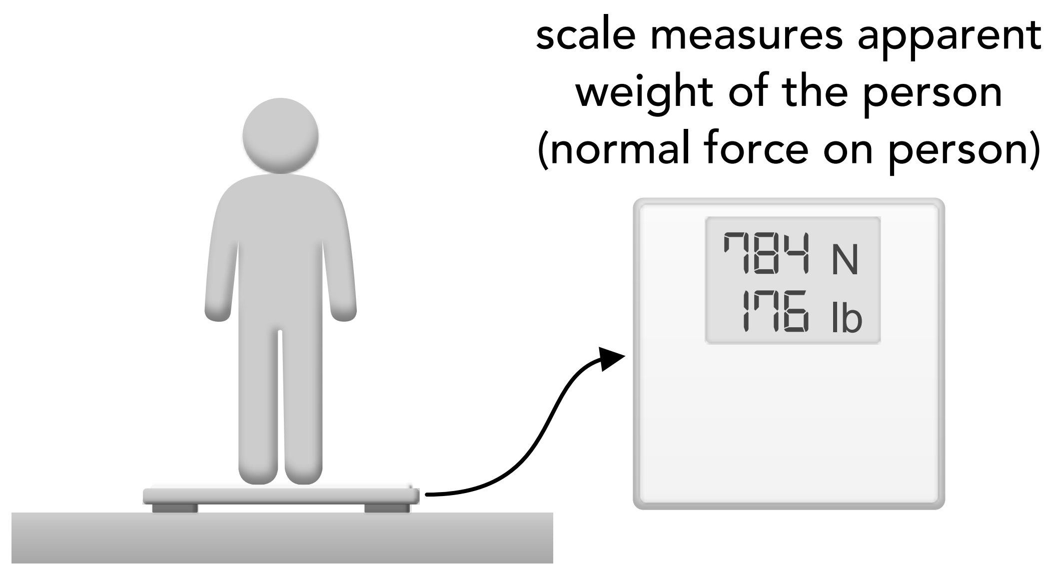
$a_y = 0\ \text{m/s}^2$



$F_n - F_g = m(0\ \text{m/s}^2)$

$F_n = F_g$

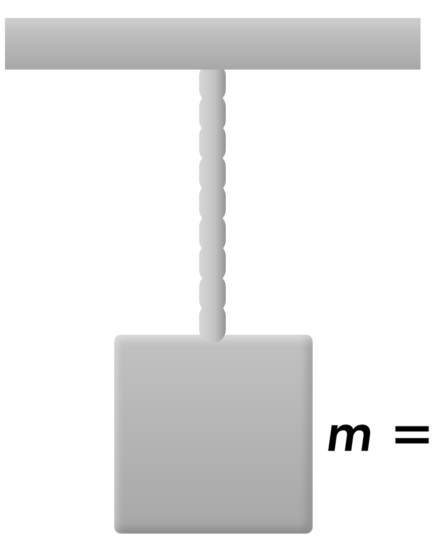
$F_n = 784\ \text{N}$



$F_g = 784\ \text{N}$ ⟵ weight

$F_n = 784\ \text{N}$ ⟵ apparent weight

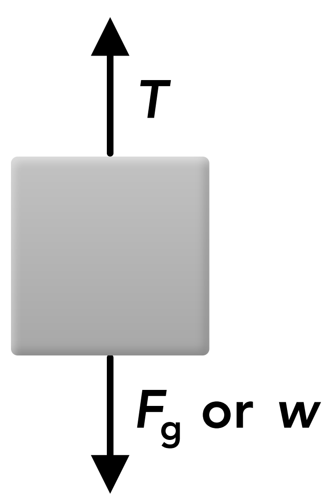
The apparent weight (tension force) is equal to the actual weight if the acceleration is zero



$m = 10\ \text{kg}$

$F_g = mg = 98\ \text{N}$

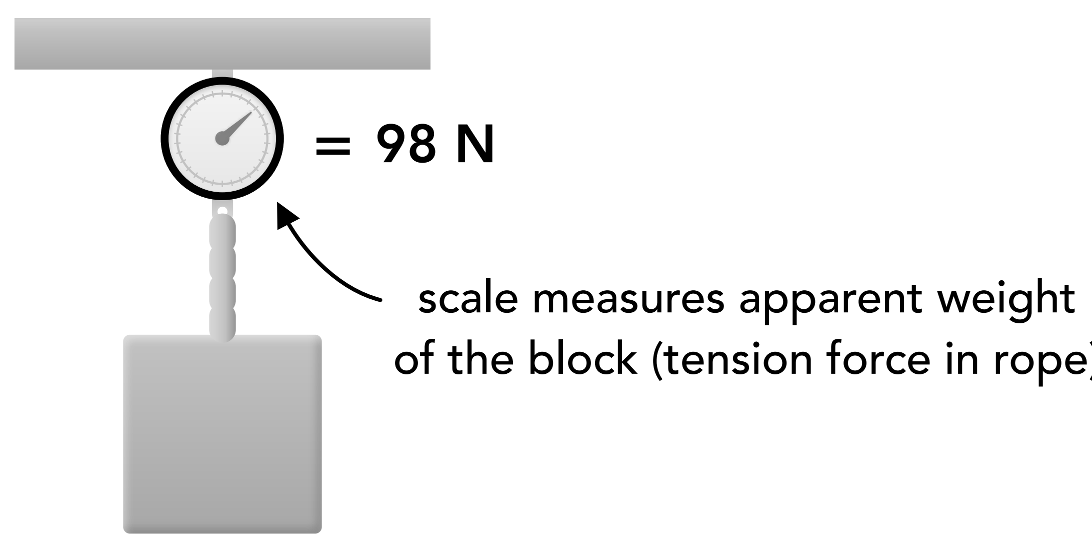
$a_y = 0\ \text{m/s}^2$



$T - F_g = m(0\ \text{m/s}^2)$

$T = F_g$

$T = 98\ \text{N}$



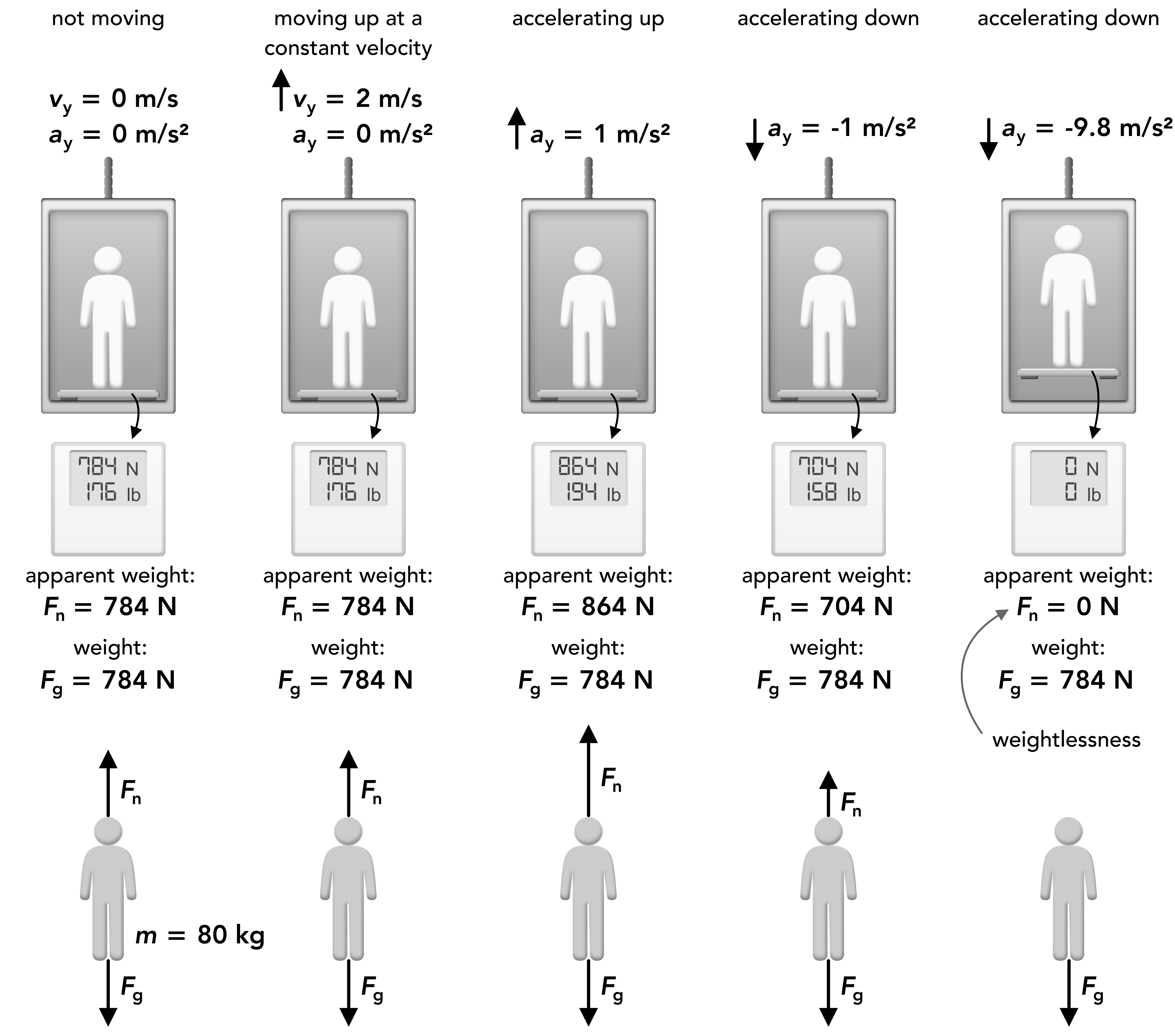
$F_g = 98\ \text{N}$ ⟵ weight

$T = 98\ \text{N}$ ⟵ apparent weight

- **If an object is accelerating up or down then the apparent weight is not equal to the weight**. The net force in the vertical direction is not zero so the normal force (or the tension force) is likely not equal to the weight force.
- Imagine you're standing on a scale in an elevator. If the elevator is not moving (the acceleration is zero) your apparent weight is equal to your weight. If the elevator is moving at a constant velocity (the acceleration is still zero) your apparent weight is still equal to your weight. But if you and the elevator are accelerating up or down then your apparent weight is not equal to your actual weight - you'll feel lighter or heavier than your true weight.
- For each of the following examples the elevator has a different motion. Look through the free body diagrams and Newton's 2nd law equations to see how the acceleration affects the apparent weight of the person.

The apparent weight of the person changes if the elevator is accelerating.

The actual weight of person is always: $F_g = mg = 784 \text{ N}$



$$\sum F_y = ma_y$$
$$F_n - F_g = (80 \text{ kg})(0 \text{ m/s}^2)$$
$$F_n = F_g$$
$$F_n = 784 \text{ N}$$

$$\sum F_y = ma_y$$
$$F_n - F_g = (80 \text{ kg})(0 \text{ m/s}^2)$$
$$F_n = F_g$$
$$F_n = 784 \text{ N}$$

$$\sum F_y = ma_y$$
$$F_n - F_g = (80 \text{ kg})(1 \text{ m/s}^2)$$
$$F_n = F_g + 80 \text{ N}$$
$$F_n = 864 \text{ N}$$

$$\sum F_y = ma_y$$
$$F_n - F_g = (80 \text{ kg})(-1 \text{ m/s}^2)$$
$$F_n = F_g - 80 \text{ N}$$
$$F_n = 704 \text{ N}$$

$$\sum F_y = ma_y$$
$$F_n - F_g = (80 \text{ kg})(-9.8 \text{ m/s}^2)$$
$$F_n = F_g - 784 \text{ N}$$
$$F_n = 0 \text{ N}$$