5 A curve has equation $x^3 - 3x^2y + y^2 + 1 = 0$.

(a) Show that $\frac{dy}{dx} = \frac{6xy - 3x^2}{2y - 3x^2}$. **[4]**

(b) Find the equation of the normal to the curve at the point (1, 2). **[4]**

6 Let $f(x) = 2x^3 + 3x$. Use differentiation from first principles to show that $f'(x) = 6x^2 + 3$. **[6]**

7 **In this question you must show detailed reasoning.**

A sequence $u_1, u_2, u_3 \ldots$ is defined by $u_n = 25 \times 0.6^n$.

Use an algebraic method to find the smallest value of N such that $\sum_{n=1}^{\infty} u_n - \sum_{n=1}^{N} u_n < 10^{-4}$. **[8]**

8 A cylindrical tank is initially full of water. There is a small hole at the base of the tank out of which the water leaks.

The height of water in the tank is x m at time t seconds. The rate of change of the height of water may be modelled by the assumption that it is proportional to the square root of the height of water.

When $t = 100$, $x = 0.64$ and, at this instant, the height is decreasing at a rate of $0.0032\,\text{ms}^{-1}$.

(a) Show that $\frac{dx}{dt} = -0.004\sqrt{x}$. **[2]**

(b) Find an expression for x in terms of t. **[4]**

(c) Hence determine at what time, according to this model, the tank will be empty. **[2]**

9 **(a)** Express $3\cos 3x + 7\sin 3x$ in the form $R\cos(3x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. **[3]**

(b) Give full details of a sequence of three transformations needed to transform the curve $y = \cos x$ to the curve $y = 3\cos 3x + 7\sin 3x$. **[4]**

(c) Determine the **greatest** value of $3\cos 3x + 7\sin 3x$ as x varies and give the smallest positive value of x for which it occurs. **[2]**

(d) Determine the **least** value of $3\cos 3x + 7\sin 3x$ as x varies and give the smallest positive value of x for which it occurs. **[2]**