2D MOTION & VECTORS

Vectors

1. A vectors can be described using which of the following? (Select all that apply)
   - A An arrow with a length and direction
   - B A single value
   - C A magnitude and an angle
   - D Two components, each with a direction

2. Which of the following always has a positive value? (Select all that apply)
   - A The angle of a vector
   - B The magnitude of a vector
   - C The components of a vector
   - D The reference angle of a vector

3. If vector $\vec{A}$ has a magnitude of 12 and vector $\vec{B}$ has a magnitude of 27, which of the following could be a possible magnitude of vector $\vec{C}$ if $\vec{C} = \vec{A} + \vec{B}$? (Select all that apply)
   - A 40
   - B 30
   - C 20
   - D 10

4. A vector will have no x component in which of the following cases? (Select all that apply)
   - A The vector is parallel to the y axis
   - B The vector has an angle of **0°**
   - C The vector is parallel to the x axis
   - D The vector has an angle of **90°**

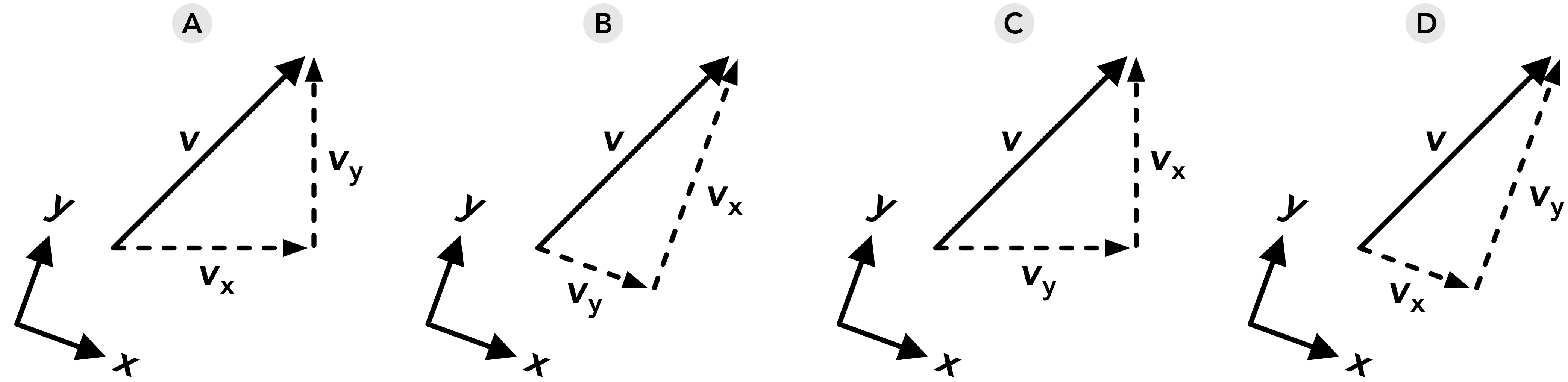
5. At what angle will a vector's x and y components have equal magnitude and sign (+/-)?
   - A **0°**
   - B **45°**
   - C **135°**
   - D **180°**

6. A vector has an angle of 155°. At what other angle would the vector have the same x component? (Select all that apply)
   - A 25°
   - B 65°
   - C 205°
   - D -155°

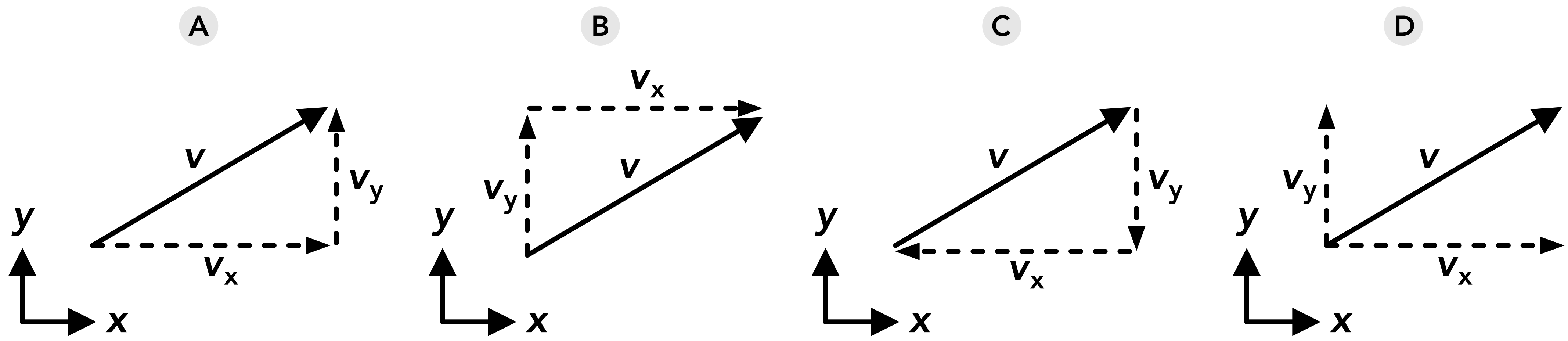
7. A vector has an angle of 60°. If the vector rotates to an angle of 30° the magnitude of the y component will
   - A increase
   - B decrease
   - C stay the same
   - D cannot be determined

8. A vector has an angle of 120°. If the vector rotates to an angle of 170° the magnitude of the *x* component will
   - A increase
   - B decrease
   - C stay the same
   - D cannot be determined

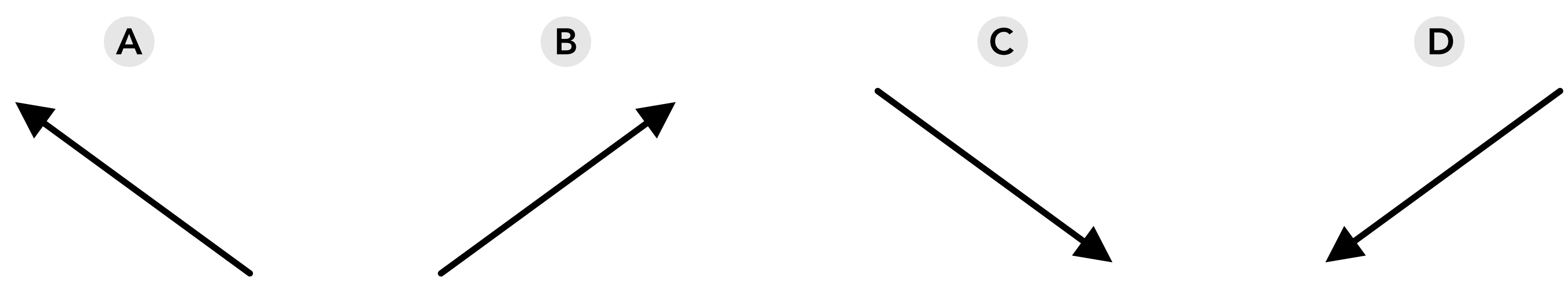
9. Which of the following shows the correct *x* and *y* components of vector $\vec{v}$ based on the axes shown?

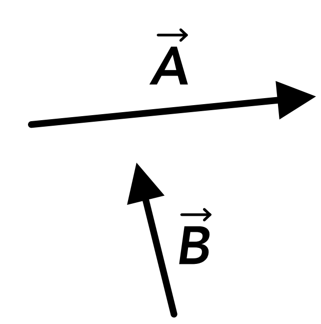


10. Which of the following is a correct way to draw the components of vector $\vec{v}$? (Select all that apply)

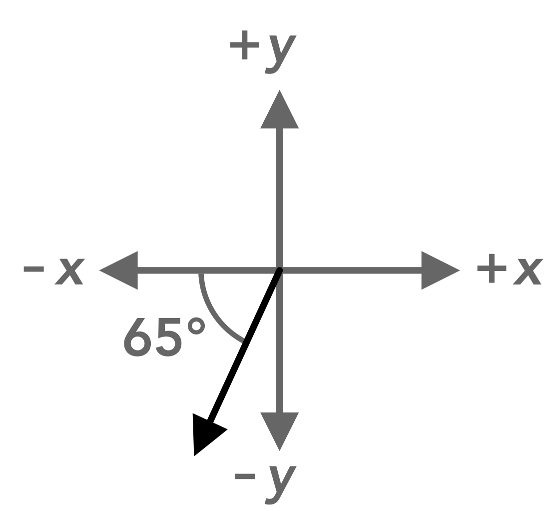


11. Which of the following would be the resultant vector of $\vec{A} + \vec{B}$ shown on the right?





12. What is the angle of the vector shown on the right using convention?
    What is the reference angle of the vector?



13. Which of the following could be considered two-dimensional (2D) motion? (Select all that apply)

A A ball which is thrown directly upwards and then falls straight down
B A soccer ball kicked upwards at an angle
C A car driving up an inclined ramp
D A car driving on a flat, straight road

14. A person is standing at a position with coordinates (7, 11) on a coordinate system where "right" is the positive ***x*** direction and "up" is the positive ***y*** direction. They walk 15 units to the right, then 6 units up, then 3 units to the left, then 2 units down. What are their new coordinates?

A $(25, 19)$
B $(19, 15)$
C $(-5, 7)$
D $(20, 9)$

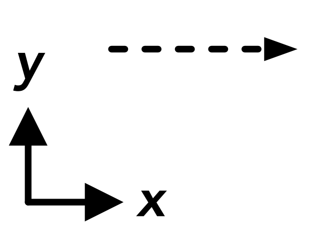
15. A person moves from coordinates (2 m, 3 m) to coordinates (-10 m, -9 m). What is the magnitude of their displacement?

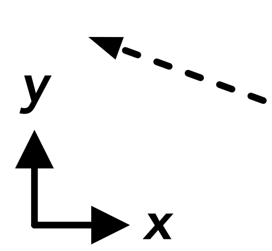
A -12 m
B 12 m
C -17 m
D 17 m

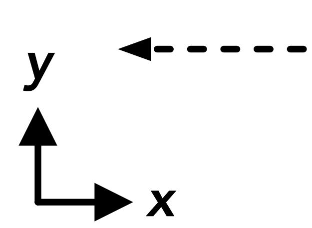
16. A car drives 1.5 km in the negative ***x*** direction and then drives 2.6 km in the positive ***y*** direction. What is the angle of the car's total displacement vector (using convention)?

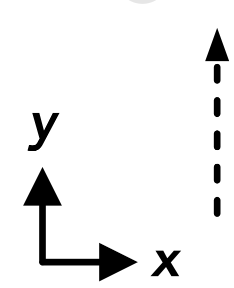
A -60°
B -120°
C 120°
D 60°

17. A person walks 25 m at an angle of 160°. Which of the following is the direction of the ***x*** component of their displacement vector?

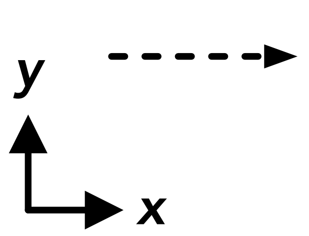
A


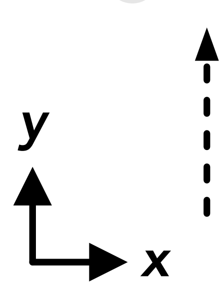
B


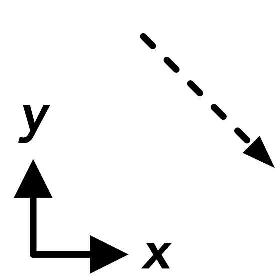
C


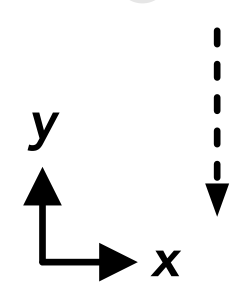
D


18. A person walks 18 m at an angle of -45°. Which of the following is the direction of the ***y*** component of their displacement vector?

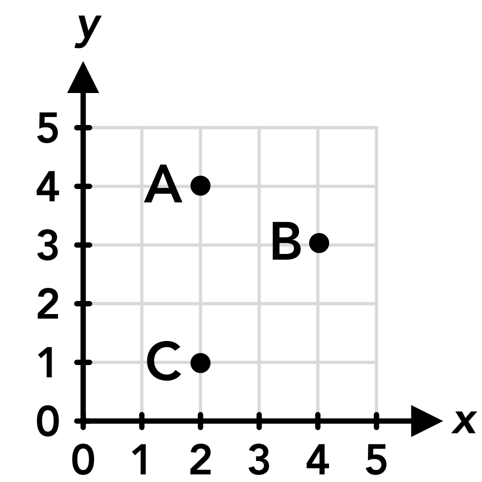
A


B


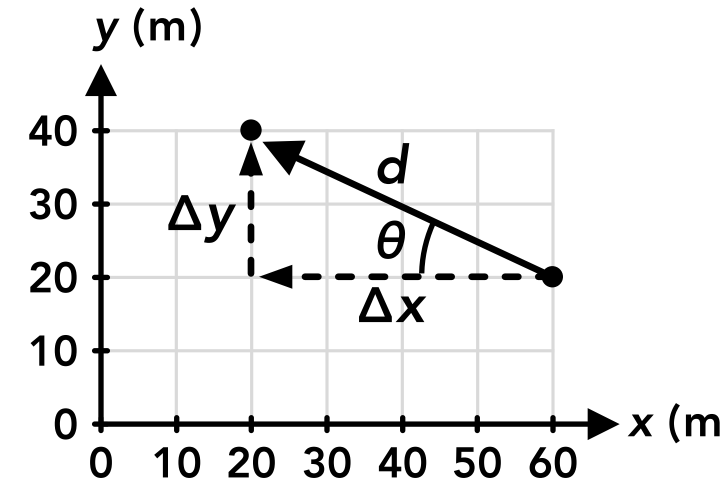
C


D


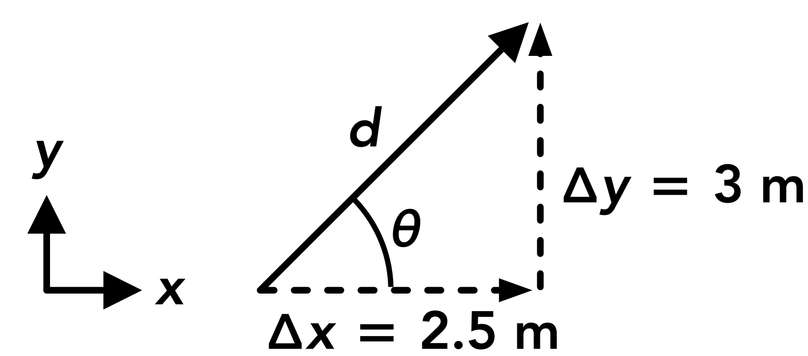
19. What are the coordinates of points A, B and C shown on the right?



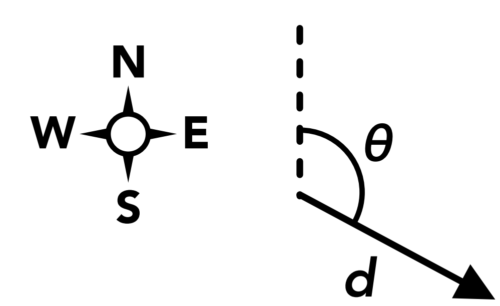
20. A person walks a displacement shown by the vector ***d*** on the right. What are the values of the ***x*** component, ***y*** component, magnitude ***d*** and angle ***θ*** of their displacement vector?



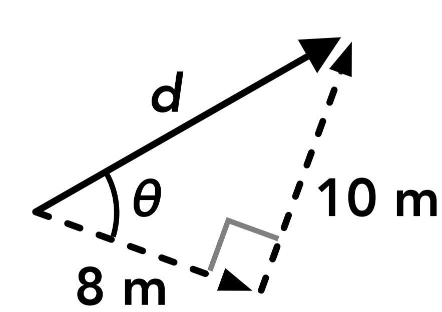
21. What are the magnitude and angle of the vector shown on the right?



22. If you walk along the displacement vector ***d*** shown on the right, where ***d*** is 2 km and ***θ*** is 130° clockwise from due north, how far east and how far south did you walk?



23. What are the magnitude and angle of the vector shown on the right?



24. Person A is at a position with coordinates (2, 3), person B is at coordinates (8, 6), and person C is at coordinates (4, 8). Who is person C closer to: person A or person B?

25. If you walk 2 km at an angle of 40°, and then walk 3 km at an angle of 10°, what is the magnitude and angle of your total displacement?

26. A person walks 45 m west, then walks 22 m north, and then walks 8 m east. What is the magnitude of their total displacement?

27. An ant walks 35 cm straight along the ground towards a wall and then walks 18 cm straight up the wall. What is the angle of the ant's total displacement relative to the ground?

Velocity

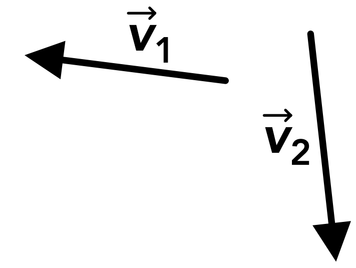
28. An object's ***x*** velocity component and ***y*** velocity component are
    - A always independent of each other
    - B independent of each other, unless one of them is changing
    - C never independent of each other in 2D motion
    - D none of the above

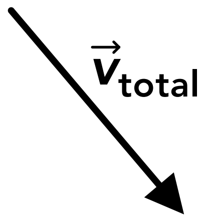
29. An instantaneous velocity vector shows which of the following?
    - A The path of an object's motion over a period of time
    - B The distance an object travels over a period of time
    - C The direction of motion at an instant in time
    - D The instantaneous change in an object's velocity

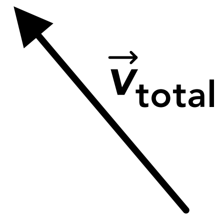
30. A car has a velocity vector ***v*** with an angle of 30°. If only the ***y*** component of the velocity is doubled, the car's new velocity vector will have
    - A the same magnitude and direction
    - B a larger magnitude and the same direction
    - C the same magnitude and a different direction
    - D a larger magnitude and a different direction

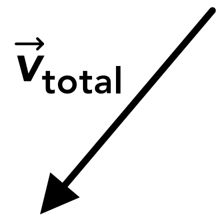
31. A biker is riding at 5 m/s at an angle of -50°. If the biker maintains their speed but turns so their velocity vector is at an angle of -80°, the magnitude of the ***x*** component of the velocity vector
    - A increases
    - B decreases
    - C stays the same
    - D cannot be determined

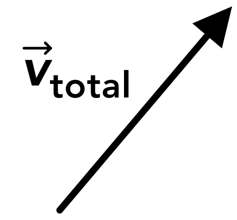
32. Which of the following shows the resultant velocity vector of $\vec{v}_1 + \vec{v}_2$ shown on the right?



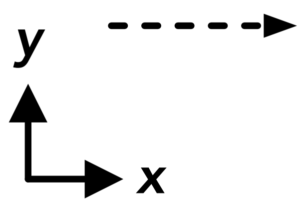
A


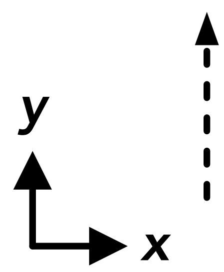
B


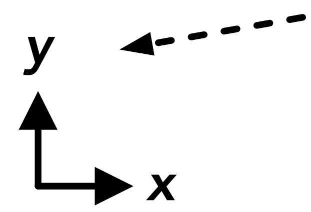
C


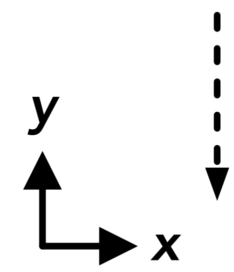
D


33. A boat is moving at 3 m/s at an angle of 190°. What is the direction of the ***y*** component of its velocity vector?

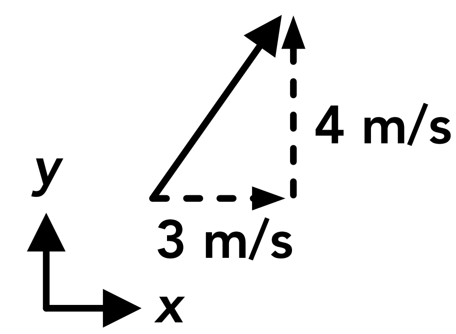
A


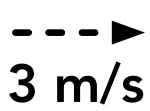
B


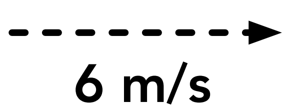
C


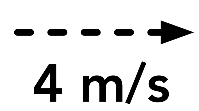
D


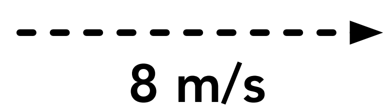
34. A runner has a velocity vector as shown on the right, and then a very strong wind causes their ***y*** velocity component to double. What is their new ***x*** velocity component?



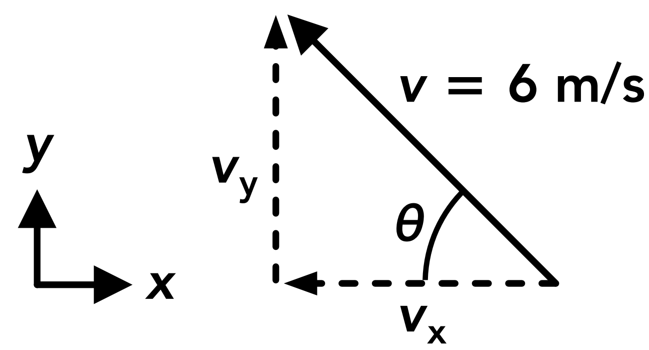
A


B


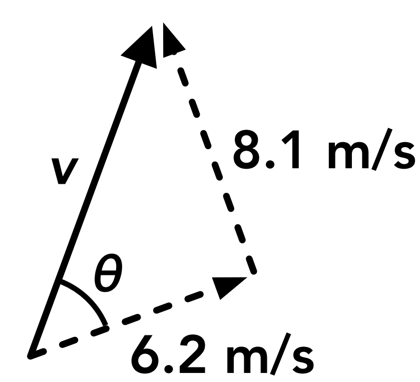
C


D


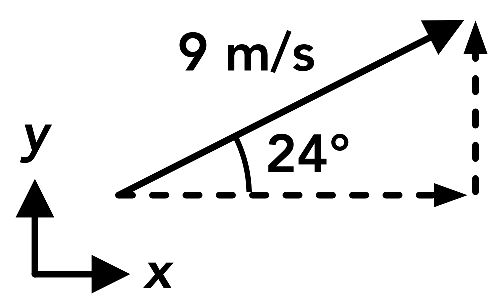
35. What are the values of the components v_x and v_y for the vector shown on the right if the angle θ is 40°?



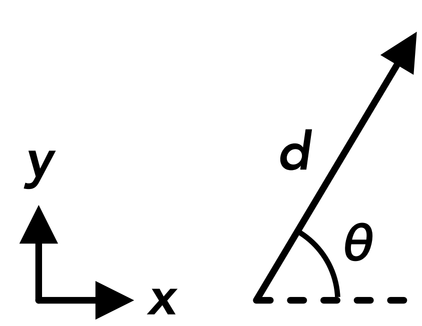
36. A car has a velocity vector with components shown on the right. What are the magnitude ***v*** and angle θ of the vector?



37. If an object has a constant velocity shown by the vector on the right, how far will it travel in the ***y*** direction over a period of 2.6 seconds?



38. If an object travels a displacement of 42 m at an angle of 58° as shown with the vector on the right, what is the ***x*** velocity component if it takes 8 seconds to travel that displacement?



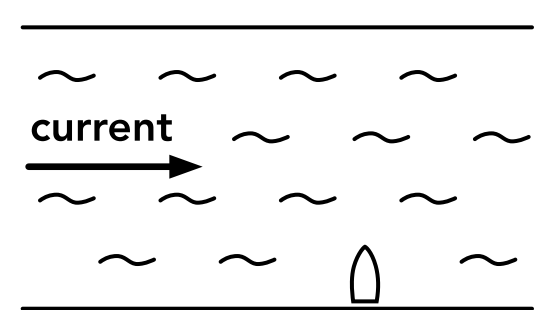
39. A boat is moving at a constant 7 m/s and an angle of 38° as seen from above, using a coordinate system on the ground. How long will it take the boat to move 32 m in the ***x*** direction?

40. A person walking at a constant velocity moves from a position with coordinates (9 m, 2 m) to coordinates (2 m, 5 m) in a period of 3 seconds. What is the magnitude and conventional angle of the person's velocity?

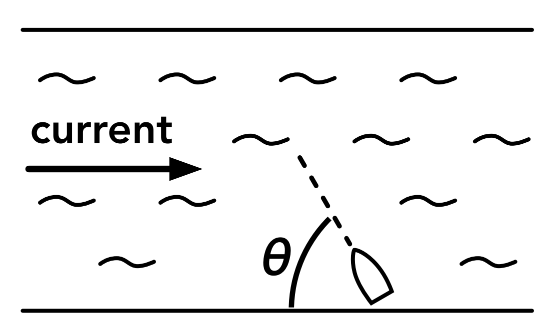
41. An object has a velocity of 4 m/s at an angle of 100°. How far does the object travel in the ***y*** direction over a period of 12 seconds?

42. A person runs at a constant speed of 2 m/s from the bottom to the top of a ramp with a height of 2 m and an angle of 15° relative to the ground. How long does it take them to reach the top?

43. A boat can travel at a speed of 3.5 m/s on its own. If the boat points straight across a river that is 62 m across and there is a sideways current of 1.5 m/s, how far does the boat move sideways by the time it reaches the other side?



44. A boat can travel at a speed of 3.5 m/s on its own. The driver wants to cross a river, and there is a sideways current of 1.5 m/s. If the driver wants to move straight across the river (so the boat does not move left or right), what angle should the driver point the boat? Find the reference angle between the direction the boat is pointing and the starting shoreline.



Answers

1. A, C, D
2. B, D
3. B, C
4. A, D
5. B
6. C, D
7. B
8. A
9. D
10. A, B, D
11. B
12. Conventional angle = 245° or -115°
    Reference angle = 65°
13. B, C
14. B
15. D
16. C
17. C
18. D
19. A = (2, 4), B = (4, 3), C = (2, 1)
20. **Δx = -40 m, Δy = 20 m**
    d = 44.7 m, θ = 26.6°
21. 3.9 m, 50.2°
22. 1.5 km east, 1.3 km south
23. 12.8 m, 51.3°
24. Person B
25. 4.8 km, 22°
26. 43.1 m
27. 27.2°
28. A
29. C
30. D
31. B
32. C
33. D
34. A
35. **v_x = -4.6 m/s, v_y = 3.9 m/s**
36. 10.2 m/s, 52.6°
37. 9.5 m
38. 2.8 m/s
39. 5.8 s
40. 2.5 m/s, 156.8°
41. 47.3 m
42. 3.9 s
43. 26.6 m
44. 64.6°

Answers - Vectors

1. **Answer: A, C, D**
   A vector can be described using an arrow with a length and direction, with a magnitude and an angle, and with two components. A vector cannot be described using a single value because it must have both a magnitude and a direction.

2. **Answer: B, D**
   The magnitude of a vector is always positive. The reference angle of a vector is the smallest angle between the vector and the horizontal (or another specified axis) and is always positive. The conventional angle of a vector can be positive or negative (counterclockwise is positive and clockwise is negative). The components of a vector can be positive or negative depending on which direction they are pointing relative to the coordinate system.

3. **Answer: B, C**
   When two vectors are added they form a resultant vector. The maximum magnitude of the resultant vector would be when the vectors point in the same direction and the resultant vector magnitude is the sum of the individual vector magnitudes. The minimum magnitude of the resultant vector would be when the two vectors point in opposite directions and the resultant vector magnitude is the difference between the individual vector magnitudes (one vector magnitude is subtracted from the other). In this problem, the minimum magnitude of the resultant vector is 27 - 12 = 15, and the maximum magnitude is 27 + 12 = 39, so 20 and 30 are possible magnitudes of the resultant vector (they both fall between 15 and 39).

4. **Answer: A, D**
   If a vector is parallel to the y axis then the vector does not have an x component. If a vector has an angle of 90° the vector is parallel to the y axis and again does not have an x component. If a vector is parallel to the x axis (which is the same as having an angle of 0°) then the vector has an x component that is equal in magnitude to full vector itself, and the vector does not have a y component.

5. **Answer: B**
   The x and y components of a vector with a magnitude of v are given below for each angle. At an angle of 45° the components would both be positive and would be equal in magnitude.
   0°: $v_x = v\cos(0°) = 1v$ $v_y = v\sin(0°) = 0v$
   45°: $v_x = v\cos(45°) = 0.71v$ $v_y = v\sin(45°) = 0.71v$
   135°: $v_x = v\cos(135°) = -0.71v$ $v_y = v\sin(135°) = 0.71v$
   180°: $v_x = v\cos(180°) = -1v$ $v_y = v\sin(180°) = 0v$

6. **Answer: C, D**
   The x component of a vector with a magnitude of v and an angle of 155° would be $v_x = v\cos(155°) = -0.91v$
   The x component of a vector with an angle of 205° or an angle of -155° would have the same x component (with the same magnitude and direction or sign).
   25°: $v_x = v\cos(25°) = 0.91v$
   65°: $v_x = v\cos(65°) = 0.42v$
   205°: $v_x = v\cos(205°) = -0.91v$
   -155°: $v_x = v\cos(-155°) = -0.91v$

7. **Answer: B**
   The y component of a vector with a magnitude of v and an angle of 60° and 30° are given below. If the vector magnitude stays the same but the angle changes from 60° to 30° the magnitude of the y component decreases.
   60°: $v_y = v\sin(60°) = 0.87v$ magnitude of $v_y = |v_y| = 0.87v$
   30°: $v_y = v\sin(30°) = 0.5v$ magnitude of $v_y = |v_y| = 0.5v$

8. **Answer: A**
   The x component of a vector with a magnitude of v and an angle of 120° and 170° are given below. If the vector magnitude is the same but the angle changes from 120° to 170° the magnitude of the x component increases.
   120°: $v_x = v\cos(120°) = -0.5v$ magnitude of $v_x = |v_x| = 0.5v$
   170°: $v_x = v\cos(170°) = -0.98v$ magnitude of $v_x = |v_x| = 0.98v$

9. **Answer: D**
   The components must be parallel to their respective axes and point in the same direction as the vector.

10. **Answer: A, B, D**
    The components must be parallel to their respective axes and point in the same direction as the vector. The components can be drawn in different positions. In option C the components point in the opposite direction as the vector.

11. **Answer: B**
    Vectors $\vec{A}$ and $\vec{B}$ can be added together using the tip-to-tail method: the tail (starting end) of vector $\vec{B}$ is placed at the tip (arrow end) of vector $\vec{A}$. Then the resultant vector is drawn from the start of vector $\vec{A}$ to the end of vector $\vec{B}$.

12. **Answer: Conventional angle = 245° or -115°, reference angle = 65°**
    Using convention, the positive angle of a vector is measured counterclockwise from the positive x axis and the negative angle is measured clockwise from the positive x axis. The reference angle is the smallest angle between the vector and the horizontal (positive or negative x axis).

Answers - Position and displacement

13. **Answer: B, C**
Options B and C are considered 2D motion because they include an object which is moving in the horizontal and vertical directions at the same time. Options A and D are considered 1D motion because they include objects that only move in one dimension (either the vertical or horizontal directions).

14. **Answer: B**
The x coordinate of their final position is: 7 + 15 - 3 = 19
The y coordinate of their final position is: 11 + 6 - 2 = 15.

15. **Answer: D**
The x component of their displacement vector is: $\Delta x = x_f - x_i = (-10\text{ m}) - (2\text{ m}) = -12\text{ m}$
The y component of their displacement vector is: $\Delta y = y_f - y_i = (-9\text{ m}) - (3\text{ m}) = -12\text{ m}$
The magnitude of their displacement vector is: $d = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(-12\text{ m})^2 + (-12\text{ m})^2} = 17.0\text{ m}$

16. **Answer: C**
The $x$component of the total displacement vector is -1.5 km and the y component is 2.6 km.
The reference angle between the vector and the x component must be found first: $\theta = \tan^{-1}\left(\frac{2.6}{1.5}\right) = 60°$
The conventional angle (counterclockwise from the positive x axis) is: $180° - 60° = 120°$

17. **Answer: C**
The magnitude and direction of the x component can be found as shown below. The x component is negative so the component points in the negative x direction (and must be parallel to the x axis).
$\Delta x = (25\text{ m})\cos(160°) = -23.5\text{ m}$

18. **Answer: D**
The magnitude and direction of the y component can be found as shown below. The y component is negative so the component points in the negative y direction (and must be parallel to the y axis).
$\Delta y = (18\text{ m})\sin(-45°) = -12.7\text{ m}$

19. **Answer: A = (2, 4), B = (4, 3), C = (2, 1)**
The coordinates for each point list the position values as (x position, y position).

20. **Answer: $\Delta x = -40\text{ m}$, $\Delta y = 20\text{ m}$, $d = 44.7\text{ m}$, $\theta = 26.6$**
The magnitude and angle of the vector is not given so the trig functions cannot be used to find the components. Each component is equal to the final coordinate value minus the initial coordinate value for that axis (x or y).
$\Delta x = x_f - x_i = (20\text{ m}) - (60\text{ m}) = -40\text{ m}$
$\Delta y = y_f - y_i = (40\text{ m}) - (20\text{ m}) = 20\text{ m}$
The magnitude and angle of the vector can be found using the components, the Pythagorean theorem and the inverse tangent function.
$d^2 = \Delta x^2 + \Delta y^2 \qquad d = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(-40\text{ m})^2 + (20\text{ m})^2} = 44.7\text{ m}$
$\theta = \tan^{-1}\left(\frac{20}{40}\right) = 26.6°$

21. **Answer: d = 3.9 m, θ = 50.2°**

The magnitude can be found using the Pythagorean theorem:

$d^2 = \Delta x^2 + \Delta y^2 \qquad d = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(2.5\text{ m})^2 + (3\text{ m})^2} = 3.9\text{ m}$

The angle can be found using the inverse tangent function:

$$\theta = \tan^{-1}\left(\frac{3\text{ m}}{2.5\text{ m}}\right) = 50.2°$$

22. **Answer: 1.5 km east, 1.3 km south**

We can first find the reference angle between the vector and the east direction: **130° − 90° = 40°**

The eastward component of the displacement vector is: **(2 km)cos(40°) = 1.5 km east**

The southward component of the displacement vector is: **(2 km)sin(40°) = 1.3 km south**

23. **Answer: d = 12.8 m, θ = 51.3°**

The magnitude and angle of the vector can be found using the Pythagorean theorem and the inverse tangnent function.

$d = \sqrt{(8\text{ m})^2 + (10\text{ m})^2} = 12.8\text{ m}$

$$\theta = \tan^{-1}\left(\frac{10\text{ m}}{8\text{ m}}\right) = 51.3°$$

24. **Answer: Person B**

The straight line distance between any two people is the magnitude of the displacement vector between them. The distance can be found using the Pythagorean theorem, where each component is the difference between the coordinates of each person.

The distance between person C and person A is: $d = \sqrt{(x_C - x_A)^2 + (y_C - y_A)^2} = \sqrt{(4 - 2)^2 + (8 - 3)^2} = 5.4$

The distance between person C and person B is: $d = \sqrt{(x_C - x_B)^2 + (y_C - y_B)^2} = \sqrt{(4 - 8)^2 + (8 - 6)^2} = 4.5$

25. **Answer: 4.8 km, 22°**

First, the components of each individual displacement vector must be found and added together.

$\Delta x_1 = (2\text{ km})\cos(40°) = 1.53\text{ km} \qquad \Delta y_1 = (2\text{ km})\sin(40°) = 1.29\text{ km}$

$\Delta x_2 = (3\text{ km})\cos(10°) = 2.95\text{ km} \qquad \Delta y_2 = (3\text{ km})\sin(10°) = 0.52\text{ km}$

$\Delta x_{total} = (1.53\text{ km}) + (2.95\text{ km}) = 4.48\text{ km} \qquad \Delta y_{total} = (1.29\text{ km}) + (0.52\text{ km}) = 1.81\text{ km}$

Then the magnitude and angle of the total displacement vector can be found using the total x and y components, the Pythagorean theorem and the inverse tangent function.

$d = \sqrt{(4.48\text{ km})^2 + (1.81\text{ km})^2} = 4.8\text{ km}$

$$\theta = \tan^{-1}\left(\frac{1.81}{4.48}\right) = 22°$$

26. **Answer: 43.1 m**

If we represent east as the positive x direction and north as the positive y direction, the total x component is the sum of the west and east displacements and the total y component is the north displacement.

$\Delta x = (-45\text{ m}) + (8\text{ m}) = -37\text{ m}$

$\Delta y = 22\text{ m}$

The magnitude of the total displacement vector can be found using the Pythagorean theorem.

$d = \sqrt{(-37\text{ m})^2 + (22\text{ m})^2} = 43.1\text{ m}$

27. **Answer: 27.2°**
The horizontal component of the total displacement vector is 35 cm and the vertical component of the total displacement vector is 18 cm. The angle of the total displacement vector relative to the horizontal (the ground) can be found using the inverse tangent function.

$$\theta = \tan^{-1}\left(\frac{18}{35}\right) = 27.2°$$

Answers - Velocity

28. **Answer: A**
The ***x*** and ***y*** components of an object's motion are always independent of each other. The motion in one direction does not affect the motion in the other direction.

29. **Answer: C**
An instantaneous velocity vector shows the direction of an object's motion (and the magnitude of the velocity) at an instant in time. It does not show anything about the motion over a period of time. An instantaneous acceleration vector would show the instantaneous change in an object's velocity.

30. **Answer: D**
The ***x*** component of the velocity vector will not change because the ***x*** and ***y*** components are independent.
The angle of the vector is related to the components by: $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$
The magnitude of the vector is related to the components by: $v = \sqrt{v_x^2 + v_y^2}$
If the ***y*** component is doubled (or increases by any amount) the magnitude (length) of the vector will increase and the angle (direction) of the vector will change.

31. **Answer: B**
The ***x*** component of the velocity vectors at each angle are given below.
-50°: $v_x = (5\text{ m/s})\cos(-50°) = 3.2\text{ m/s}$
-80°: $v_x = (5\text{ m/s})\cos(-80°) = 0.9\text{ m/s}$

32. **Answer: C**
Vectors $\vec{v}_1$ and $\vec{v}_2$ can be added together using the tip-to-tail method: the tail (starting end) of vector $\vec{v}_2$ is placed at the tip (arrow end) of vector $\vec{v}_1$. Then the resultant vector is drawn from the start of vector $\vec{v}_1$ to the end of vector $\vec{v}_2$.

33. **Answer: D**
The magnitude and direction of the ***y*** component can be found as shown below. The ***y*** component is negative so the component points in the negative ***y*** direction (and must be parallel to the ***y*** axis).
$v_y = (3\text{ m/s})\sin(190°) = -0.5\text{ m/s}$

34. **Answer: A**
The ***x*** and ***y*** components of the velocity are independent of each other so the ***x*** component does not change.

35. **Answer: $v_x = -4.6\text{ m/s}$, $v_y = 3.9\text{ m/s}$**
Using the reference angle of 40° and the directions of the components relative to the ***x*** and ***y*** axes:
$v_x = (6\text{ m/s})\cos(40°) = 4.6\text{ m/s}$ v_x points in the negative ***x*** direction so $v_x = -4.6$ m/s
$v_y = (6\text{ m/s})\sin(40°) = 3.9\text{ m/s}$ v_y points in the positive ***y*** direction so $v_y = 3.9$ m/s
Using the conventional angle of 140° (180° - 40° = 140°):
$v_x = (6\text{ m/s})\cos(140°) = -4.6\text{ m/s}$
$v_y = (6\text{ m/s})\sin(140°) = 3.9\text{ m/s}$

36. **Answer:** v **= 10.2 m/s,** θ **= 52.6°**

The magnitude and angle can be found by using the Pythagorean theorem and the inverse tangent function.

$$v = \sqrt{(6.2 \text{ m/s})^2 + (8.1 \text{ m/s})^2} = 10.2 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{8.1}{6.2}\right) = 52.6°$$

37. **Answer: 9.5 m**

First we can find the ***y*** component of the velocity vector (the velocity of the object in the ***y*** direction):

$$v_y = (9 \text{ m/s})\sin(24°) = 3.66 \text{ m/s}$$

The displacement of the object in the ***y*** direction can be found using the kinematic equation for velocity:

$$v_y = \frac{\Delta y}{\Delta t} \qquad (3.66 \text{ m/s}) = \frac{\Delta y}{(2.6 \text{ s})} \qquad \Delta y = 9.5 \text{ m}$$

38. **Answer: 2.8 m/s**

First we can find the ***x*** component of the displacement vector (the displacement of the object in the ***x*** direction):

$$\Delta x = (42 \text{ m})\cos(58°) = 22.26 \text{ m}$$

The velocity of the object in the ***x*** direction can be found using the kinematic equation for velocity:

$$v_x = \frac{\Delta x}{\Delta t} = \frac{(22.6 \text{ m})}{(8 \text{ s})} = 2.8 \text{ m/s}$$

Alternatively, we could have found the velocity vector using the magnitude of the displacement and the time, and then found the ***x*** component of the velocity vector.

39. **Answer: 5.8 s**

First we can find the ***x*** component of the velocity vector (the velocity of the boat in the ***x*** direction):

$$v_x = (7 \text{ m/s})\cos(38°) = 5.52 \text{ m/s}$$

The time it takes the boat to move 32 m in the ***x*** direction can be found using the equation for velocity:

$$v_x = \frac{\Delta x}{\Delta t} \qquad (5.52 \text{ m/s}) = \frac{(32 \text{ m})}{\Delta t} \qquad \Delta t = 5.8 \text{ s}$$

40. **Answer: 2.5 m/s, 156.8°**

The velocity vector is in the same direction as the displacement vector. We can first find the components of the displacement vector:

$$\Delta x = (2 \text{ m}) - (9 \text{ m}) = -7 \text{ m}$$

$$\Delta y = (5 \text{ m}) - (2 \text{ m}) = 3 \text{ m}$$

The magnitude of the displacement vector is: $d = \sqrt{(-7 \text{ m})^2 + (3 \text{ m})^2} = 7.62 \text{ m}$

The magnitude of the velocity vector can be found using the kinematic equation for velocity:

$$v = \frac{d}{\Delta t} = \frac{7.62 \text{ m}}{3 \text{ s}} = 2.5 \text{ m/s}$$

The reference angle between the velocity vector and the ***x*** component is: $\theta = \tan^{-1}\left(\frac{3}{7}\right) = 23.2°$

The conventional angle of the velocity vector (counterclockwise from the +***x*** axis) is: $180° - 23.2° = 156.8°$

41. **Answer: 47.3 m**

First we can find the ***y*** component of the velocity vector: $v_y = (4 \text{ m/s})\sin(100°) = 3.94 \text{ m/s}$

The displacement in the ***y*** direction can be found using the kinematic equation for velocity:

$$v_y = \frac{\Delta y}{\Delta t} \qquad (3.94 \text{ m/s}) = \frac{\Delta y}{(12 \text{ s})} \qquad \Delta y = 47.3 \text{ m}$$

42. **Answer: 3.9 s**

The magnitude of the displacement vector (the length of the ramp) can be found using right triangle trig:

$$\sin(15°) = \frac{2 \text{ m}}{d} \qquad d = 7.73 \text{ m}$$

The period of time can be found using the kinematic equation for velocity:

$$v = \frac{d}{\Delta t} \qquad (2 \text{ m/s}) = \frac{(7.73 \text{ m})}{\Delta t} \qquad \Delta t = 3.9 \text{ s}$$

43. **Answer: 26.6 m**

The component of the boat's velocity vector that is straight across the river is 3.5 m/s because the boat points straight across the river (the current does not affect this velocity component). The boat also has a sideways velocity component equal to the speed of the river, but the time it takes the boat to cross the river only depends on the 3.5 m/s velocity component straight across the river. Using ***y*** for the direction straight across the river:

$$v_y = \frac{\Delta y}{\Delta t} \qquad (3.5 \text{ m/s}) = \frac{(62 \text{ m})}{\Delta t} \qquad \Delta t = 17.71 \text{ s}$$

The amount the boat moves to the right during the time it takes to cross the river depends on that time and the rightwards component of the velocity from the current. Using ***x*** for the rightwards direction:

$$v_x = \frac{\Delta x}{\Delta t} \qquad (1.5 \text{ m/s}) = \frac{\Delta x}{(17.71 \text{ s})} \qquad \Delta x = 26.6 \text{ m}$$

44. **Answer: 64.6°**

In order for the boat to move straight across the river (and not move left or right) the boat must point at an angle where the leftwards component of the boat's velocity vector is equal to the rightwards current velocity so that the total velocity vector (the sum of the boat's velocity vector and the current velocity) points straight across the river.

The sidewards component of the boat's velocity is $v_x = -1.5 \text{ m/s}$ (1.5 m/s to the left)

The magnitude of the boat's velocity vector is $v = 3.5 \text{ m/s}$

The reference angle between the direction the boat is pointing and the ***x*** (sideways) component is:

$$\theta = \cos^{-1}\left(\frac{1.5}{3.5}\right) = 64.6°$$