

HOMOGENEOUS DIFFERENTIAL EQUATIONS WORKSHEET

1. Solve the differential equations

$$(a) y'' - 2y' - 8y = 0$$

$$(b) 2y'' - 5y' - 3y = 0$$

$$(c) 3y'' - 10y' + 8y = 0$$

$$(d) 4 \frac{d^2 y}{dx^2} - 36y = 0$$

2. Solve the differential equations

$$(a) y'' - 4y' + 4y = 0$$

$$(b) y'' + 8y' + 16y = 0$$

$$(c) 4 \frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} + y = 0$$

$$(d) 9 \frac{d^2 y}{dx^2} - 12 \frac{dy}{dx} + 4y = 0$$

3. Solve the differential equations

$$(a) \frac{d^2 y}{dx^2} + y = 0$$

$$(b) y'' + 2y' + 5y = 0$$

$$(c) 2y'' + y' + 3y = 0$$

$$(d) -2 \frac{d^2 y}{dx^2} + 3 \frac{dy}{dx} - 2y = 0$$

4. The movement of the needle of an instrument used for measuring the strength of an electric current can be modeled by the differential equation

$$4 \frac{d^2 \theta}{dt^2} + 5 \frac{d\theta}{dt} + \theta = 0$$

where θ represents the angle turned from a standard position and t represents time.

Find the general solution of the differential equation.

ANSWERS

1. (a) $y = Ae^{-2x} + Be^{4x}$ (b) $y = Ae^{-x} + Be^{\frac{3}{2}x}$ (c) $y = Ae^{-\frac{2}{3}x} + Be^{4x}$
(d) $y = Ae^{-3x} + Be^{3x}$

2. (a) $y = e^{2x}(Ax + B)$ (b) $y = e^{4x}(Ax + B)$ (c) $y = e^{-\frac{1}{2}x}(Ax + B)$
(d) $y = e^{\frac{2}{3}x}(Ax + B)$

3. (a) $y = A \cos x + B \sin x$ (b) $y = e^x(A \cos 2x + B \sin 2x)$
(c) $y = e^{-\frac{1}{4}x} \left(A \cos \left(\frac{\sqrt{23}}{4} x \right) + B \sin \left(\frac{\sqrt{23}}{4} x \right) \right)$ (d) $y = e^{\frac{3}{4}x} \left(A \cos \left(\frac{\sqrt{7}}{4} x \right) + B \sin \left(\frac{\sqrt{7}}{4} x \right) \right)$

4. $\theta = Ae^{-\frac{1}{4}t} + Be^{-t}$
