A curve has equation $x^2 + 4y^2 = k^2$ where $k$ is a positive constant.

a. Verify that $x = k \cos \theta$, $y = \frac{1}{2}k \sin \theta$ are parametric equations for this curve

b. For $k = 3$, state the range of the curve $x^2 + 4y^2 = k^2$

The curve shows the path of a particle C in a vertical plane. The path of C is modelled by the parametric equations

\[ x = 10 \cos \theta + 5 \cos 2\theta, \quad y = 10 \sin \theta + 5 \sin 2\theta, \quad 0 \leq \theta < 2\pi \]

Find the coordinates of the x-intercepts

The sketch shows the arch ABCD of a bridge. The section from B to C is part of the curve OBCE with parametric equations

\[ x = a(\theta - \sin \theta), \quad y = a(1 - \cos \theta) \]

for $0 \leq \theta \leq 2\pi$ where $a$ is a constant.

a. Find, in terms of $a$, the length of the straight line OE

b. Find, in terms of $a$, the maximum height of the arch.

c. Show that $BF = \frac{3}{2}a$ and find OF in terms of $a$, giving your answer exactly.

d. Given that the straight line distance AD is 20 metres, calculate the value of $a$