NOTES:

5: Quadratic Equations and Graphs

Quadratic Equations:

1, An equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$, is known as a quadratic equation. The constants, a and b are known as the coefficients of x^2 and x respectively whereas the constant c is known as the constant term.

2. To solve a quadratic equation in the form of $ax^2 + bx + c = 0$, we need to find the value(s) of x that satisfies the equation. The value(s) of x found are known as the **roots** of the equation.

3. There are basically four ways to solve a quadratic equation:

(i) Factorization (covered in Sec 2)

- Factoring $ax^2 + bx + c = 0$ into $(ex + f)(gx + h) = 0$
- Solving for x by substituting $ex + f = 0$, $gx + h = 0$ (more below)
- Learn More: https://www.youtube.com/watch?v=c40QJYnKltM&t=1s

(ii) Graphical method (covered in Sec 2)

- since $ax^2 + bx + c = 0$; we substitute $y = 0$;
- on the graph we draw $y = 0$ and $y = ax^2 + bx + c$,
- the intersection points of both line/curve, $A(x, y)$ and $B(x, y)$'s x-coordinate will be the roots of the equation.

(iii) Completing the square (covered in Sec 3)

- Factoring $ax^2 + bx + c = 0$ into $(x + k)^2 + h = 0$
- Divide all terms by a; Let $\frac{b}{a} = B$, $\frac{c}{a} = C$ to avoid confusion.
- equation becomes $x^2 + Bx + C = 0$
- Change your equation to $(x + \frac{B}{2})^2 - (\frac{B}{2})^2 + C = 0$. Simplify the constant terms to be one, h and let $\frac{B}{2}$ be k.
- After your equation is factored into $(x + k)^2 + h = 0$, you can use $(-k, ah)$ as the coordinate of the turning point (minimum/maximum point) of the quadratic curve.
- To find the solution, $x = -k \pm \sqrt{-h}$
- Learn More: https://www.youtube.com/watch?v=7cwcuYIA3hM&t=1s

(iv) Quadratic formula (covered in Sec 3)

- Finding the value of x in $ax^2 + bx + c = 0$ with any arbitrary values of a, b, c.
- Use the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Solving Quadratic Equations By Factorization

4. The following table illustrates the steps we can use to solve a quadratic equation, e.g. $3x^2 = x + 2$, by factorization.

Step	Method	Illustration
1	Rewrite the given equation in the general form $ax^2 + bx + c = 0$	$3x^2 = x + 2$ $\rightarrow 3x^2 - x - 2 = 0$
2	Factorize the expression on the LHS. $ax^2 + bx + c = 0$ $\rightarrow (ex + f)(gx + h) = 0$	$3x^2 - x - 2 = 0$ $3(x^2 - \frac{x}{3} - \frac{2}{3}) = 0$ $\rightarrow 3(x + f)(x + h) = 0$ $\rightarrow fh = -\frac{2}{3}, f + h = -\frac{1}{3}$ $\therefore f = \frac{2}{3}, h = -1$ $3(x + \frac{2}{3})(x - 1) = 0$ $(3x + 2)(x - 1) = 0$
3	Set each factor equal to 0. $(ex + f) = 0$ or $(gx + h) = 0$ (Since two factors P and Q are such that $P \times Q = 0 \rightarrow P = 0$ or $Q = 0$ or both P and Q are equal to zero as stated in the Zero Product Property.)	$3x + 2 = 0$ or $x - 1 = 0$
4	Solve the two linear equations in Step 3 to obtain the solutions or roots of the equation. $(ex + f) = 0 \rightarrow x = -\frac{f}{e}$ $(gx + h) = 0 \rightarrow x = -\frac{h}{g}$	$x = -(\frac{2}{3})$ $= -\frac{2}{3}$ $x = -(\frac{-1}{1})$ $= 1$ $\rightarrow x = -\frac{2}{3}$ or $x = 1$

5	Check your answers by substituting both solutions into the original equation.	When $x = -\frac{2}{3}$, LHS = RHS = $1\frac{2}{3}$ When $x = 1$, LHS = RHS = 3

Quadratic Word Problems:
5. The following table illustrates the steps we can use to solve word problems involving quadratic equations.

Steps	Process
1	Read the question, highlight important information and look out for the concepts required to solve the question.
2	Understand the problem by using a sketch or diagram where necessary.
3	Assign a variable to the unknown quantity and state what it represents.
4	Set up algebraic expressions for any other unknowns in terms of the assigned variable in Step 3.
5	Use the known information and data to write down a quadratic equation.
6	Solve the equation for the unknown variable and reject any solution(s) that does not fulfil the definition of the variable, e.g. negative values of money, time, length, age etc. will be rejected.
7	Check that the solution satisfies the conditions of the original problem.
8	Answer the original problem with a statement.

Vieta's Formulas [not in syllabus]:
Vieta's is useful in solving many questions relating to quadratics, and polynomials in general. It relates the coefficient of a polynomial to its sum and product of their roots. Here's how you can solve a possible secondary two question with these formulas.

Let the roots of a standard quadratic equation $ax^2 + bx + c$ be α and β, where the question gives you one root, β. If a question wants you to find the second root, α, you can use the formulas below:

- $\frac{-b}{a}$ is the formula to find α + β (sum of roots); we can find α by $\frac{-b}{a} - \beta$
- $\frac{c}{a}$ is the formula to find αβ (product of roots); we can find α by $\frac{c}{a} \div \beta$

*α and β are just Greek alphabets for the variables, so you can replace it with m, n (or any other alphabet for that matter). It can just get confusing with many variables like $a,\ b,\ c,\ x,\ y,\ m,\ n$ however, with α and β being more standard for roots.

Graphs of Quadratic Equations:

6. Quadratic graphs are graphs of the equation $y = ax^2 + bx + c$, where a, b and c are constants with $a \neq 0$.

Some examples of quadratic graphs are $y = 2x^2$, $y = 5 - 3x^2$, $y = x^2 - 5x + 6$.

7. The shape of a quadratic graph $y = ax^2 + bx + c$ is a **parabola**, which is either a ∪-shaped or ∩-shaped curve, depending on the value of *a* which is the coefficient of x^2.

- If $a > 0$, the quadratic curve is ∪**-shaped** and it has a **minimum point**.
  To remember easily, just know that if a is positive, the graph is a smiley-face.

- If $a < 0$, the quadratic curve is ∩**-shaped** and it has a **maximum point.**
  To remember easily, just know that if a is negative, the graph is a frowny-face.

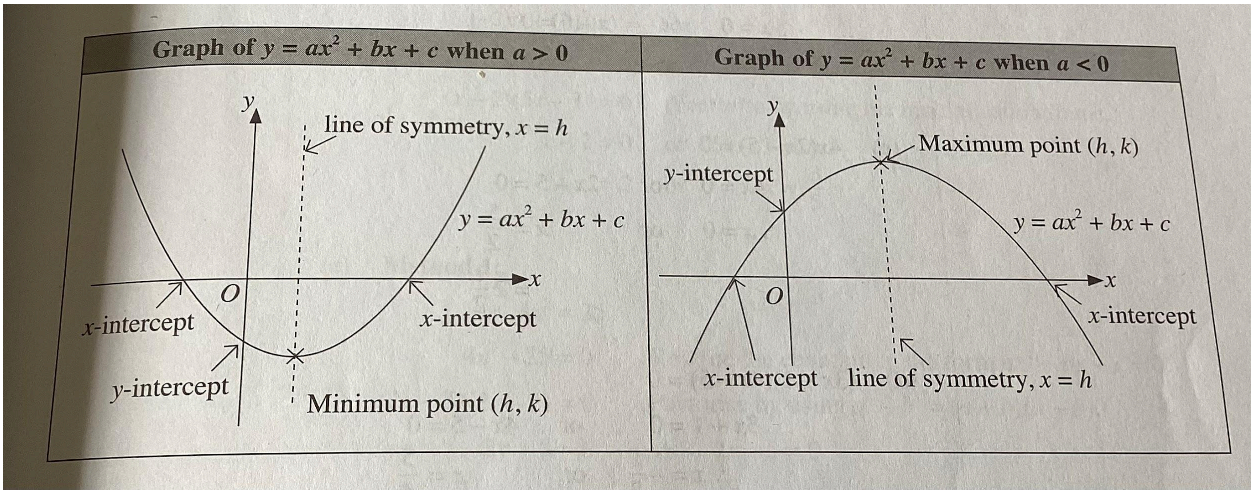
8. The quadratic curve is symmetrical about the line of symmetry. The **line of symmetry** is a vertical line of the form $x = h$ which **passes** through the **maximum** or **minimum point** (h, k) of the curve, also known as the **turning point** or the **vertex**.

9. To sketch a quadratic curve, we need to take note of the following:

(i) **Coefficient of x^2 (aka a):** For $a > 0$, the graph is ∪-shaped whereas for $a < 0$, the graph is ∩-shaped.
(ii) **y-intercept:** Substitute $x = 0$ into the equation of the curve to find the value of y.
(iii) **x-intercept:** Substitute $y = 0$ into the equation of the curve to find the value of x.
(iv) **Maximum or minimum point (h, k):**

- Find the equation of the line of symmetry by using $x = h = \frac{x_1 + x_2}{2}$, where x_1 and x_2 are the x-intercepts.
- To find the value of k, substitute $x = h$ into the equation of the curve.

Examples of quadratic curves on the next page.



*must be shown in drawing

10. To draw the graph of a quadratic equation, we can use the following steps:

1) Construct a table of values of x and y for the equation of the quadratic curve, $y = ax^2 + bx + c$. Ensure that the points are well-spread.

2) Select an appropriate scale for both axes, e.g. x-axis and y-axis, to allow the largest possible graph to be drawn for accurate results (usually given in questions)

3) Plot the points on the graph paper and join them up to form a smooth ∪-shaped or ∩-shaped curve.

4) Label the axes and the equation of the curve clearly on your graph as well as the part(s) of the questions that were read.

Version 2