Test – Complex Numbers

IB Math Minds

■ **Part I** – no calculator on questions 1-5

1. Write each – completely simplified – expression in the form $a+bi$, where $a, b \in \mathbb{R}$.

(a) $\dfrac{2-4\mathrm{i}}{1+3\mathrm{i}}$ (b) $\dfrac{2}{\mathrm{i}}\left(\dfrac{1}{\sqrt{3}}-\dfrac{3\mathrm{i}}{\sqrt{3}}\right)^2$

2. Given that $\dfrac{w+4}{w-2}=\mathrm{i}$, find w in the form $a+bi$, where $a, b \in \mathbb{R}$.

3. Given that $p, q \in \mathbb{R}$ and that $x=1+2\mathrm{i}$ is a solution of the equation $x^3+px^2+qx+20=0$ find the value of p and the value of q.

4. Find all complex solutions to $z^5+32=0$. Express the solutions in modulus-argument form, i. e. in the form $z=r\,\mathrm{cis}\,\theta$

5. Find $(-1-\mathrm{i})^{11}$ in Cartesian form $a+bi$, where $a, b \in \mathbb{R}$.

■ **Part II** – calculator allowed on questions 6-10

6. If $w_1=k-k\sqrt{3}\,\mathrm{i}$ and $w_2=4\mathrm{i}$, where k is a real constant, express w_1 and w_2 in the form $r\,\mathrm{cis}\,\theta$, and hence find an expression for $\left(\dfrac{w_1}{w_2}\right)^3$ in terms of k and i.

7. Find the four 4th roots of -16 and express them exactly in Cartesian form $a+bi$, where $a, b \in \mathbb{R}$

8. (a) Show that $(a-1)(a^2+a+1)=a^3-1$

(b) Given that $z=e^{\mathrm{i}\left(\frac{2\pi}{3}\right)}$

- show that $z^3=1$ and $1+z+z^2=0$; and
- express each of the following expressions in terms of z (write in **simplest** form)

(i) z^8 (ii) $(1-z)^2+4z$

9. (a) By using de Moivre’s theorem, or otherwise, find the roots of the equation $z^4+4=0$ and express them in Cartesian form.

(b) Hence, or otherwise, express z^4+4 as the product of two quadratic polynomials in z with real coefficients.

10. (a) Use de Moivre’s theorem to prove that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

(b) Hence, find the exact solutions of the equation $3x-4x^3=\dfrac{1}{2}$.