HEALTH AND MORTALITY DELTA:
ASSESSING THE WELFARE COST OF HOUSEHOLD INSURANCE CHOICE

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ABSTRACT

We develop a pair of risk measures, health and mortality delta, for the universe of life and health insurance products. A life-cycle model of insurance choice simplifies to replicating the optimal health and mortality delta through a portfolio of insurance products. We estimate the model to explain the observed variation in health and mortality delta implied by the ownership of life insurance, annuities including private pensions, and long-term care insurance in the Health and Retirement Study. For the median household aged 51 to 57, the lifetime welfare cost of market incompleteness and suboptimal choice is 3.2% of total wealth.

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Retail financial advisors and insurance companies offer a wide variety of insurance products that includes life insurance, annuities, and long-term care insurance. They offer each of these products in a full range of maturities and payout structures. Examples include term life insurance with guaranteed term up to 30 years, universal and whole life insurance, immediate annuities, and deferred annuities whose income is deferrable for a year or longer. This variety begs for a risk measure that allows households to assess the degree of complementarity and substitutability between various products and, ultimately, to choose an optimal portfolio of products. Such risk measures already exist in other parts of the retail financial industry. For example, beta measures an equity product's exposure to aggregate market risk, while duration measures a fixed-income product's exposure to interest-rate risk. The existence of such risk measures, based on sound economic theory, has proven to be tremendously valuable for quantifying and managing financial risk for both households and institutions.

This paper develops a pair of risk measures for the universe of life and health insurance products, which we refer to as health and mortality delta. Health delta measures the differential payoff that a product delivers in poor health, while mortality delta measures the differential payoff that a product delivers at death. A life-cycle model of insurance choice implies optimal consumption as well as optimal health and mortality delta, which are determined by household preferences and state variables (i.e., age, birth cohort, health, and wealth). An optimal portfolio of insurance products, not necessarily unique, aggregates health and mortality delta over individual products to replicate the optimal health and mortality delta predicted by the life-cycle model.

We use our risk measures to assess how close the observed demand for private insurance is to the optimal demand, given the provision of public insurance through Social Security and Medicare. For each household in the Health and Retirement Study, we calculate the health and mortality delta implied by its ownership of term and whole life insurance, annuities including private pensions (i.e., defined benefit plans), and long-term care insurance. We estimate household preferences, allowing the bequest motive to vary across households, to minimize the welfare cost implied by the deviations of observed demand from the optimal demand predicted by the life-cycle model. We achieve sharp identification of relative risk aversion, the average bequest motive, and the complementarity of consumption and health. Insurance choice, which embeds the desired path of wealth in future health states, is much more informative than the realized path of savings for identifying these preference parameters.

The life-cycle model explains 68% of the variation in observed health delta, and 83% of the variation in observed mortality delta. Consistent with economic intuition, we find that married households and those with living children have stronger bequest motives. We also find stronger bequest motives for more educated and wealthier households, consistent with the notion that bequests are a luxury good. Overall, these household characteristics explain 66% of the variation in bequest motives.

Although the life-cycle model explains most of the variation in observed health and mortality delta across households, it fails to explain the variation within a household over time. The model prescribes that households decrease their health and mortality delta over the life cycle by rebalancing from life insurance to annuities. Observed health and mortality delta are much more persistent than the predictions of the life-cycle model, due to the default path of annuitization from private pensions and the lack of rebalancing. We uncover a new puzzle that is distinct from the so-called "annuity puzzle", which concerns the low level of annuitization relative to a life-cycle model with no bequest motive. The unexplained variation in the degree to which households are annuitized, rather than the average level at which they are annuitized, is puzzling from the perspective of life-cycle theory.

For each household, we estimate the welfare cost of deviations from the optimal demand, which we interpret as the joint cost of market incompleteness (due to private information, borrowing constraints, or other frictions outside the model) and suboptimal choice. For the median household aged 51 to 57, the lifetime welfare cost is 3.2% of total wealth, defined as the sum of financial and housing wealth and the present value of future income minus out-of-pocket health expenses. Our estimate is an order of magnitude larger than the welfare cost of under-diversification in stock and mutual fund portfolios (e.g., Calvet et al., 2007, estimate it to be 0.5% of disposable income for the median Swedish household). Most of the welfare cost is explained by the deviations from optimal mortality delta, instead of the deviations from optimal health delta. That is, choices over life insurance and annuities have a much larger welfare impact than choices over long-term care insurance.

This paper is not the first attempt to understand the demand for life insurance (Bernheim, 1991; Inkmann and Michaelides, 2012), annuities (Brown, 2001; Inkmann et al., 2011), or long-term care insurance (Brown and Finkelstein, 2008; Lockwood, 2013). Relative to the previous literature, an important methodological contribution is to examine insurance choice comprehensively as a portfolio-choice problem, instead of one product at a time. By collapsing insurance choice into a pair of risk measures, we explicitly account for the complementarity and the substitutability between various products. In particular, annuities and private pensions can partially substitute for long-term care insurance, by insuring that households have sufficient income to cover late-life health expenses as long as they live. Therefore, one cannot study the demand for long-term care insurance without simultaneously thinking about annuities and private pensions.

The remainder of the paper proceeds as follows. In Section I, we develop a life-cycle model in which households face health and mortality risk and chooses from a complete set of insurance products that includes life insurance, annuities, and supplemental health insurance. In Section II, we derive the optimal demand for insurance and a key formula for measuring the welfare cost of deviations from the optimal demand. In Section III, we calibrate the life-cycle model based on the Health and Retirement Study. In Section IV, we estimate household preferences and compare the observed demand to the optimal demand predicted by the life-cycle model. We also estimate the welfare cost of deviations from the optimal demand. In Section V, we illustrate how a portfolio of existing insurance products can replicate the optimal health and mortality delta predicted by the life-cycle model. Section VI concludes with practical implications of our study for retail financial advisors and insurance companies.

I. A Life-Cycle Model with Health and Mortality Risk

We develop a life-cycle model in which a household faces health and mortality risk that affects life expectancy, health expenses, and the marginal utility of consumption or wealth. The household can accumulate financial and housing wealth and also purchase a complete set of insurance products that includes life insurance, annuities, and supplemental health insurance.

Complete markets is a natural starting point, given the rich menu of insurance products that retail financial advisors and insurance companies already offer. In Section V, we show that a realistic portfolio of existing insurance products replicates the optimal health and mortality delta predicted by a calibrated version of the life-cycle model. Even if actual markets are incomplete, our framework is a useful benchmark for quantifying the importance of market incompleteness that may arise for various reasons, including private information and borrowing constraints.

A. Health and Mortality Risk

In our model, health refers to any information that is verifiable through medical underwriting that involves a health examination and a review of medical history. For tractability, we do not model residual private information, such as self-assessments of health, that could affect the demand for insurance. In Section IV, however, we show that residual private information does not explain much of the observed demand for insurance.

A.1. Health Transition Probabilities

A household consists of an insured and other members who share common resources. The insured lives for at most T periods and dies with certainty in period $T+1$. In each period

$t \in \{1, \ldots, T\}$, the insured's health is in one of three states, indexed as $h_t \in \{1, 2, 3\}$. The health states are ordered so that $h_t = 1$ corresponds to death, $h_t = 2$ corresponds to poor health, and $h_t = 3$ corresponds to good health.

Our empirical framework is based on three states because this is the minimum number necessary to model both health and mortality risk. The three-state model can be interpreted as a discrete-time analog of a continuous-time model in which a continuous process drives health risk, and a jump process drives mortality risk. As we discuss in Section III, we limit our estimation sample to households that have adequate health insurance coverage, for whom the primary out-of-pocket health expense is nursing home care. This limits potential heterogeneity in health insurance coverage that would require additional health states.

The insured's health evolves from period t to $t+1$ according to a Markov chain with a 3×3 transition matrix π_t. We denote the (i, j)th element of the transition matrix as

$$\pi_t(i, j) = \Pr(h_{t+1} = j | h_t = i). \tag{1}$$

Conditional on being in health state i in period t, $\pi_t(i, j)$ is the probability of being in health state j in period $t+1$. Death is an absorbing state so that $\pi_t(1, 1) = 1$. Let $\mathbf{e}_i$ denote a 3×1 vector with the ith element equal to one, and the other elements equal to zero. We define an n-period transition probability as

$$\pi_t^n(i, j) = \mathbf{e}_i' \prod_{s=0}^{n-1} \pi_{t+s} \mathbf{e}_j. \tag{2}$$

Conditional on being in health state i in period t, $\pi_t^n(i, j)$ is the probability of being in health state j in period $t+n$.

We define an n-period mortality rate as

$$p_t(n|i) = \begin{cases} \mathbf{e}_i' \pi_t \mathbf{e}_1 & \text{if } n = 1 \\ \mathbf{e}_i' \prod_{s=0}^{n-2} \pi_{t+s} \begin{bmatrix} \mathbf{0} & \mathbf{e}_2 & \mathbf{e}_3 \end{bmatrix} \pi_{t+n-1} \mathbf{e}_1 & \text{if } n > 1 \end{cases}. \tag{3}$$

Conditional on being in health state i in period t, $p_t(n|i)$ is the probability of being alive in period $t+n-1$ but dead in period $t+n$. We also define an n-period survival probability as

$$q_t(n|i) = 1 - \pi_t^n(i, 1). \tag{4}$$

Conditional on being in health state i in period t, $q_t(n|i)$ is the probability of being alive in period $t+n$.

A.2. Out-of-Pocket Health Expenses

The household has employer-provided health insurance while working and Medicare in retirement, which cover the basic health expenses. However, the household may face out-of-pocket health expenses that are not covered by basic health insurance, for which it could purchase supplemental health insurance. For example, Medicare does not cover nursing home care, for which the household could purchase long-term care insurance.

In the absence of supplemental health insurance, the household faces an out-of-pocket health expense M_t in each period t.[1] The distribution of out-of-pocket health expenses depends on age and health, where $M_t(j)$ denotes its realization for health state j. Naturally, poor health is associated with higher out-of-pocket health expenses. We assume that end-of-life health expenses incur in the last period prior to death. There is no health expense at death so that $M_t(1) = 0$.

B. Insurance Products

The household can accumulate financial and housing wealth, which earns gross interest R. In addition, the household can purchase life insurance, annuities, and supplemental health insurance of all maturities.

B.1. Life Insurance

Let $\mathbf{1}_t(j)$ denote an indicator function that is equal to one if the insured is in health state j in period t. Term life insurance of maturity n, issued in period t, pays out a death benefit of

$$D_{L,t+s}(n - s|h_{t+s}) = \mathbf{1}_{t+s}(1), \tag{5}$$

upon death of the insured in any period $s \in \{1, \dots, n\}$. In each period t, $T-t$ is the maximum maturity because the insured dies with certainty in period $T+1$. For our purposes, universal or whole life insurance is a special case of term life insurance with the maximum maturity.

The pricing of life insurance depends on the insured's age and health at issuance of the policy.[2] Naturally, younger and healthier individuals with longer life expectancy pay a lower

[1]To focus on insurance choice, we abstract from the endogenous choice of health expenditure (see Picone et al., 1998; Yogo, 2009; Hugonnier et al., 2013).

[2]The insurer could charge a premium that is independent of health in a pooling equilibrium (e.g., group life insurance). In that case, we would have to solve for a pooling price at which the insurer breaks even, given the aggregate demand for a given product. This extension of our framework is conceptually straightforward but computationally challenging. We refer to a related literature that studies the welfare implications of pooled pricing and private information in annuity (Einav et al., 2010b) and health insurance markets (Einav et al., 2010a; Bundorf et al., 2012).

premium. Conditional on being in health state h_t in period t, the price of n-period life insurance per unit of death benefit is

$$P_{L,t}(n|h_t) = \sum_{s=1}^{n} \frac{p_t(s|h_t)}{R_L^s}, \tag{6}$$

where $R_L \leq R$ is the discount rate. The pricing of life insurance is actuarially fair when $R_L = R$, while $R_L < R$ implies a markup.

B.2. Deferred Annuities

A deferred annuity of maturity n, issued in period t, pays out a constant income of

$$D_{A,t+s}(n-s|h_{t+s}) = \begin{cases} 0 & \text{if } s < n \\ 1 - \mathbf{1}_{t+s}(1) & \text{if } s \geq n \end{cases}, \tag{7}$$

in each period $s \in \{1, \ldots, T-t\}$ that the insured is alive. In each period t, $T-t$ is the maximum maturity because the insured dies with certainty in period $T+1$. For our purposes, an immediate annuity is a special case of deferred annuities with the minimum maturity (i.e., $n = 1$).

The pricing of annuities depends on the insured's age and health at issuance of the policy.[3] Naturally, younger and healthier individuals with longer life expectancy pay a higher premium. Conditional on being in health state h_t in period t, the price of an n-period annuity per unit of income is

$$P_{A,t}(n|h_t) = \sum_{s=n}^{T-t} \frac{q_t(s|h_t)}{R_A^s}, \tag{8}$$

where $R_A \leq R$ is the discount rate.

B.3. Supplemental Health Insurance

Supplemental health insurance of maturity n, issued in period t, covers

$$D_{H,t+s}(n-s|h_{t+s}) = \mathbf{1}_{t+s}(2)(M_{t+s}(2) - M_{t+s}(3)), \tag{9}$$

[3] In the United States, annuities can be purchased without medical underwriting at a price that depends only on gender and age. However, those with a serious health condition can purchase medically underwritten annuities at a lower price that reflects their impaired mortality.

in each period $s \in \{1, \ldots, n\}$ that the insured is in poor health. Insofar as health expenses include nursing home stays and home health care, we also interpret this product as long-term care insurance. A unit of this product represents full coverage, equating health expenses across all health states in which the insured is alive. In each period t, $T - t$ is the maximum maturity because the insured dies with certainty in period $T + 1$.

The pricing of supplemental health insurance depends on the insured's age and health at issuance of the policy. Naturally, younger and healthier individuals with lower expected health expenses pay a lower premium. Conditional on being in health state h_t in period t, the price of n-period supplemental health insurance per unit of coverage is

$$P_{H,t}(n|h_t) = \sum_{s=1}^{n} \frac{\pi_t^s(h_t, 2)(M_{t+s}(2) - M_{t+s}(3))}{R_H^s}, \tag{10}$$

where $R_H \leq R$ is the discount rate.

C. Health and Mortality Delta for Insurance Products

For each insurance product $i = \{L, A, H\}$ of maturity n, we define its health delta in period t as

$$\Delta_{i,t}(n) = P_{i,t+1}(n-1|2) + D_{i,t+1}(n-1|2) - (P_{i,t+1}(n-1|3) + D_{i,t+1}(n-1|3)). \tag{11}$$

Health delta measures the differential payoff that a policy delivers in poor health relative to good health in period $t + 1$. Similarly, we define its mortality delta in period t as

$$\delta_{i,t}(n) = D_{i,t+1}(n-1|1) - (P_{i,t+1}(n-1|3) + D_{i,t+1}(n-1|3)). \tag{12}$$

Mortality delta measures the differential payoff that a policy delivers at death relative to good health in period $t + 1$.

Figure 1 illustrates the relation between the payoffs of a policy and its health and mortality delta. Section III explains how we estimate the payoffs based on the Health and Retirement Study, which is not essential for the purposes of this illustration. The solid line represents the payoffs of a policy in the three possible health states in the subsequent period. Health delta is the payoff of a policy in poor health relative to good health, which is minus the slope of the dashed line if the horizontal distance between good and poor health is one. Mortality delta is the payoff of a policy at death relative to good health, which is minus the slope of the dotted line if the horizontal distance between good health and death is one.

Long-term life insurance and supplemental health insurance have positive health delta, while deferred annuities have negative health delta. That is, long-term life insurance is a substitute for supplemental health insurance in terms of health delta. This is because the expected payoff from long-term life insurance rises in poor health when the insured has shorter life expectancy, just like supplemental health insurance. In contrast, deferred annuities are complements of supplemental health insurance in terms of health delta. This is because the expected payoff from deferred annuities falls in poor health when the insured has shorter life expectancy, which is the opposite of supplemental health insurance.

Life insurance has positive mortality delta, while deferred annuities and long-term health insurance have negative mortality delta. That is, deferred annuities and long-term health insurance are complements of life insurance in terms of mortality delta. This is because deferred annuities and long-term health insurance lose their value entirely at death, which is the opposite of life insurance. Therefore, deferred annuities and long-term health insurance are both effective ways to transfer wealth to future states in which the insured remains alive and faces high health expenses.

Figure 1 highlights the importance of studying insurance products together, instead of one product at a time. Long-term life insurance not only insures mortality risk, but also has positive exposure to health delta. Deferred annuities not only insure longevity risk, but also have negative exposure to health delta. Finally, long-term health insurance not only insures health risk, but also has negative exposure to mortality delta.

D. Budget Constraint

In each period t that the insured is alive, the household starts with initial wealth A_t. The household receives income Y_t, pays health expenses M_t, and consumes C_t. We define consumption broadly to include the service flow from owner-occupied or rental housing. The household saves the wealth remaining after health expenses and consumption in bonds, housing, life insurance, annuities, and supplemental health insurance.[4] Let F_t denote the face value of bonds with price $1/R$ per unit. Similarly, let $G_t \geq 0$ denote the quantity of housing with price $1/R$ per unit. Finally, let $B_{i,t}(n) \geq 0$ denote the face value for each insurance product i of maturity n. The household's savings in period t is

$$A_t + Y_t - M_t - C_t = \frac{F_t + G_t}{R} + \sum_{i=\{L,A,H\}} \sum_{n=1}^{T-t} P_{i,t}(n) B_{i,t}(n). \tag{13}$$

We model a mortgage or a home equity loan as a short position in bonds. We also assume

[4] To focus on insurance choice, we abstract from various frictions that pin down the portfolio choice between financial assets and housing (see Cocco, 2005; Yao and Zhang, 2005; Yogo, 2009; Nakajima and Telyukova, 2012).

that the household can borrow from its savings in insurance products at a gross interest rate R. Thus, a loan from insurance products is a short position in bonds, which is a simple way to model actual features of these products. The premiums on long-term life insurance and long-term care insurance are typically paid as constant periodic payments, instead of a lump-sum payment up front. Periodic payments are essentially equivalent to borrowing against the value of the policy because the present value of the periodic payments is equal to the value of the policy at issuance. Whole life insurance typically has an explicit option to borrow from the cash surrender value of the policy. Finally, households can take out a loan from annuities in a defined contribution plan.

The intertemporal budget constraint is

$$A_{t+1} = F_t + G_t + \sum_{i=\{L,A,H\}} \sum_{n=1}^{T-t} (P_{i,t+1}(n-1) + D_{i,t+1}(n-1)) B_{i,t}(n). \tag{14}$$

That is, wealth in the subsequent period is equal to the value of bonds and housing plus the (realized and expected) payoffs from life insurance, annuities, and supplemental health insurance. Let $A_{t+1}(j)$ denote wealth if health state j is realized in period $t+1$. In particular, wealth that is bequeathed if the insured dies in period $t+1$ is

$$A_{t+1}(1) = F_t + G_t + \sum_{n=1}^{T-t} B_{L,t}(n). \tag{15}$$

That is, wealth at the insured's death is equal to the value of bonds and housing plus the death benefit from life insurance. The household must have non-negative wealth at the insured's death (i.e., $A_{t+1}(1) \geq 0$).

E. Objective Function

The household maximizes expected utility over consumption while alive and the bequest upon death. The household's objective function in health state $h_t \in \{2, 3\}$ is

$$U_t(h_t) = \left\{ \omega(h_t)^\gamma C_t^{1-\gamma} + \beta \left[\pi_t(h_t, 1) \omega(1)^\gamma A_{t+1}(1)^{1-\gamma} + \sum_{j=2}^{3} \pi_t(h_t, j) U_{t+1}(j)^{1-\gamma} \right] \right\}^{1/(1-\gamma)}, \tag{16}$$

with the terminal value

$$U_T(h_T) = \omega(h_T)^{\gamma/(1-\gamma)} C_T. \tag{17}$$

The parameter $\beta \in (0, 1)$ is the subjective discount factor. The parameter $\gamma > 1$ is relative risk aversion, or the inverse of the elasticity of intertemporal substitution. The health state-dependent utility parameter $\omega(h_t) \geq 0$ allows the marginal utility of consumption or wealth to vary across health states. The presence of a bequest motive is parameterized as $\omega(1) > 0$, in contrast to its absence $\omega(1) = 0$. In Section IV, we allow the bequest motive to vary across households. Consumption and health are complements if the marginal utility of consumption is lower in poor health, which is parameterized as $\omega(2) < \omega(3)$. For example, the marginal utility of housing services may fall with physical disability (Davidoff, 2010). Otherwise, consumption and health are substitutes if $\omega(2) > \omega(3)$.

II. Optimal Demand for Insurance

We derive the optimal demand for insurance under complete markets. When markets are complete, there are potentially many combinations of insurance products that achieve the same consumption and wealth allocations. Therefore, we characterize the unique solution to the life-cycle problem in terms of optimal consumption and optimal health and mortality delta. We then derive a key formula for measuring the welfare cost of deviations from the optimal demand.

A. Optimal Health and Mortality Delta

We define health delta in period t as the difference in realized wealth between poor and good health in period $t + 1$:

$$\Delta_t = A_{t+1}(2) - A_{t+1}(3). \tag{18}$$

Similarly, we define mortality delta in period t as the difference in realized wealth between death and good health in period $t + 1$:

$$\delta_t = A_{t+1}(1) - A_{t+1}(3). \tag{19}$$

Proposition 1. *The solution to the life-cycle problem under complete markets is*

$$C_t^* = c_t(h_t)\left(A_t + \sum_{s=0}^{T-t}\frac{\mathbf{E}_t[Y_{t+s} - M_{t+s}|h_t]}{R^s}\right), \tag{20}$$

$$\begin{aligned}\Delta_t^* =& \frac{(\beta R)^{1/\gamma}C_t^*}{\omega(h_t)}\left(\frac{\omega(2)}{c_{t+1}(2)} - \frac{\omega(3)}{c_{t+1}(3)}\right)\\ &- \left(\sum_{s=1}^{T-t}\frac{\mathbf{E}_{t+1}[Y_{t+s} - M_{t+s}|2]}{R^{s-1}} - \sum_{s=1}^{T-t}\frac{\mathbf{E}_{t+1}[Y_{t+s} - M_{t+s}|3]}{R^{s-1}}\right),\end{aligned} \tag{21}$$

$$\delta_t^* = \frac{(\beta R)^{1/\gamma}C_t^*}{\omega(h_t)}\left(\omega(1) - \frac{\omega(3)}{c_{t+1}(3)}\right) + \sum_{s=1}^{T-t}\frac{\mathbf{E}_{t+1}[Y_{t+s} - M_{t+s}|3]}{R^{s-1}}. \tag{22}$$

The average propensity to consume in health state $h_t \in \{2, 3\}$ is

$$c_t(h_t) = \left[1 + \frac{\pi_t(h_t, 1)(\beta R)^{1/\gamma}\omega(1)}{R\omega(h_t)} + \sum_{j=2}^{3}\frac{\pi_t(h_t, j)(\beta R)^{1/\gamma}\omega(j)}{R\omega(h_t)c_{t+1}(j)}\right]^{-1}, \tag{23}$$

with the terminal value $c_T(h_T) = 1$.

In Appendix A, we show that the optimal policy equates the marginal utility of consumption or wealth across all future health states (Yaari, 1965). The expression for optimal health delta Δ_t^* shows that three forces drive the household's desire to insure poor health relative to good health. First, the household would like to deliver relatively more wealth to the health state in which the marginal utility of consumption is high, determined by the relative magnitudes of $\omega(2)$ and $\omega(3)$. Second, the household would like to deliver relatively more wealth to the health state in which the average propensity to consume is low, determined by the relative magnitudes of $c_{t+1}(2)$ and $c_{t+1}(3)$. Naturally, the household consumes more slowly out of wealth in good health associated with longer life expectancy. Finally, the household would like to deliver relatively more wealth to the health state in which lifetime disposable income (i.e., income minus out-of-pocket health expenses) is low. Naturally, the household has lower lifetime disposable income in poor health associated with shorter life expectancy, higher health expenses, and potentially lower income.

The same three forces also explain the expression for optimal mortality delta δ_t^*. First, the household would like to deliver relatively more wealth to death if the bequest motive $\omega(1)$ is high. Second, the household would like to deliver relatively more wealth to death if the average propensity to consume in good health $c_{t+1}(3)$ is high. Finally, the household would like to deliver relatively more wealth to death if lifetime disposable income is high in good health.

B. *Optimal Portfolio of Insurance Products*

Proposition 2. *Given an optimal consumption policy, a feasible portfolio policy that satisfies the budget constraint* (13) *is optimal if it satisfies the equations*

$$\Delta_t^* = \sum_{i=\{L,A,H\}} \sum_{n=1}^{T-t} \Delta_{i,t}(n) B_{i,t}(n), \tag{24}$$

$$\delta_t^* = \sum_{i=\{L,A,H\}} \sum_{n=1}^{T-t} \delta_{i,t}(n) B_{i,t}(n). \tag{25}$$

Proposition 2 shows that health and mortality delta are sufficient for constructing an optimal portfolio of insurance products. Health delta $\Delta_{i,t}(n)$ measures the marginal contribution that insurance product i of maturity n has to the overall health delta. A portfolio, not necessarily unique, that satisfies equation (24) delivers the optimal amount of wealth to poor health in period $t+1$. Similarly, mortality delta $\delta_{i,t}(n)$ measures the marginal contribution that insurance product i of maturity n has to the overall mortality delta. A portfolio, not necessarily unique, that satisfies equation (25) delivers the optimal amount of wealth to death in period $t+1$.

A common wisdom in the life-cycle literature is that the household can simply accumulate financial and housing wealth to "self-insure" late-life health expenses (Hubbard et al., 1994). However, this result relies on an unrealistic modeling assumption that excludes insurance products from the investment opportunity set. Proposition 2 implies that a portfolio of only bonds and housing without insurance products is optimal only if optimal health and mortality delta are equal to zero. By Proposition 1, optimal health and mortality delta are equal to zero only in the knife-edge case that the utility weight on poor health $\omega(2)$ is sufficiently low and the bequest motive $\omega(1)$ is sufficiently high, so that the desired path of wealth is identical across all future health states.

C. *Welfare Cost of Deviations from Optimal Health and Mortality Delta*

Suppose that the household's demand for insurance were to deviate from the optimal demand in Proposition 1. In Appendix A, we estimate the welfare cost of such deviations from the optimal demand through a second-order Taylor approximation around the known value function under complete markets. By the envelope theorem, the welfare cost is second order for sufficiently small deviations from the optimal demand (Cochrane, 1989).

Proposition 3. *Let V_t^* denote the value function associated with the optimal path $\{\Delta_{t+s-1}^*(i), \delta_{t+s-1}^*(i)\}_{s=1}^n$ of health and mortality delta under complete markets. Let V_t de-*

note the value function associated with an alternative path $\{\Delta_{t+s-1}(i), \delta_{t+s-1}(i)\}_{s=1}^{n}$ *of health and mortality delta that satisfies the budget constraint. The welfare cost of deviations from optimal health and mortality delta is*

$$
\begin{aligned}
L_t(n) =& \frac{V_t}{V_t^*} - 1 \\
\approx& \frac{1}{2} \sum_{s=1}^{n} \sum_{i=2}^{3} \left[\frac{\partial^2 L_t(n)}{\partial \Delta_{t+s-1}(i)^2} (\Delta_{t+s-1}(i) - \Delta^*_{t+s-1}(i))^2 \right. \\
&+ \frac{\partial^2 L_t(n)}{\partial \delta_{t+s-1}(i)^2} (\delta_{t+s-1}(i) - \delta^*_{t+s-1}(i))^2 \\
&\left. + 2 \frac{\partial^2 L_t(n)}{\partial \Delta_{t+s-1}(i) \partial \delta_{t+s-1}(i)} (\Delta_{t+s-1}(i) - \Delta^*_{t+s-1}(i))(\delta_{t+s-1}(i) - \delta^*_{t+s-1}(i)) \right],
\end{aligned} \tag{26}
$$

where the expressions for the second partial derivatives are given in Appendix A.

The observed demand for insurance may deviate from the optimal demand for two reasons. First, markets may be incomplete due to private information, borrowing constraints, or other frictions outside the model. Second, the observed demand may be suboptimal, given the complexity of the portfolio-choice problem and the lack of academic guidance. Because these two hypotheses are not mutually exclusive and difficult to distinguish based on the available data, we do not quantify their relative importance. Instead, we focus on estimating the joint cost of market incompleteness and suboptimal choice in this paper.

III. Calibrating the Life-Cycle Model

We calibrate the life-cycle model based on the Health and Retirement Study, which is a representative panel of older households in the United States since 1992. This household survey is uniquely suited for our study because it contains household-level data on health outcomes, health expenses, income, and wealth as well as ownership of life insurance, annuities, private pensions, and long-term care insurance. Some of these critical variables are missing in other household surveys such as the Panel Study of Income Dynamics and the Survey of Consumer Finances.

We calibrate the life-cycle model so that each period corresponds to two years, matching the frequency of interviews in the Health and Retirement Study. The life-cycle model starts at age 51, corresponding to the youngest age at which households enter the survey. We assume that the primary respondent dies with certainty at age 111, so that there are 30 periods (60 years) in total. We set the riskless interest rate to 2% annually, which is the average real return on the one-year Treasury note during our sample period. Our remaining measurement assumptions in this section are consistent with our maintained assumption of complete markets.

A. *Estimation Sample*

We focus on households whose primary respondent is male and aged 51 or older at the time of interview. We also require that income as well as financial and housing wealth are positive. Finally, we limit our sample to households that have adequate health insurance coverage, for whom the primary out-of-pocket health expense is nursing home care. To do so, we first eliminate households whose primary respondent is on Medicaid. We then select only those households whose primary respondent has employer-provided or individual health insurance. For households aged 65 and older, this criterion includes those that have supplemental coverage through Medicare Advantage (Part C), Medicare Part D, Medigap, or long-term care insurance. However, it excludes those that are solely on traditional Medicare (Parts A and B). Overall, this criterion eliminates only 17% of otherwise eligible households at age 51, and 29% of otherwise eligible households at age 65. We believe that the uniformity of health insurance coverage within the resulting sample trades off favorably with a narrower concept of health risk and a smaller sample size. We also refer the reader to an earlier version of this paper, in which we did not limit the sample based on health insurance coverage.

Life insurance is written on the life of an insured, while resources like income and wealth are shared by the members of a household. Because the primary respondent is typically married at the time of first interview, we must make some measurement assumptions when mapping the data to the life-cycle model. We measure health outcomes and the ownership of life insurance, annuities including private pensions, and long-term care insurance for only the primary respondent. We measure health expenses, income, and wealth at the household level. These measurement assumptions are consistent with the life-cycle model insofar as the budget constraint holds for the household, and the primary respondent purchases life insurance to leave a bequest for surviving members when he dies.

B. *Definition of the Health States*

In Table I, we use a probit model to predict future mortality based on doctor-diagnosed health problems and its interaction with age. Doctor-diagnosed health problems are statistically significant predictors of future mortality. For example, the marginal effect of cancer on the mortality rate is 10.43 with a t-statistic of 7.10. This means that males with cancer are 10.43 percentage points more likely to die within two years, holding everything else constant. Past age 51, each additional ten years in age is associated with an increase of 2.28 percentage points in the mortality rate. Based on the estimated probit model, we predict the mortality rate for the primary respondent at each interview.

Mortality rates and health expenses are not perfectly correlated, and they both contain important information about true unobserved health. Therefore, we define three health states based on both factors.

1. Death.
2. Poor health: The predicted mortality rate is higher than the median conditional on age and birth cohort. In addition, out-of-pocket health expenses are higher than the median conditional on age, birth cohort, and the ownership of long-term care insurance.
3. Good health: Alive and not in poor health.

Our definition of poor health conditions on age and birth cohort because mortality rates and health expenses vary significantly across these groups. In Appendix C, we show that our results are robust to an alternative definition of poor health, in which out-of-pocket health expenses must be higher than the 75th percentile conditional on age, birth cohort, and the ownership of long-term care insurance.

C. Description of the Sample

To verify our definition of the health states, Panel A of Table II reports the prevalence of doctor-diagnosed health problems and difficulty with activities of daily living by age group and health state. Within each age group, males in poor health are more likely to have doctor-diagnosed health problems. For example, among males aged 65 to 78, 28% of those in poor health have had cancer, which is higher than 11% of those in good health. Older males, especially those in poor health, are more likely to have difficulty with activities of daily living. For example, among males aged 79 and older, 24% of those in poor health have some difficulty dressing, which is higher than 13% of those in good health.

Panel B of Table II reports health care utilization by age group and health state. Within each age group, males in poor health are more likely to have used health care in the two years prior to the interview. For example, among males aged 79 and older, 14% of those in poor health have stayed at a nursing home, which is higher than 5% of those in good health. This is consistent with the fact that males in poor health have higher out-of-pocket health expenses than those in good health.

Panel C of Table II reports the ownership rates of life insurance, annuities including private pensions, and long-term care insurance by age group and health state. Among males aged 51 to 64, 72% of those in poor health and 71% of those in good health own term life insurance. Although the ownership rate for life insurance falls in age, it remains remarkably high for older males. Among males aged 65 to 78, 55% of those in poor health and 59% of those in good health receive annuity income from a private source that is not Social Security. For the same age group, only 18% of those in good health and 20% of those in poor health own long-term care insurance.

Panel D of Table II reports the face value of life insurance, annuity and pension income, and financial and housing wealth by age group and health state. Among males aged 51 to 64 that own term life insurance, the median face value is $78.4k for those in poor health and $81.0k for those in good health. Among males aged 65 to 78, the median annual annuity and pension income is $11.9k for those in poor health and $12.8k for those in good health. For the same age group, median financial and housing wealth is $233.9k for those in poor health and $257.6k for those in good health.

D. Health and Mortality Risk

D.1. Health Transition Probabilities

After defining the three health states, we estimate the transition probabilities between the health states through an ordered probit model. The outcome variable is the health state at two years from the present interview. The explanatory variables are dummies for present health and 65 or older, a quadratic polynomial in age, log income, the interaction of the dummies with the age polynomial and log income, and cohort dummies. The dummy for 65 or older accounts for potential changes in household behavior that arise from eligibility for Social Security and Medicare. Figure 2 reports the estimated transition probabilities by age and birth cohort, which are the predicted probabilities from the ordered probit model.

To better understand the health dynamics implied by the estimated transition probabilities, Panel A of Table III reports the long-run health transition probabilities (i.e., equation (2)) for males born between 1936 and 1940 and in good health at age 51. The probability of dying prior to age 65 is 0.24, and the probability of being in poor health at age 65 is 0.22. Panel B reports the average life expectancy conditional on age and health for the same group. Males in poor health at age 65 are expected to live for 17 more years, which is shorter than 20 years for those in good health.

D.2. Out-of-Pocket Health Expenses

In Appendix B, we use a panel regression model to estimate how out-of-pocket health expenses depend on dummies for present health and 65 or older, a quadratic polynomial in age, log income, and the interaction of the dummies with the age polynomial and log income. Our measure of out-of-pocket health expenses is comprehensive, including nursing home and end-of-life health expenses. We exclude households that own long-term care insurance in our estimation because the relevant measure in the life-cycle model is out-of-pocket health expenses in the absence of additional coverage.

Panel C of Table III reports out-of-pocket health expenses by age and health for males born between 1936 and 1940. For comparison, Panel D reports average income by age, which includes Social Security but excludes annuities and private pensions.[5] Households in poor health at age 51 have out-of-pocket health expenses of \$2.3k per year, which is higher than \$0.4k for those in good health. Out-of-pocket health expenses rise rapidly in old age (De Nardi et al., 2010). Households in poor health at age 93 have out-of-pocket health expenses of \$22.1k per year, which is higher than \$3.6k for those in good health. Since income at age 93 is \$17.7k per year, households in poor health must cover part of their health expenses through savings.

We use the estimated models for health transition probabilities and out-of-pocket health expenses to simulate paths of out-of-pocket health expenses over the life cycle. Figure 3 reports the distribution of realized out-of-pocket health expenses for males born between 1936 and 1940 and in good health at age 51. The distribution of lifetime out-of-pocket health expenses has wide range and positive skewness, much like the wealth distribution. Lifetime out-of-pocket health expenses have a long right tail that can exceed \$250k, which represents a health catastrophe. Households would have to accumulate significant wealth to "self-insure" this tail, which is less efficient than insurance through deferred annuities or long-term care insurance that have survival-contingent payoffs.

Panel E of Table III reports the present value of future disposable income (i.e., income minus out-of-pocket health expenses) by age and health. Households in good health at age 93 have −\$30.1k in lifetime disposable income because the present value of future health expenses exceeds the present value of future income. A younger household can insure this late-life risk by purchasing deferred annuities or long-term care insurance.

[5]In Appendix B, we use a panel regression model to estimate how income depends on a dummy for 65 or older, a quadratic polynomial in age, and the interaction of the dummy with the age polynomial. Our specification does not include present health and its interaction with the age polynomial because we found those coefficients to be statistically insignificant.

E. Additional Measurement Assumptions

E.1. Pricing of Insurance

We do not observe the premiums that households pay for life insurance, annuities, and long-term care insurance. Therefore, our baseline calibration assumes that insurance is actuarially fair conditional on age, birth cohort, and health. That is, we set the discount rate on insurance products to be the same as the riskless interest rate of 2% (i.e., $R_L = R_A = R_H = R$). Insurance may not be actuarially fair in practice for various reasons including private information, imperfect competition, regulation, and financial frictions (Koijen and Yogo, 2012). In Appendix C, we show that our results are robust to an alternative calibration in which insurance is more expensive than actuarially fair.

The impact of private information on the pricing of insurance is ambiguous because adverse selection on health may be offset by advantageous selection on another dimension of private information such as preferences (de Meza and Webb, 2001). In life insurance markets, Cawley and Philipson (1999) find no evidence for private information. Although the pricing of annuities depends on gender and age only, Finkelstein and Poterba (2004) find evidence for separation along contract dimensions such as payout structure. In long-term care insurance markets, Finkelstein and McGarry (2006) find no significant relation between insurance ownership and future long-term care utilization, consistent with the absence of private information. However, they argue that private information about health may be offsetting unobserved preferences for insurance. Given the ambiguous nature of both the theoretical predictions and the empirical findings, the absence of private information is a natural starting point for our baseline calibration.

E.2. Insurance Coverage

Because we do not observe the maturity of term life insurance, we need a measurement assumption to map it to the life-cycle model. We assume that term life insurance matures in two years and that whole life insurance matures at death. This assumption is motivated by the fact that (annually renewable) group policies account for a large share of term life insurance. In Appendix C, we show that our results are robust to an alternative calibration in which term life insurance has long-term coverage until age 65. We also assume that annuity income starts at age 65, which is the full Social Security retirement age, and terminates at death. Finally, we assume that the ownership of long-term care insurance corresponds to owning one unit of short-term supplemental health insurance in the life-cycle model. Thus, a household that owns long-term care insurance is fully insured against uncertainty in health expenses for the subsequent period.

Conditional on ownership, households report the face value of term and whole life insurance. Measurement error in the face value of these policies could contaminate our estimates of health and mortality delta. In Appendix B, we use a panel regression model to estimate how the face values of term and whole life insurance depend on dummies for present health and 65 or older, a quadratic polynomial in age, log income, and the interaction of the dummies with the age polynomial and log income. Instead of the observed face values, we use the predicted values with household fixed effects under the assumption that measurement error is transitory. We apply the same procedure to annuity and pension income.

We model all payoffs from insurance products to be real. We normalize the death benefit of life insurance and annuity income to be \$1k per unit in 2005 dollars. Modeling nominal payoffs for insurance products would introduce inflation risk, which is beyond the scope of this paper. Moreover, a cost-of-living-adjustment rider that effectively eliminates inflation risk is sometimes available for life insurance, annuities, and long-term care insurance. In the data, we deflate the face value of life insurance as well as pension and annuity income by the consumer price index to 2005 dollars.

F. *Health and Mortality Delta Implied by Household Insurance Choice*

For each household at each interview, we calculate the health and mortality delta implied by its ownership of term and whole life insurance, annuities including private pensions, and long-term care insurance. The household's health delta is determined by positive health delta from whole life insurance and long-term care insurance, which is offset by negative health delta from annuities including private pensions. The household's mortality delta is determined by positive mortality delta from term and whole life insurance, which is offset by negative mortality delta from annuities including private pensions.

Figure 4 reports the health and mortality delta for each household-interview observation, together with the mean and standard deviation by age. Average health delta is negative throughout the life cycle. This implies that annuities have a predominant effect on the average household's health delta. Average mortality delta is positive for younger households and negative for older households. This implies that life insurance has a predominant effect on younger households' mortality delta, while annuities have a predominant effect for older households. The cross-sectional variation in mortality delta is significantly higher than that in health delta throughout the life cycle.

When we calculate the health delta for each household based solely on its ownership of annuities including private pensions, it explains 98% of the variation in the overall health delta. When we calculate the mortality delta for each household in a similar way, it explains 56% of the variation in the overall mortality delta. In addition, Panel C of Table II reports that private pensions, rather than the active purchase of individual annuities, account for most of private annuitization. Together, these facts imply that most of the variation in observed health and mortality delta is driven by heterogeneity in the ownership of private pensions and the default path of annuitization conditional on ownership

IV. Explaining Household Insurance Choice

We first estimate household preferences based on the observed demand for insurance. We then compare the observed demand to the optimal demand predicted by the life-cycle model. Finally, we estimate the welfare cost of deviations from the optimal demand.

A. Estimation Methodology

Proposition 1 shows that the subjective discount factor is not separately identified from relative risk aversion since it enters through the term $(\beta R)^{1/\gamma}$. Therefore, we calibrate the subjective discount factor to $\beta = 0.96$ annually, which is a common practice in the life-cycle literature. We also normalize the utility weight for good health to $\omega(3) = 1$. We estimate the remaining preference parameters, which are relative risk aversion, the bequest motive, and the utility weight for poor health. For convenience, we denote these parameters as $\theta = [\gamma, \omega(1), \omega(2)]'$.

Heterogeneity in bequest motives is a natural explanation for the significant variation in observed mortality delta across households. That is, households with higher mortality delta in Figure 4 simply have stronger bequest motives, and those with lower mortality delta have weaker bequest motives. Therefore, we model the bequest motive as heterogeneous across households, but constant within a household over time. As discussed in Section III, most of the variation in observed mortality delta arises from heterogeneity in the ownership of private pensions. Therefore, a positive relation between mortality delta and the bequest motive means that households with weaker bequest motives tend to have jobs with private pension benefits.

For each household $i \in \{1, \dots, I\}$, let $j = 1, \dots, J_i$ denote its observations at different interviews. Let $L_{i,j}(\theta)$ denote the per-period welfare cost for household i at interview j, implied by equation (26) for $n = 1$. We estimate household preferences through a two-step procedure. In the first step, we estimate average preferences by minimizing the average per-period welfare cost:

$$\frac{1}{I} \sum_{i=1}^{I} \frac{1}{J_i} \sum_{j=1}^{J_i} L_{i,j}(\theta). \tag{27}$$

We do so through continuous-updating generalized method of moments:

$$\widehat{\theta} = \arg\min_{\theta} m(\theta)' W(\theta)^{-1} m(\theta), \tag{28}$$

where

$$m(\theta) = \frac{1}{I} \sum_{i=1}^{I} \frac{1}{J_i} \sum_{j=1}^{J_i} \frac{\partial L_{i,j}(\theta)}{\partial \theta} \tag{29}$$

is the moment function and

$$W(\theta) = \frac{1}{I} \sum_{i=1}^{I} \frac{1}{J_i} \sum_{j=1}^{J_i} \frac{\partial L_{i,j}(\theta)}{\partial \theta} \frac{\partial L_{i,j}(\theta)}{\partial \theta'} \tag{30}$$

is the weighting matrix. As we show in Appendix A, the welfare cost $L_{i,j}(\theta)$ is known in closed form, so that we can calculate its derivative numerically.

In the second step, we estimate the bequest motive for each household i by minimizing the average per-period welfare cost:

$$\widehat{\omega}_i(1) = \arg\min_{\omega_i(1)} \frac{1}{J_i} \sum_{j=1}^{J_i} L_{i,j}(\widehat{\gamma}, \omega_i(1), \widehat{\omega}(2)). \tag{31}$$

Finally, we regress the logarithm of estimated bequest motives $\widehat{\omega}_i(1)$ on household characteristics such as age, marital status, the presence of children, and financial and housing wealth. This last step is a simple diagnostic to describe the conditional distribution of bequest motives and to see whether it is consistent with economic intuition.

B. *Estimating Household Preferences*

Table IV reports our estimates of household preferences. Our estimate of relative risk aversion is 2.17 with a standard error of 0.01. Our point estimate is somewhat lower, and our standard error is much smaller than previous estimates based on the Health and Retirement Study. In particular, our point estimate is in the lower range of the confidence interval in De Nardi et al. (2010), which is estimated from the realized path of savings instead of insurance choice (i.e., the desired path of wealth in future health states). Our point estimate is also lower than that in Barsky et al. (1997), which is based on survey responses to hypothetical income gambles. Higher risk aversion would imply higher welfare cost of insurance choice because Proposition 3 shows that the welfare cost is approximately linear in relative risk aversion.

Our estimate of the utility weight for poor health is 0.74 with a standard error of 0.01. The left panel of Figure 4 explains why we find that consumption and health are complements. Average health delta is negative throughout the life cycle because few households own long-term care insurance, and many more own annuities or private pensions. These ownership patterns reveal that the average household desires to deliver less wealth to poor future health states, which must be justified through a low marginal utility of consumption in poor health. Put differently, we should see a lot more demand for long-term care insurance if consumption and health were less complementary. Finkelstein et al. (2013) also find evidence for complementarity of consumption and health, based on the relation between realized permanent income (a proxy for consumption) and health instead of insurance choice (i.e., the desired path of consumption in future health states).

The average bequest motive from the first-step estimation is 5.20 with a standard error of 0.03. That is, the average household has a strong bequest motive that is equivalent to more than 5 periods (10 years) of consumption. The presence of a bequest motive is consistent with the survey evidence (Laitner and Juster, 1996; Ameriks et al., 2011). The right panel of Figure 4 explains why we find such a strong bequest motive. Average mortality delta is positive for younger households because many own life insurance, and only slightly negative for older households because many do not own annuities or private pensions. As emphasized by Bernheim (1991) and Brown (2001), an intentional bequest motive can simultaneously justify a strong demand for life insurance and a weak demand for annuities. The fact that our sample includes married men, who may want to leave wealth for a surviving spouse, partly explains why we find such a strong bequest motive.

Figure 5 reports the distribution of estimated bequest motives from the second-step estimation. The distribution of bequest motives has wide range and positive skewness, much like the wealth distribution. 64% of households have bequest motives that are less than 5 periods (10 years) of consumption, and 91% of households have bequest motives that are less than 10 periods (20 years) of consumption.

In Table V, we regress the logarithm of estimated bequest motives on household characteristics. Consistent with economic intuition, married households and those with living children have stronger bequest motives. More educated and wealthier households also have stronger bequest motives, consistent with the notion that bequests are a luxury good. The bequest motive increases by 53% per 100% increase in financial and housing wealth. Overall, these household characteristics explain 66% of the variation in bequest motives.

The remaining 34% of the variation in bequest motives may be explained by unobserved characteristics (Fang and Kung, 2012). Yet another possibility is that household-specific bequest motives capture some variation in observed mortality delta due to market incompleteness or suboptimal choice, rather than preference heterogeneity. There are two patterns in Table V that suggest this possibility. First, the bequest motive is negatively related to self-reported health status, which suggests that adverse selection explains some of the variation in observed mortality delta. Second, the bequest motive is positively related to age, which is counter to economic intuition that it weakens over the life cycle.

C. Observed versus Optimal Demand for Insurance

The left panel of Figure 6 is a scatter plot of the observed health delta for each household-interview observation against the optimal health delta predicted by the life-cycle model. The right panel is an analogous scatter plot for mortality delta. If the life-cycle model were perfect, the slope of the regression line would be one, and the R^2 would be 100%. The life-cycle model has significant explanatory power. For health delta, the slope of the regression line is 0.68, and the R^2 is 68%. For mortality delta, the slope of the regression line is 0.99, and the R^2 is 83%.

The 45-degree line in the left panel of Figure 6 divides the sample into two groups. Above the 45-degree line are households that have too much whole life insurance or long-term care insurance, whose health delta is higher than the optimal health delta. Below the 45-degree line are households that have too much annuities or private pensions, whose health delta is lower than the optimal health delta.

The 45-degree line in the right panel of Figure 6 also divides the sample into two groups. Above the 45-degree line are households that are under-annuitized, whose mortality delta is higher than the optimal mortality delta. Below the 45-degree line are households that are over-annuitized, whose mortality delta is lower than the optimal mortality delta. This figure uncovers a new puzzle that is distinct from the so-called "annuity puzzle". The unexplained variation in the degree to which households are annuitized, rather than the average level at which they are annuitized, is puzzling from the perspective of life-cycle theory.

In Table VI, we use a panel regression model with household fixed effects to explain the deviations from optimal health and mortality delta. Our explanatory variables are dummies for present health and 65 or older, a quadratic polynomial in age, log financial and housing wealth, and the interaction of the dummies with the age polynomial and log financial and housing wealth. We control for financial and housing wealth to identify variation in age that is independent of variation in household resources over the life cycle. The positive and significant coefficients on age mean that the life-cycle model fails to explain the variation in health and mortality delta within a household over time. In order to resolve this puzzle, the bequest motive would have to strengthen over the life cycle, which is counter to economic intuition.

As we discuss in Section V, the model prescribes that households, especially those younger than 65, decrease their health and mortality delta over the life cycle by rebalancing from life insurance to annuities. Observed health and mortality delta are much more persistent than the predictions of the life-cycle model, due to the default path of annuitization from private pensions and the lack of rebalancing. Households may not be able to rebalance due to market incompleteness that may arise for various reasons, including private information and borrowing constraints. Alternatively, households may not rebalance due to suboptimal choice.

D. Welfare Cost of Household Insurance Choice

D.1. Per-Period Welfare Cost

We now estimate the per-period welfare cost of household insurance choice by applying Proposition 3 for $n = 1$. Conceptually, the per-period welfare cost assumes that the household deviates from the optimal health and mortality delta in the present period, then follows the optimal path for the remaining lifetime. While the per-period welfare cost is not our primary measure of interest, we can estimate it based on the observed health and mortality delta alone, without an auxiliary model for predicting the path of future health and mortality delta.

Panel A of Table VII reports the median per-period (two-year) welfare cost by age group. The per-period welfare cost for households aged 51 to 57 is precisely estimated to be 0.03% of total wealth. Through equation (26) for $n = 1$, we can decompose this welfare cost into the sum of three parts. The deviations from optimal health delta account for 0.01% of the welfare cost, and so do the deviations from optimal mortality delta. The interaction between health and mortality delta explains the remainder of the welfare cost, which is 0.01%. The per-period welfare cost is virtually constant in age, which implies that the life-cycle model fits uniformly well across age.

D.2. Lifetime Welfare Cost

We now estimate the lifetime welfare cost of household insurance choice by applying Proposition 3 for $n = T - t$. This is essentially a present-value calculation that accumulates the per-period welfare cost over the life cycle. This calculation requires an auxiliary model for predicting the path of future health and mortality delta. In Appendix D, we estimate such a model based on the joint transition probabilities for health and insurance ownership.

Panel B of Table VII reports the median lifetime welfare cost by age group. The lifetime welfare cost for households aged 51 to 57 is 3.21% of total wealth with a standard error of 0.27%. By the homogeneity of preferences, this is a large welfare cost that is equivalent to a

3.21% reduction in lifetime consumption. To put our estimate into perspective, Calvet et al. (2007) find that the welfare cost of under-diversification in stock and mutual fund portfolios is 0.51% of disposable income for the median Swedish household. Through equation (26) for $n = T - t$, we can decompose this welfare cost into the sum of three parts. The deviations from optimal health delta account for 0.46% of the welfare cost, while the deviations from optimal mortality delta account for 3.52%. The interaction between health and mortality delta explains the remainder of the welfare cost, which is -0.77%. The lifetime welfare cost is higher for younger households, for whom the per-period welfare cost accumulates over a longer expected lifetime.

V. Optimal Portfolio of Existing Insurance Products

We illustrate how a portfolio of existing insurance products can replicate the optimal health and mortality delta predicted by the life-cycle model. Our illustration is for a male born between 1936 and 1940 and in good health at age 51. The household faces the health transition probabilities, out-of-pocket health expenses, and income that are reported in Table III. The household's initial wealth is \$95.4k at age 51, which is chosen to match average financial and housing wealth for this cohort. In addition to bonds, the household can save in short-term life insurance, deferred annuities, and long-term care insurance (i.e., short-term supplemental health insurance). Figure 1 reports the health and mortality delta for these insurance products at age 51. The household's preference parameters are those that we estimate in the Health and Retirement Study, reported in Table IV.

Panel A of Table VIII reports the optimal health and mortality delta, which we calculate through Proposition 1. The optimal health delta is $-\$1.9$k at age 51, which implies that the household desires an additional \$1.9k in good health relative to poor health at age 53. As equation (21) shows, three offsetting forces determine the optimal health delta. First, the household has preference for consumption in good health over poor health (i.e., $\omega(2) < \omega(3)$ in Table IV), which lowers the optimal health delta. Second, the household saves less in poor health because of shorter life expectancy (i.e., $c_{t+1}(2) > c_{t+1}(3)$), which lowers the optimal health delta. Third, the household has lower lifetime disposable income in poor health, which raises the optimal health delta. The first two forces more than offset the third, so that the optimal health delta is overall negative at age 51.

The optimal mortality delta is \$268.3k at age 51, which implies that the household desires to leave an additional \$268.3k at death relative to good health at age 53. As equation (22) shows, three offsetting forces determine the optimal mortality delta. First, the household has preference for bequest over consumption in good health (i.e., $\omega(1) > \omega(3)$ in Table IV), which raises the optimal mortality delta. Second, the household must save for future consumption in good health (i.e., $c_{t+1}(3) < 1$), which lowers the optimal mortality delta. Third, the household has higher lifetime disposable income in good health, which raises the optimal mortality delta. The first and third forces more than offset the second, so that the optimal mortality delta is overall positive at age 51.

Panel B of Table VIII reports a portfolio of life insurance, deferred annuities, and long-term care insurance that replicates the optimal health and mortality delta, which we calculate through Proposition 2. The optimal portfolio at age 51 consists of 293.1 units of life insurance (i.e., death benefit of \$293.1k), 5.8 units of deferred annuities (i.e., income of \$5.8k per period), no long-term care insurance, and 68.1 units of bonds. Panel C reports the cost of the optimal portfolio, which is the sum of \$7.1k in life insurance, \$22.8k in deferred annuities, and \$65.5k in bonds.

The left panel of Figure 7 shows that the optimal health delta has a slightly U-shaped profile over the life cycle. To replicate the optimal health delta, the household needs long-term care insurance at age 86 and older when out-of-pocket health expenses start to rise rapidly. Since one unit of long-term care insurance eliminates all uncertainty in health expenses in the subsequent period, the positions reported in Panel B of Table VIII imply that the household demands only partial coverage throughout the life cycle. Full coverage is not optimal because consumption and health are complements, and the shorter life expectancy in poor health naturally offsets the higher health expenses.

The right panel of Figure 7 shows that the optimal mortality delta falls over the life cycle. To replicate the optimal mortality delta, the household needs life insurance when young to generate positive mortality delta, then switch to deferred annuities when old to generate negative mortality delta. The optimal position in deferred annuities increases from 5.8 units at age 51 to 32.7 units at age 93. A practical implication of Figure 7 is that an insurance company may want to package life insurance and deferred annuities into a product that automatically replicates the life-cycle profile of optimal mortality delta, eliminating the need for active rebalancing.

Figure 7 shows the optimal health and mortality delta for a bequest motive of 5.20. To understand the role of the bequest motive, the same figure shows the optimal health and mortality delta under no bequest motive, holding the other preference parameters constant. A weaker bequest motive shifts down the optimal mortality delta, which would imply a higher demand for deferred annuities. A weaker bequest motive also shifts down the optimal health delta because it raises the average propensity to consume in poor health.

In this illustration, the household is exposed to reclassification risk because it has access to only short-term insurance products. For example, a household in good health at age

51 has to pay a higher premium for life insurance and supplemental health insurance if its health worsens at age 53. As emphasized by Cochrane (1995), the household can insure reclassification risk in a world with health state-contingent securities. Our illustration here shows that an optimal portfolio of short-term insurance products essentially replicates health state-contingent securities, thereby insuring reclassification risk.

VI. Conclusion

We find a large welfare cost of deviations from the optimal demand for insurance. We have several reasons to suspect that this is a consequence of suboptimal choice for many households. First, the variation in observed demand is mostly driven by heterogeneity in the ownership of private pensions and the default path of annuitization conditional on ownership. Second, we calibrate the life-cycle model to the Health and Retirement Study and find that a typical household can replicate the optimal health and mortality delta through existing insurance products. Finally, there has been little academic guidance on optimal portfolio choice for insurance products, unlike for equity and fixed-income products. Due to the lack of academic guidance, existing financial calculators (available from insurance companies) make recommendations for life insurance, annuities, and long-term care insurance in isolation, instead of as a comprehensive financial decision.

To improve household insurance choice, retail financial advisors and insurance companies should report the health and mortality delta of their insurance products, just as mutual fund companies already report the market beta of their equity products and the duration of their fixed-income products. We hope that these risk measures will facilitate standardization, identify overlap between existing products, identify risks that are not insured by existing products, and ultimately lead to new product development. One such product that we find particularly promising is a life-cycle product that automatically shifts from life insurance to annuities as a function of age, so that households achieve the optimal mortality delta over the life cycle without active rebalancing. This product would be analogous to life-cycle funds that automatically shift from equity to fixed income as a function of age, which have proven to be tremendously successful in the mutual fund industry.

Smarter default plans for employer-provided insurance and retirement accounts is yet another way to improve household insurance choice, especially for the financially illiterate. The default plan for group life insurance could start with a higher death benefit for younger employees, and let it gradually decline to no coverage at retirement age. Defined contribution plans could annuitize a share of savings by default, thereby mimicking defined benefit plans. Thus, a combination of group life insurance and annuitization through retirement accounts could replicate the optimal mortality delta over the life cycle, without active decisions on the part of employees. These simple changes to the default plan only affect the allocation of wealth across future health states, without changing the overall level of savings. Therefore, these changes potentially improve welfare for free without additional cost to the employer.

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Table I: Predicting Future Mortality with Observed Health Problems

A probit model is used to predict death within two years from the present interview. This table reports the marginal effects on the mortality rate (in percentage points) with heteroskedasticity-robust t-statistics in parentheses. The sample consists of males aged 51 and older in the Health and Retirement Study from 1992 to 2010. The omitted cohort is males born prior to 1911.

Explanatory variable	Marginal effect	t-statistic
Doctor-diagnosed health problems:		
High blood pressure	0.70	(1.64)
Diabetes	4.49	(5.32)
Cancer	10.43	(7.10)
Lung disease	6.07	(4.57)
Heart problems	1.98	(3.39)
Stroke	3.62	(2.84)
$(\text{Age} - 51)/10$	2.28	(11.16)
$\times$ High blood pressure	-0.03	(-0.18)
$\times$ Diabetes	-0.61	(-2.85)
$\times$ Cancer	-1.39	(-6.29)
$\times$ Lung disease	0.03	(0.11)
$\times$ Heart problems	0.08	(0.44)
$\times$ Stroke	-0.02	(-0.08)
Birth cohort:		
1911–1915	-1.24	(-3.77)
1916–1920	-1.83	(-6.73)
1921–1925	-2.56	(-10.94)
1926–1930	-3.02	(-12.63)
1931–1935	-3.34	(-10.54)
1936–1940	-3.62	(-9.38)
1941–1945	-3.11	(-10.29)
1946–1950	-3.20	(-13.49)
1951–1955	-2.84	(-9.99)
Correctly predicted (%):		
Both outcomes	94	
Dead only	66	
Alive only	94	
Observations	38,913	

Table II: Health Problems, Health Care Utilization, and Insurance Ownership

Term life insurance refers to individual and group policies that have only a death benefit. Whole life insurance refers to policies that build cash value, from which the policyholder can borrow or receive cash upon surrender. The sample consists of males aged 51 and older in the Health and Retirement Study from 1992 to 2010.

Age	51–64		65–78		79–	
Health	Poor	Good	Poor	Good	Poor	Good
Panel A: Doctor-diagnosed health problems and difficulty with activities of daily living (%)						
High blood pressure	57	30	67	47	65	47
Diabetes	20	8	35	14	28	15
Cancer	8	3	28	11	33	21
Lung disease	7	3	19	6	22	7
Heart problems	23	9	54	22	74	35
Stroke	5	2	15	5	31	10
Some difficulty bathing	3	1	5	2	21	9
Some difficulty dressing	6	3	10	5	24	13
Some difficulty eating	1	0	3	1	12	4
Panel B: Health care utilization (%)						
Monthly doctor visits	9	3	16	6	21	11
Hospital stay	25	11	42	23	55	34
Outpatient surgery	22	16	28	23	27	26
Nursing home stay	0	0	2	1	14	5
Home health care	3	1	10	5	22	10
Special facilities and services	8	4	11	6	16	10
Prescription drugs	80	51	95	77	97	86
Panel C: Insurance ownership rate (%)						
Term life insurance	72	71	57	59	48	48
Whole life insurance	35	34	32	31	29	28
Annuities including private pensions	45	48	55	59	58	63
Annuities excluding private pensions	1	1	4	4	6	7
Long-term care insurance	8	9	18	20	18	18
Panel D: Insurance coverage conditional on ownership (median in thousands of 2005 dollars)						
Term life insurance	78.4	81.0	22.8	23.2	10.7	10.0
Whole life insurance	42.8	42.8	23.2	23.3	16.6	15.0
Annual annuity and pension income	0.0	0.0	11.9	12.8	9.0	9.4
Financial and housing wealth	152.8	175.7	233.9	257.6	234.2	244.3
Observations	7,702	12,234	4,672	10,228	1,717	3,796

Table III: Health Dynamics, Out-of-Pocket Health Expenses, and Income

Panels A and B are based on the estimated model for health transition probabilities. Panels C and D are based on the estimated models for out-of-pocket health expenses and income, respectively. Panel E reports the present value of future disposable income, based on the estimated health transition probabilities and a riskless interest rate of 2%. The reported estimates are for males with average income, born between 1936 and 1940, and in good health at age 51

	Age						
Health	51	58	65	72	79	86	93
Panel A: Long-run health transition probabilities							
Dead	0.00	0.09	0.24	0.34	0.50	0.65	0.87
Poor	0.00	0.25	0.22	0.18	0.17	0.15	0.08
Good	1.00	0.65	0.54	0.48	0.34	0.20	0.05
Panel B: Remaining life expectancy (years)							
Poor	26	22	17	13	9	6	4
Good	27	24	20	17	12	9	6
Mean	27	24	19	16	11	8	5
Panel C: Out-of-pocket health expenses (thousands of 2005 dollars per year)							
Poor	2.3	2.9	3.5	4.0	6.1	9.6	22.1
Good	0.4	0.7	1.0	1.2	1.7	2.2	3.6
Mean	0.4	1.3	1.7	2.0	3.1	5.4	15.0
Panel D: Income (thousands of 2005 dollars per year)							
Mean	56.1	48.4	30.6	25.2	20.7	18.8	17.7
Panel E: Present value of future disposable income (thousands of 2005 dollars)							
Poor	568.5	390.7	230.3	146.7	63.5	19.0	-23.8
Good	606.1	437.8	278.5	191.3	96.7	37.8	-30.1
Mean	606.1	424.6	264.4	179.3	85.7	29.7	-26.2

Table IV: Estimated Household Preferences

The subjective discount factor is calibrated to 0.96 annually, and the utility weight for good health is normalized to one. The remaining preference parameters are estimated by continuous-updating generalized method of moments with heteroskedasticity-robust standard errors in parentheses. The sample consists of males aged 51 and older in the Health and Retirement Study from 1992 to 2010.

Parameter	Symbol	Value
Subjective discount factor	β	0.96
Relative risk aversion	γ	2.17
		(0.01)
Average bequest motive	$\omega(1)$	5.20
		(0.03)
Utility weight for poor health	$\omega(2)$	0.74
		(0.01)
Utility weight for good health	$\omega(3)$	1.00
Observations		28,828

Table V: Explaining the Bequest Motive

A linear regression model is used to estimate how the logarithm of the estimated bequest motive depends on household characteristics. This table reports the coefficients with heteroskedasticity-robust t-statistics in parentheses. The sample consists of males aged 51 and older in the Health and Retirement Study from 1992 to 2010. The omitted categories for the dummies are non-high school graduate, white, good self-reported health, and born prior to 1911.

Explanatory variable	Coefficient	t-statistic
65 or older	0.14	(2.19)
Poor health	-0.01	(-0.30)
Married	0.58	(11.95)
Has living children	0.29	(5.03)
High school graduate	0.62	(10.57)
× Married	0.05	(1.64)
× Has living children	-0.16	(-3.77)
College graduate	1.44	(23.42)
× Married	-0.14	(-3.83)
× Has living children	-0.16	(-3.65)
Black	-0.48	(-10.71)
Hispanic and other	-0.30	(-5.68)
Self-reported health status:		
Poor	0.33	(10.92)
Fair	0.20	(7.30)
Very good	-0.16	(-3.96)
Excellent	-0.48	(-6.67)
$(\text{Age} - 51)/10$	0.64	(6.92)
× 65 or older	-0.22	(-2.63)
× Poor health	0.02	(0.62)
× Married	-0.10	(-2.42)
× Has living children	-0.06	(-1.31)
× High school graduate	-0.14	(-3.65)
× College graduate	-0.34	(-8.07)
× Black	0.01	(0.25)
× Hispanic and other	-0.07	(-1.04)
× Poor	-0.13	(-3.56)
× Fair	0.01	(0.20)
× Very good	0.02	(0.35)
× Excellent	0.12	(1.61)

Explanatory variable	Coefficient	t-statistic
$(\text{Age} - 51)^2/100$	-0.06	(-1.26)
× 65 or older	0.08	(1.71)
× Poor health	-0.01	(-1.32)
× Married	0.01	(0.72)
× Has living children	-0.01	(-0.69)
× High school graduate	0.03	(2.82)
× College graduate	0.05	(4.58)
× Black	-0.02	(-1.59)
× Hispanic and other	0.01	(0.59)
× Poor	0.02	(1.95)
× Fair	-0.02	(-2.19)
× Very good	0.00	(0.24)
× Excellent	-0.01	(-0.45)
Log financial and housing wealth	0.53	(21.29)
× 65 or older	0.02	(1.55)
× Poor health	-0.01	(-0.85)
× Married	0.01	(0.97)
× Has living children	-0.05	(-2.69)
× High school graduate	-0.05	(-3.85)
× College graduate	-0.06	(-4.28)
× Black	-0.05	(-2.50)
× Hispanic and other	0.06	(2.74)
× Poor	0.00	(0.18)
× Fair	-0.02	(-1.85)
× Very good	-0.01	(-0.33)
× Excellent	0.01	(0.58)
Birth cohort:		
1911–1915	0.17	(5.12)
1916–1920	0.25	(7.58)
1921–1925	0.49	(15.22)
1926–1930	0.47	(14.14)
1931–1935	0.54	(15.78)
1936–1940	0.49	(14.23)
1941–1945	0.52	(14.43)
1946–1950	0.61	(16.58)
1951–1955	0.58	(14.46)
Constant	-1.01	(-12.18)
R^2	0.66	
Observations	28,464	

Table VI: Explaining the Deviations from Optimal Health and Mortality Delta

A panel regression model with household fixed effects is used to explain the deviations from optimal health and mortality delta. The optimal health and mortality delta are predicted by the life-cycle model with household-specific bequest motives. This table reports the coefficients with heteroskedasticity-robust t-statistics in parentheses. The sample consists of males aged 51 and older in the Health and Retirement Study from 1992 to 2010.

Explanatory variable	Health delta		Mortality delta	
65 or older	-47.15	(-19.88)	-32.75	(-1.94)
Poor health	3.93	(2.76)	22.87	(2.04)
$(\text{Age} - 51)/10$	2.13	(1.81)	200.15	(18.45)
$\times$ 65 or older	8.98	(5.33)	-163.58	(-12.62)
$\times$ Poor health	2.25	(4.19)	7.60	(1.73)
$(\text{Age} - 51)^2/100$	1.89	(2.55)	-77.49	(-11.48)
$\times$ 65 or older	-4.10	(-5.22)	77.64	(11.33)
$\times$ Poor health	-0.59	(-4.16)	-2.11	(-2.07)
Log financial and housing wealth	-1.61	(-11.92)	-4.12	(-4.11)
$\times$ 65 or older	3.89	(23.42)	8.85	(6.99)
$\times$ Poor health	-0.44	(-3.54)	-2.08	(-2.15)
R^2	0.20		0.15	
Observations	28,828		28,828	

Table VII: Welfare Cost of Household Insurance Choice

The per-period welfare cost is based on the deviations from optimal health and mortality delta. The lifetime welfare cost is based on the predicted path of future health and mortality delta, described in Appendix D. The sample consists of males aged 51 and older in the Health and Retirement Study from 1992 to 2010.

	Age					
	51–57	58–64	65–71	72–78	79–85	86–
Panel A: Per-period welfare cost (median in % of total wealth)						
Total cost	0.03	0.03	0.03	0.02	0.02	0.02
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.15)
Cost due to health delta	0.01	0.01	0.02	0.01	0.01	0.01
	(0.00)	(0.01)	(0.00)	(0.00)	(0.00)	(0.26)
Cost due to mortality delta	0.01	0.01	0.01	0.01	0.01	0.01
	(0.01)	(0.01)	(0.00)	(0.01)	(0.01)	(0.18)
Panel B: Lifetime welfare cost (median in % of total wealth)						
Total cost	3.21	2.91	2.70	1.57	0.72	0.38
	(0.27)	(0.19)	(0.22)	(0.22)	(0.26)	(0.49)
Cost due to health delta	0.46	0.38	0.39	0.23	0.13	0.15
	(0.07)	(0.04)	(0.05)	(0.06)	(0.09)	(0.39)
Cost due to mortality delta	3.52	3.14	2.98	1.69	0.73	0.32
	(0.30)	(0.21)	(0.23)	(0.24)	(0.28)	(0.51)

Table VIII: Optimal Portfolio of Insurance Products

Panel A reports the optimal health and mortality delta predicted by the life-cycle model with the preference parameters in Table IV. Panel B reports a portfolio of short-term life insurance, deferred annuities, long-term care insurance (i.e., short-term supplemental health insurance), and bonds that replicates the optimal health and mortality delta. Short-term policies mature in two years, and the income from deferred annuities start at age 65. Panel C reports the cost of the optimal portfolio in thousands of 2005 dollars, averaged across the health distribution at the given age. The reported estimates are for males with average income, born between 1936 and 1940, and in good health at age 51.

	Age						
	51	58	65	72	79	86	93
Panel A: Optimal health and mortality delta (thousands of 2005 dollars)							
Health delta	-1.9	-5.9	-12.9	-16.4	-17.6	-14.7	6.3
Mortality delta	268.3	132.1	27.3	-9.1	-39.4	-58.6	-86.0
Panel B: Optimal portfolio (units)							
Life insurance	293.1	175.8	106.8	69.5	26.9	0.0	0.0
Deferred annuities	5.8	7.9	10.1	11.9	13.5	15.4	32.7
Long-term care insurance	0.0	0.0	0.0	0.0	0.0	0.1	0.6
Bonds	68.1	165.1	208.6	228.2	248.6	260.0	240.6
Panel C: Cost of the optimal portfolio (thousands of 2005 dollars)							
Life insurance	7.1	6.4	4.7	3.6	2.4	0.0	0.0
Deferred annuities	22.8	38.9	69.7	67.3	52.3	40.8	44.8
Long-term care insurance	0.0	0.0	0.0	0.0	0.0	0.9	13.0
Bonds	65.5	158.6	200.5	219.3	239.0	249.9	231.3
Total cost	95.4	203.9	274.9	290.2	293.6	291.6	289.1

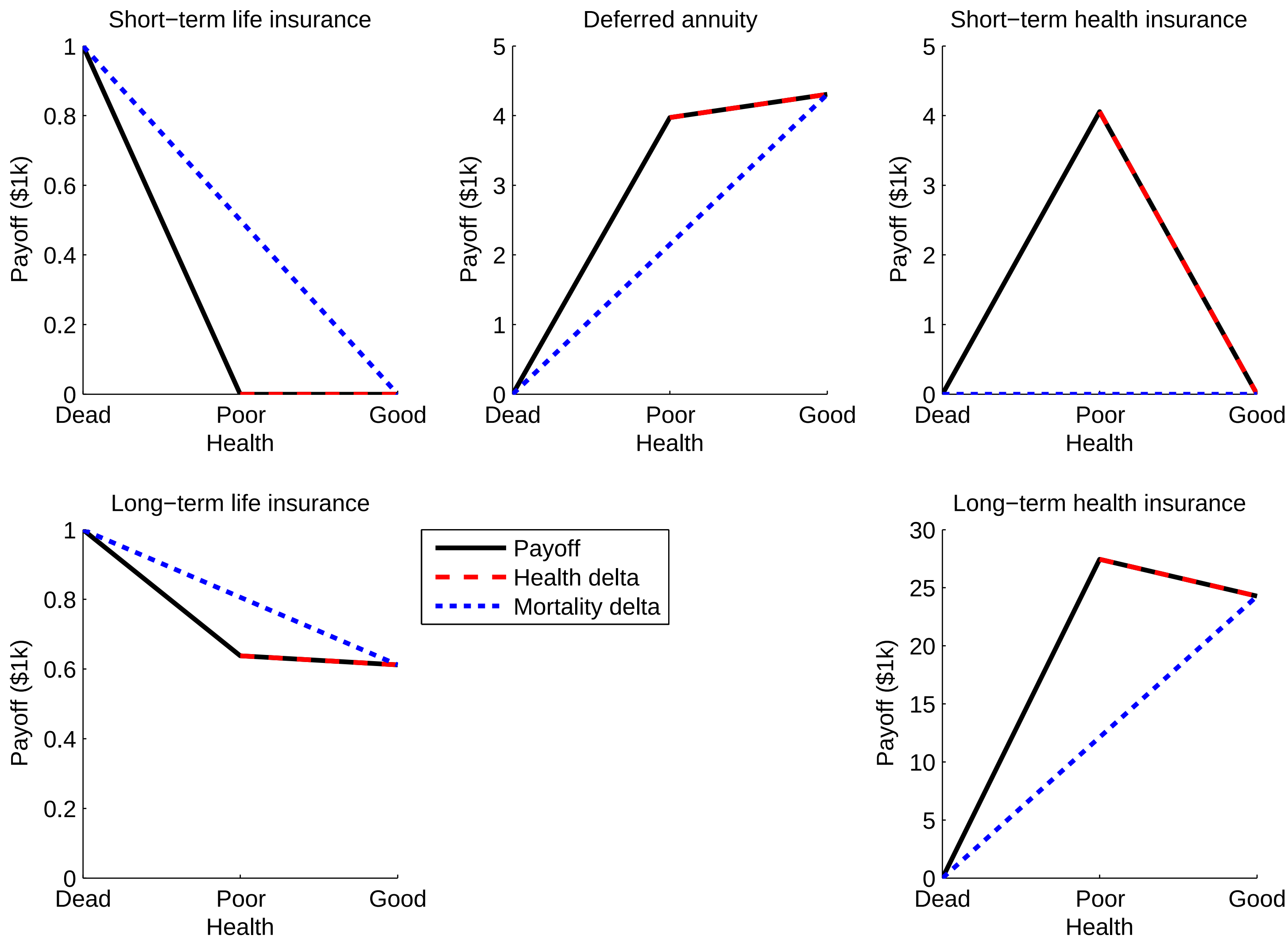

Figure 1: Health and mortality delta for insurance products
The solid line represents the payoff of each policy for the three possible health states in two years, reported in thousands of 2005 dollars. Short-term policies mature in two years (i.e., the frequency of interviews in the Health and Retirement Study), while long-term policies mature at death. The income from deferred annuities start at age 65. The reported estimates are for males with average income, born between 1936 and 1940, and in good health at age 51.

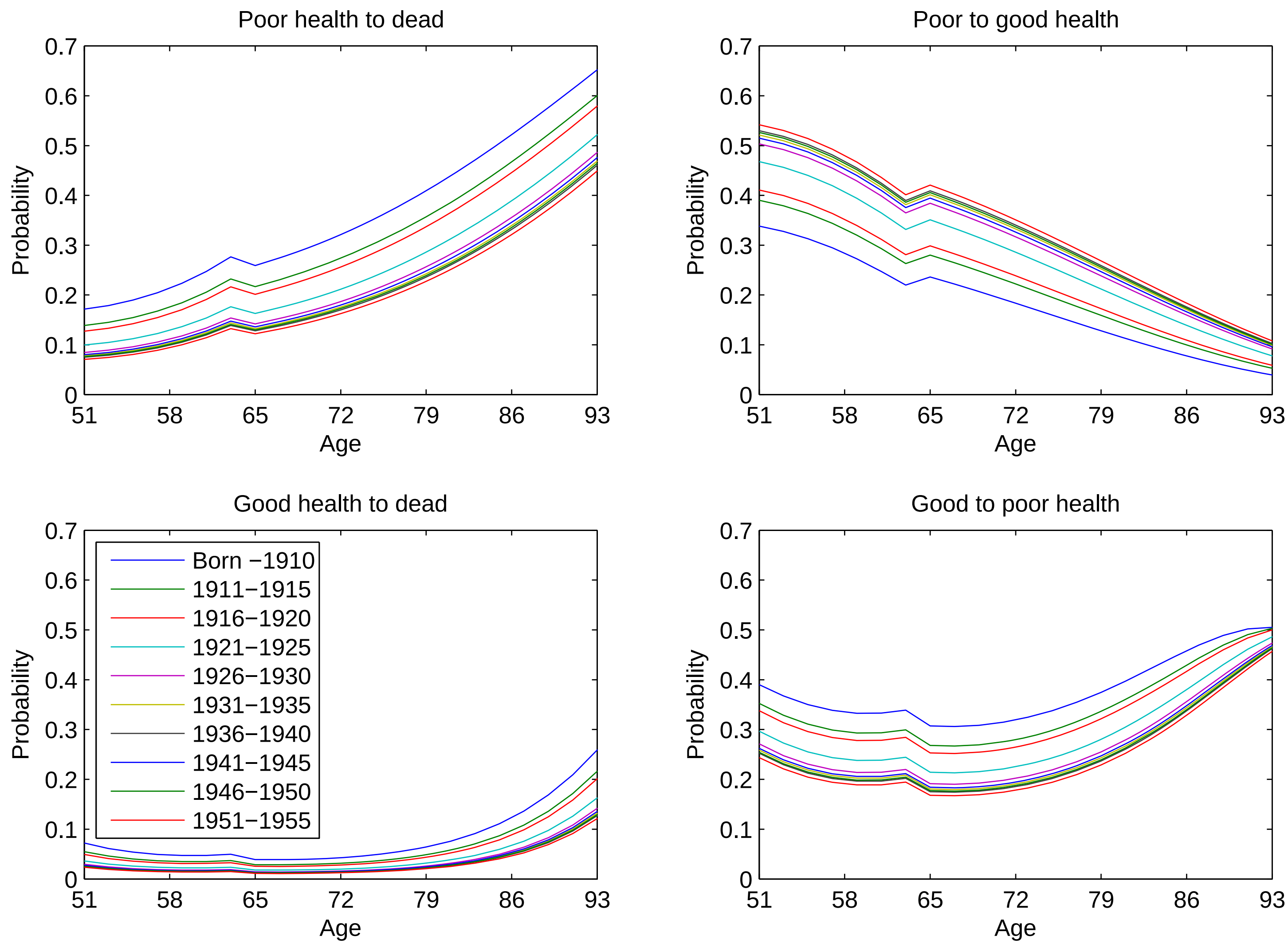

Figure 2: Estimated health transition probabilities

An ordered probit model is used to estimate how health transition probabilities depend on dummies for present health and 65 or older, a quadratic polynomial in age, log income, the interaction of the dummies with the age polynomial and log income, and cohort dummies. This figure reports the predicted transition probabilities at average income by age and birth cohort. The sample consists of males aged 51 and older in the Health and Retirement Study from 1992 to 2010.

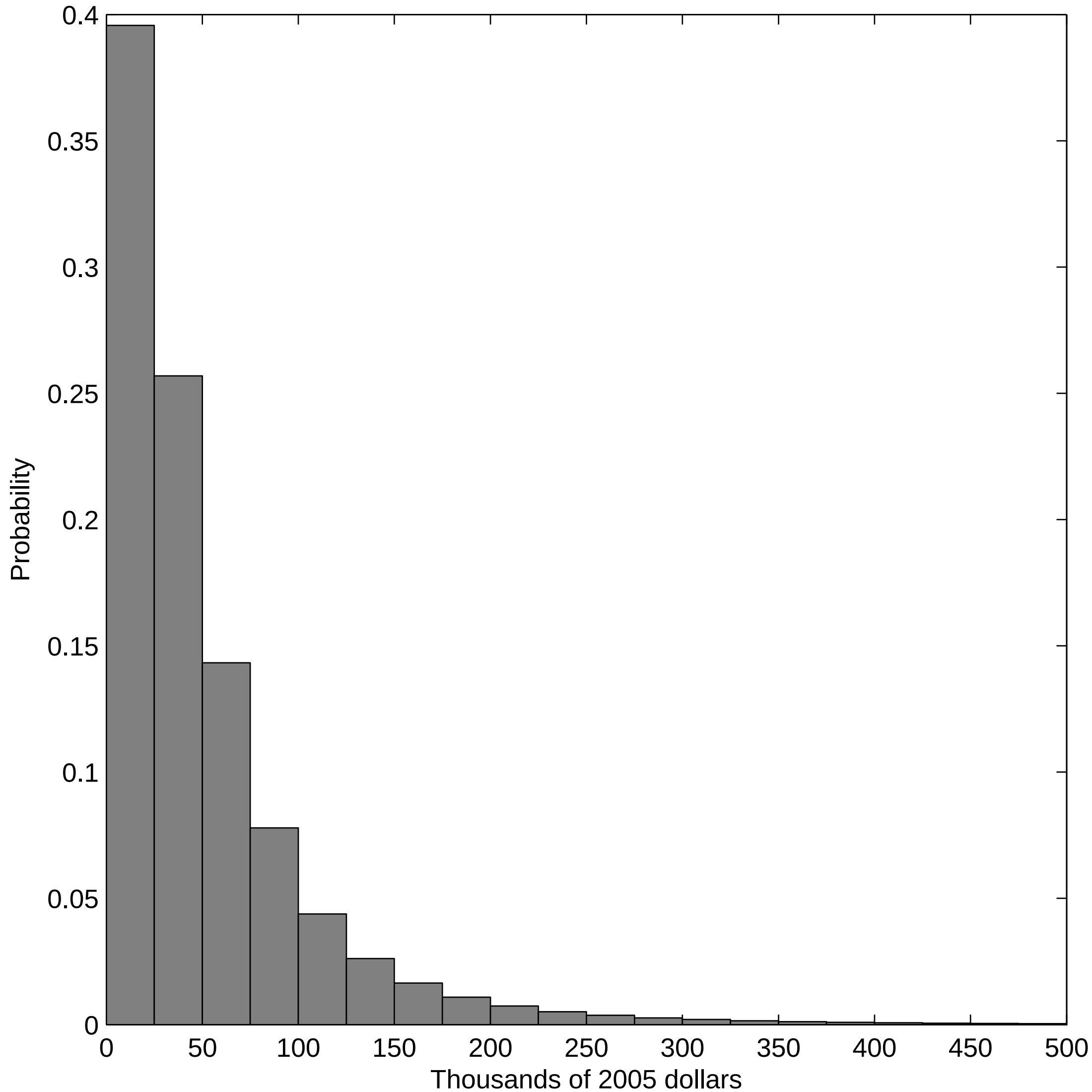

Figure 3: Lifetime out-of-pocket health expenses
The estimated models for health transition probabilities and out-of-pocket health expenses are used to simulate paths of out-of-pocket health expenses over the life cycle. This figure reports the distribution of realized out-of-pocket health expenses, discounted at a riskless interest rate of 2%. The reported distribution is for males born between 1936 and 1940 and in good health at age 51.

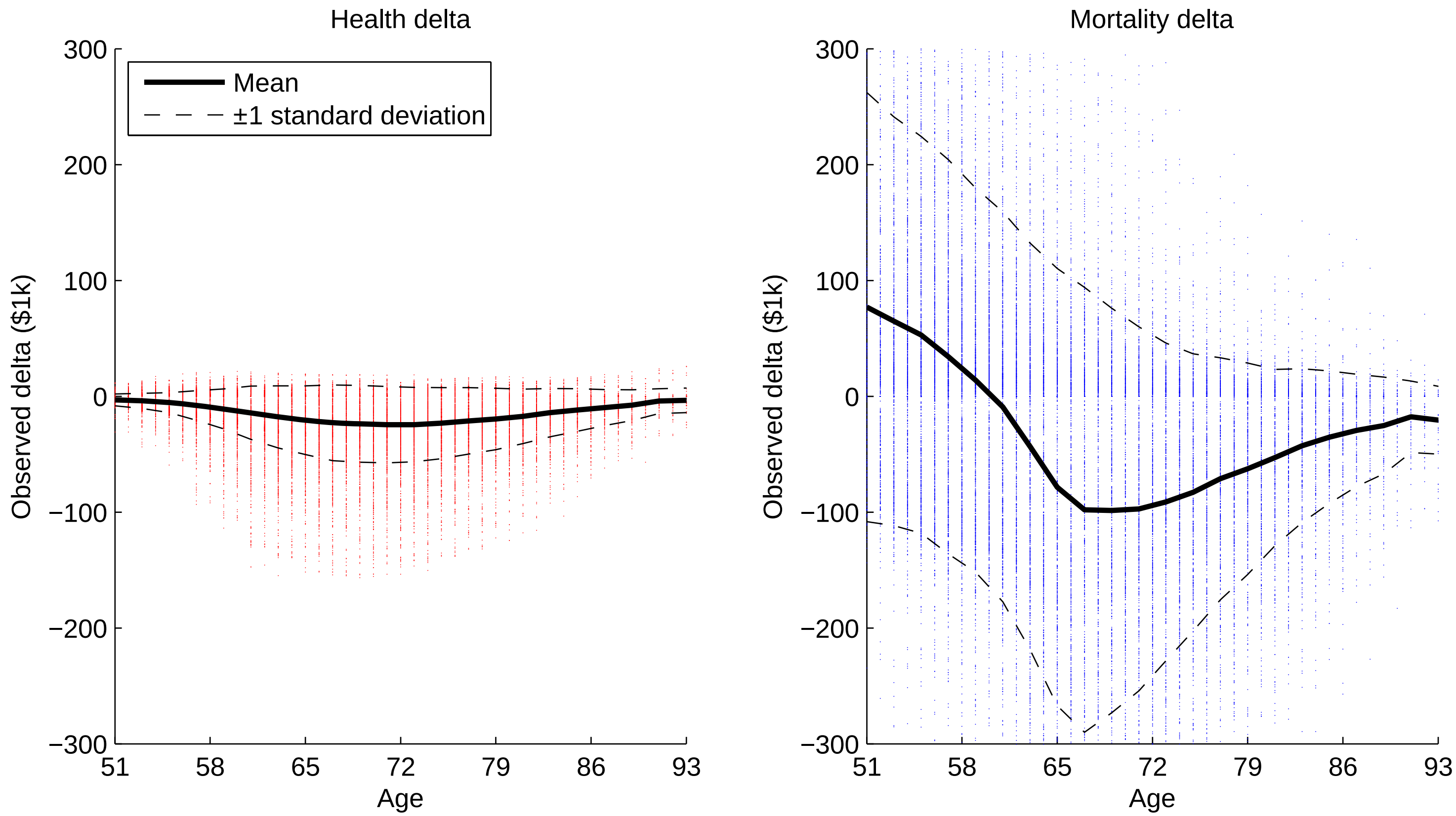

Figure 4: Observed health and mortality delta over the life cycle
Each dot in the left (right) panel represents health (mortality) delta for a household-interview observation. This figure also reports the mean and standard deviation by age, smoothed around a plus or minus one-year window. The sample consists of males aged 51 and older in the Health and Retirement Study from 1992 to 2010.

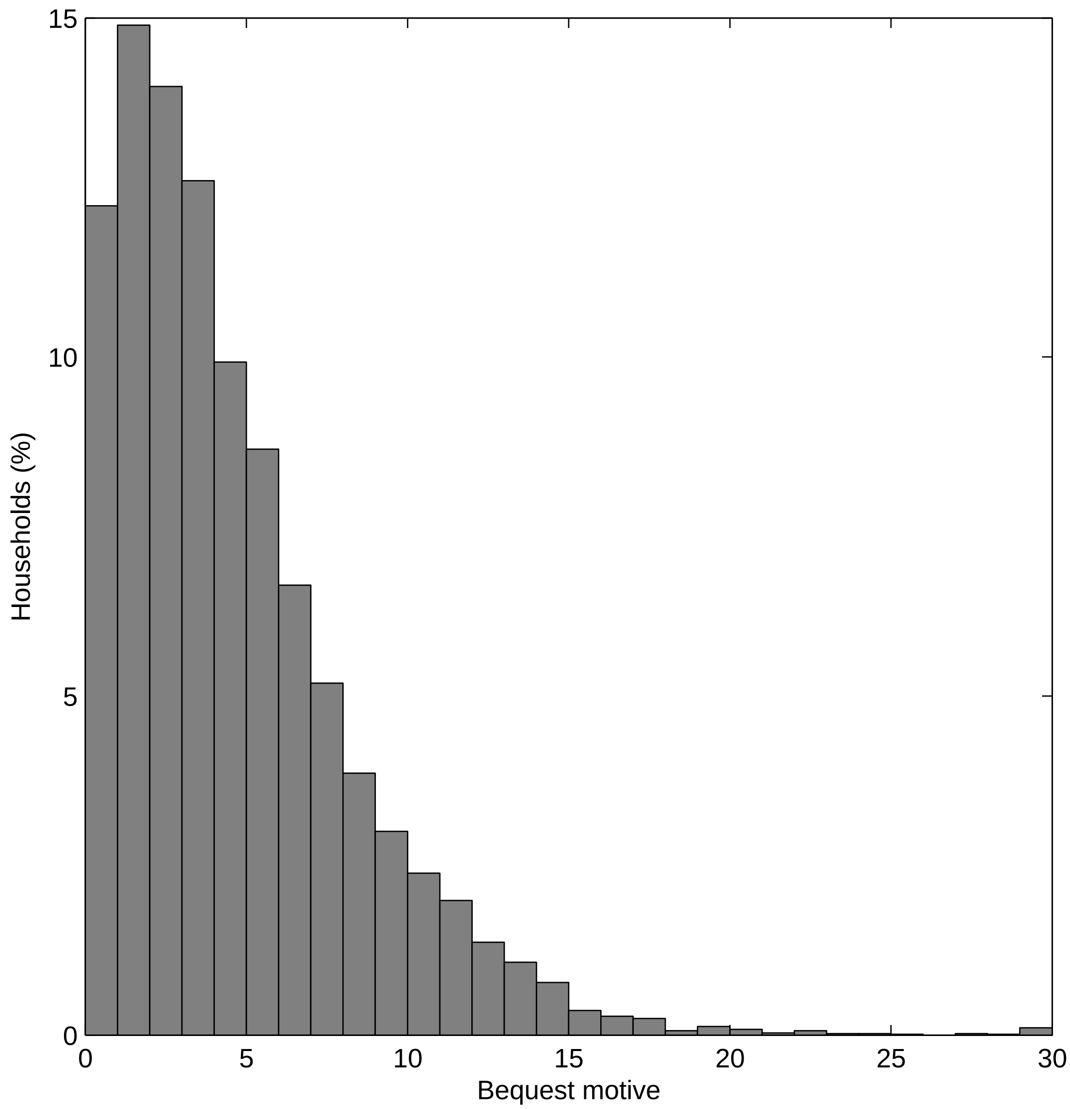

Figure 5: Estimated bequest motives
The bequest motive is estimated for each household to minimize the average per-period welfare cost. The sample consists of males aged 51 and older in the Health and Retirement Study from 1992 to 2010.

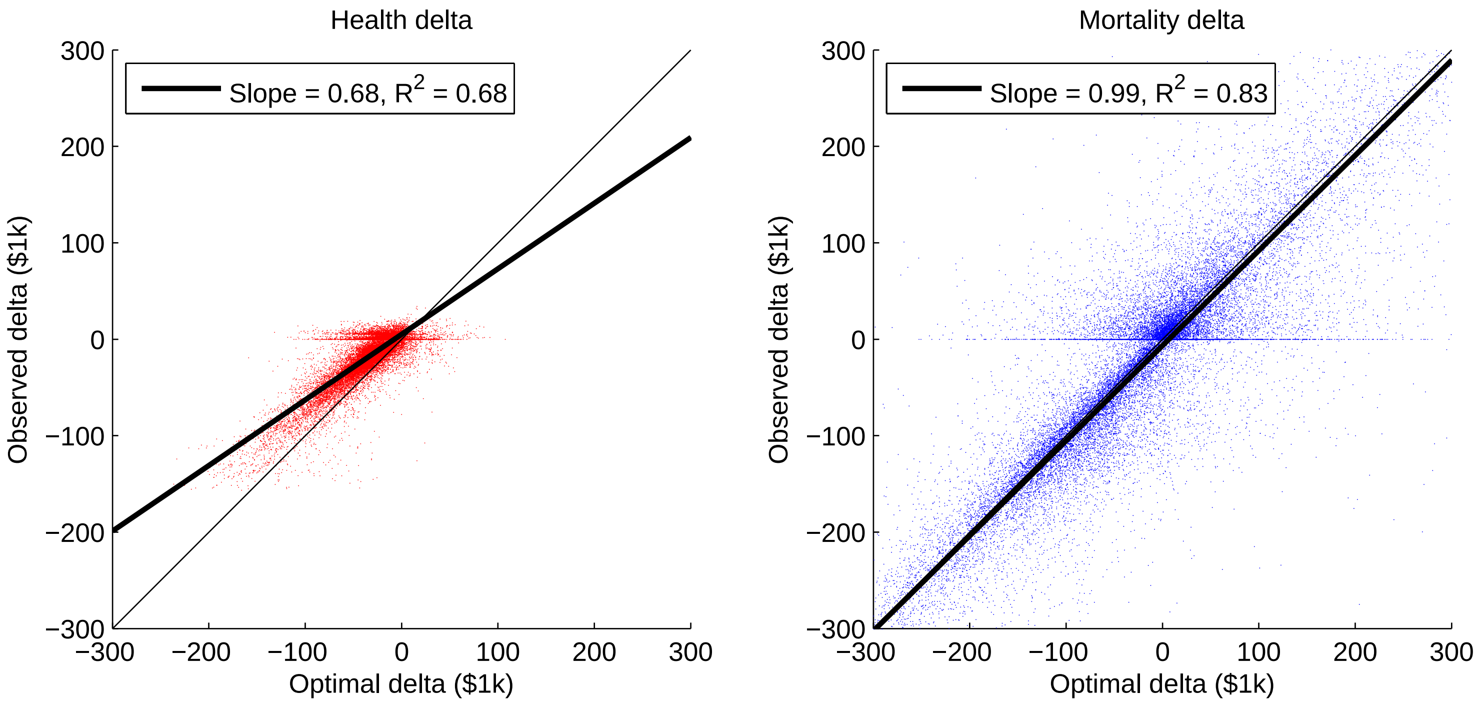

Figure 6: Observed versus optimal health and mortality delta
The optimal health and mortality delta are predicted by the life-cycle model with household-specific bequest motives. The sample consists of males aged 51 and older in the Health and Retirement Study from 1992 to 2010.

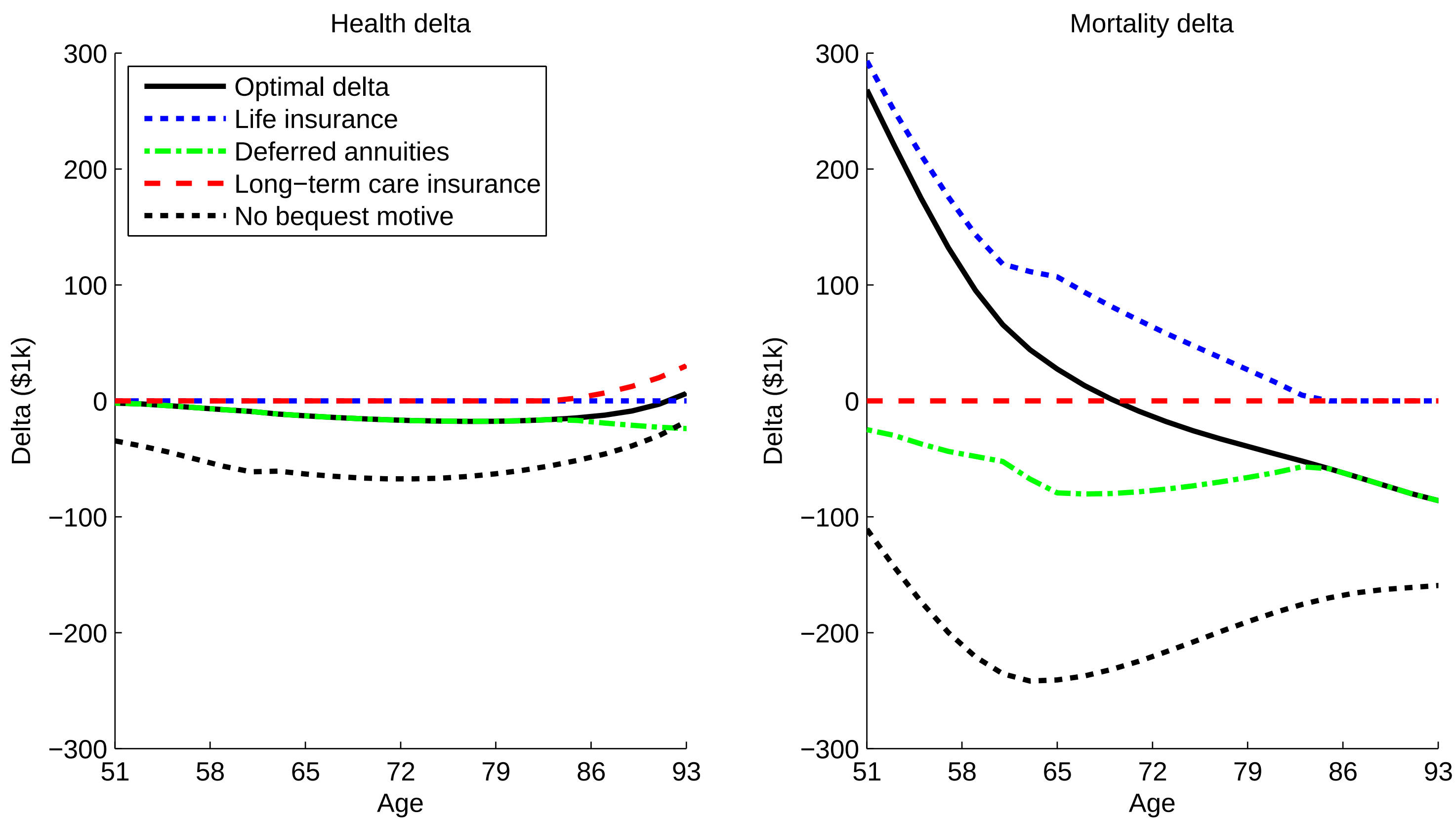

Figure 7: Optimal health and mortality delta over the life cycle

This figure reports the optimal health and mortality delta predicted by the life-cycle model with the preference parameters in Table IV. The sum of health (mortality) delta for short-term life insurance, deferred annuities, and long-term care insurance (i.e., short-term supplemental health insurance) equals the optimal health (mortality) delta. This figure also reports the optimal health and mortality delta predicted by the life-cycle model with no bequest motive. The reported estimates are for males with average income, born between 1936 and 1940, and in good health at age 51

Appendix A. Proofs of the Propositions

A. Proof of Proposition 1

The household maximizes the objective function (16) subject to the intertemporal budget constraint (14), which we rewrite as

$$A_t + Y_t - M_t - C_t = \sum_{j=1}^{3} \frac{\pi_t(h_t, j)}{R} A_{t+1}(j). \tag{A1}$$

The Bellman equation in period t is

$$\begin{aligned} V_t(h_t, A_t) = & \max_{C_t, A_{t+1}(1), A_{t+1}(2), A_{t+1}(3)} \Bigg\{ \omega(h_t)^\gamma C_t^{1-\gamma} \\ & + \beta \left[\pi_t(h_t, 1)\omega(1)^\gamma A_{t+1}(1)^{1-\gamma} + \sum_{j=2}^{3} \pi_t(h_t, j) V_{t+1}(j, A_{t+1}(j))^{1-\gamma} \right] \Bigg\}^{1/(1-\gamma)}. \end{aligned} \tag{A2}$$

The proposition claims that the optimal health state-contingent wealth policies are given by

$$A^*_{t+1}(1) = \frac{(\beta R)^{1/\gamma} \omega(1) C^*_t}{\omega(h_t)}, \tag{A3}$$

$$A^*_{t+1}(j) = \frac{(\beta R)^{1/\gamma} \omega(j) C^*_t}{\omega(h_t) c_{t+1}(j)} - \sum_{s=1}^{T-t} \frac{\mathbf{E}_{t+1}[Y_{t+s} - M_{t+s} | j]}{R^{s-1}} \text{ for } j \in \{2, 3\}. \tag{A4}$$

The proof proceeds by backward induction.

To simplify notation, we define total wealth as cash-on-hand plus the present value of future disposable income:

$$W_t = A_t + \sum_{s=0}^{T-t} \frac{\mathbf{E}_t[Y_{t+s} - M_{t+s} | h_t]}{R^s}. \tag{A5}$$

Because the household dies with certainty in period $T + 1$, optimal consumption in period T is $C^*_T = W_T$. Thus, the value function in period T is

$$V_T(h_T, A_T) = \omega(h_T)^{\gamma/(1-\gamma)} W_T. \tag{A6}$$

The first-order conditions in period $T-1$ are

$$\begin{aligned}\omega(h_{T-1})^{\gamma}C_{T-1}^{*-\gamma} =&\beta R\omega(1)^{\gamma}A_T^*(1)^{-\gamma}\\ =&\beta R\omega(j)^{\gamma}(A_T^*(j)+Y_T(j)-M_T(j))^{-\gamma} \text{ for } j\in\{2,3\}.\end{aligned} \tag{A7}$$

These equations, together with equation (A1), imply the policy functions (20), (A3), and (A4) for period $T-1$. Substituting the policy functions into the Bellman equation, the value function in period $T-1$ is

$$V_{T-1}(h_{T-1},A_{T-1})=\left(\frac{\omega(h_{T-1})}{c_{T-1}(h_{T-1})}\right)^{\gamma/(1-\gamma)}W_{T-1}. \tag{A8}$$

Suppose that the value function in period $t+1$ is

$$V_{t+1}(h_{t+1},A_{t+1})=\left(\frac{\omega(h_{t+1})}{c_{t+1}(h_{t+1})}\right)^{\gamma/(1-\gamma)}W_{t+1}. \tag{A9}$$

The first-order conditions in period t are

$$\begin{aligned}\omega(h_t)^{\gamma}C_t^{*-\gamma} =&\beta R\omega(1)^{\gamma}A_{t+1}^*(1)^{-\gamma}\\ =&\frac{\beta R\omega(j)^{\gamma}}{c_{t+1}(j)^{\gamma}}\left(A_{t+1}^*(j)+\sum_{s=1}^{T-t}\frac{\mathbf{E}_{t+1}[Y_{t+s}-M_{t+s}|j]}{R^{s-1}}\right)^{-\gamma} \text{ for } j\in\{2,3\}.\end{aligned} \tag{A10}$$

These equations, together with equation (A1), imply the policy functions (20), (A3), and (A4) for period t. Substituting the policy functions into the Bellman equation, the value function in period t is

$$V_t(h_t,A_t)=\left(\frac{\omega(h_t)}{c_t(h_t)}\right)^{\gamma/(1-\gamma)}W_t. \tag{A11}$$

B. *Proof of Proposition 2*

We substitute the intertemporal budget constraint (14) into the definitions of optimal health and mortality delta (i.e., equations (18) and (19)). We then use the definitions of health and mortality delta for each insurance product (i.e., equations (11) and (12)) to derive equations (24) and (25).

C. Proof of Proposition 3

To simplify notation, let $\pi_t^0(h_t, i) = \mathbf{1}_t(i)$. Iterating forward on the intertemporal budget constraint (A1),

$$\begin{aligned} A_t + Y_t - M_t - C_t = & \sum_{s=1}^{n-1} \sum_{i=2}^{3} \frac{\pi_t^s(h_t, i)}{R^s} (C_{t+s}(i) - Y_{t+s}(i) + M_{t+s}(i)) \\ & + \sum_{s=1}^{n} \sum_{i=2}^{3} \frac{\pi_t^{s-1}(h_t, i) \pi_{t+s-1}(i, 1)}{R^s} (\delta_{t+s-1}(i) + A_{t+s}(i)) \\ & + \sum_{i=2}^{3} \left[\frac{\pi_t^{n-1}(h_t, i) \pi_{t+n-1}(i, 2)}{R^n} (\Delta_{t+n-1}(i) + A_{t+n}(i)) \right. \\ & \left. + \frac{\pi_t^{n-1}(h_t, i) \pi_{t+n-1}(i, 3)}{R^n} A_{t+n}(i) \right]. \end{aligned} \tag{A12}$$

Iterating forward on the first-order conditions (A10),

$$\begin{aligned} \left(\frac{\omega(h_t)}{c_t(h_t)} \right)^{\gamma/(1-\gamma)} V_t^{*-\gamma} = & (\beta R)^n \omega(1)^\gamma (\delta_{t+n-1}^*(i) + A_{t+n}^*(i))^{-\gamma} \\ = & (\beta R)^n \left(\frac{\omega(2)}{c_{t+n}(2)} \right)^{\gamma/(1-\gamma)} V_{t+n}(2, \Delta_{t+n-1}^*(i) + A_{t+n}^*(i))^{-\gamma} \\ = & (\beta R)^n \left(\frac{\omega(3)}{c_{t+n}(3)} \right)^{\gamma/(1-\gamma)} V_{t+n}(3, A_{t+n}^*(i))^{-\gamma}. \end{aligned} \tag{A13}$$

We consider the following perturbations of health and mortality delta that satisfy the intertemporal budget constraint:

$$\pi_{t+n-1}(i, 2) \partial \Delta_{t+n-1}(i) + \partial A_{t+n}(i) = 0, \tag{A14}$$

$$\pi_{t+n-1}(i, 1) \partial \delta_{t+n-1}(i) + \partial A_{t+n}(i) = 0. \tag{A15}$$

We rewrite the value function under complete markets as

$$
\begin{aligned}
V_t(\Delta_{t+n-1}(i), \delta_{t+n-1}(i)) = \Bigg\{ & \omega(h_t)^\gamma C_t^{1-\gamma} + \sum_{s=1}^{n-1} \beta^s \sum_{i=2}^{3} \pi_t^s(h_t, i)\omega(i)^\gamma C_{t+s}(i)^{1-\gamma} \\
& + \sum_{s=1}^{n} \beta^s \sum_{i=2}^{3} \pi_t^{s-1}(h_t, i)\pi_{t+s-1}(i,1)\omega(1)^\gamma(\delta_{t+s-1}(i) + A_{t+s}(i))^{1-\gamma} \\
& + \beta^n \sum_{i=2}^{3} \left[\pi_t^{n-1}(h_t, i)\pi_{t+n-1}(i,2)V_{t+n}(2, \Delta_{t+n-1}(i) + A_{t+n}(i))^{1-\gamma}\right. \\
& \left. + \pi_t^{n-1}(h_t, i)\pi_{t+n-1}(i,3)V_{t+n}(3, A_{t+n}(i))^{1-\gamma}\right]\Bigg\}^{1/(1-\gamma)}. \qquad \text{(A16)}
\end{aligned}
$$

Taking the partial derivative of equation (A16) with respect to $\Delta_{t+n-1}(i)$,

$$
\begin{aligned}
& \frac{\partial V_t(\Delta_{t+n-1}(i), \delta_{t+n-1}(i))}{\partial \Delta_{t+n-1}(i)} = \beta^n \pi_t^{n-1}(h_t, i)\pi_{t+n-1}(i,2)V_t^\gamma \\
& \times \left[-\pi_{t+n-1}(i,1)\omega(1)^\gamma(\delta_{t+n-1}(i) + A_{t+n}(i))^{-\gamma}\right. \\
& + (1 - \pi_{t+n-1}(i,2))\left(\frac{\omega(2)}{c_{t+n}(2)}\right)^{\gamma/(1-\gamma)} V_{t+n}(2, \Delta_{t+n-1}(i) + A_{t+n}(i))^{-\gamma} \\
& \left. - \pi_{t+n-1}(i,3)\left(\frac{\omega(3)}{c_{t+n}(3)}\right)^{\gamma/(1-\gamma)} V_{t+n}(3, A_{t+n}(i))^{-\gamma}\right]. \qquad \text{(A17)}
\end{aligned}
$$

Evaluated at the optimal policy,

$$
\frac{\partial V_t(\Delta^*_{t+n-1}(i), \delta^*_{t+n-1}(i))}{\partial \Delta_{t+n-1}(i)} = 0. \qquad \text{(A18)}
$$

Similarly, the partial derivative of equation (A16) with respect to $\delta_{t+n-1}(i)$, evaluated at the optimal policy, is

$$
\frac{\partial V_t(\Delta^*_{t+n-1}(i), \delta^*_{t+n-1}(i))}{\partial \delta_{t+n-1}(i)} = 0. \qquad \text{(A19)}
$$

The partial derivative of equation (A17) with respect to $\Delta_{t+n-1}(i)$, evaluated at the optimal policy, is

$$\frac{\partial^2 V_t(\Delta^*_{t+n-1}(i),\delta^*_{t+n-1}(i))}{\partial \Delta_{t+n-1}(i)^2} = -\gamma\beta^n \pi_t^{n-1}(h_t,i)\pi_{t+n-1}(i,2)^2 V_t^{*\gamma}$$
$$\times \Big[\pi_{t+n-1}(i,1)\omega(1)^\gamma(\delta^*_{t+n-1}(i)+A^*_{t+n}(i))^{-1-\gamma}$$
$$+\frac{(1-\pi_{t+n-1}(i,2))^2}{\pi_{t+n-1}(i,2)}\left(\frac{\omega(2)}{c_{t+n}(2)}\right)^{2\gamma/(1-\gamma)} V_{t+n}(2,\Delta^*_{t+n-1}(i)+A^*_{t+n}(i))^{-1-\gamma}$$
$$+\pi_{t+n-1}(i,3)\left(\frac{\omega(3)}{c_{t+n}(3)}\right)^{2\gamma/(1-\gamma)} V_{t+n}(3,A^*_{t+n}(i))^{-1-\gamma}\Bigg]. \tag{A20}$$

Substituting the first-order conditions (A13),

$$\frac{\partial^2 V_t(\Delta^*_{t+n-1}(i),\delta^*_{t+n-1}(i))}{\partial \Delta_{t+n-1}(i)^2} = -\frac{\gamma\pi_t^{n-1}(h_t,i)\pi_{t+n-1}(i,2)^2}{\beta^{n/\gamma}R^{n(1+1/\gamma)}V_t^*}\left(\frac{\omega(h_t)}{c_t(h_t)}\right)^{(1+\gamma)/(1-\gamma)}$$
$$\times\left[\frac{\pi_{t+n-1}(i,1)}{\omega(1)}+\frac{(1-\pi_{t+n-1}(i,2))^2 c_{t+n}(2)}{\pi_{t+n-1}(i,2)\omega(2)}+\frac{\pi_{t+n-1}(i,3)c_{t+n}(3)}{\omega(3)}\right]. \tag{A21}$$

Similarly, the second partial derivative of the value function with respect to $\delta_{t+n-1}(i)$, evaluated at the optimal policy, is

$$\frac{\partial^2 V_t(\Delta^*_{t+n-1}(i),\delta^*_{t+n-1}(i))}{\partial \delta_{t+n-1}(i)^2} = -\frac{\gamma\pi_t^{n-1}(h_t,i)\pi_{t+n-1}(i,1)^2}{\beta^{n/\gamma}R^{n(1+1/\gamma)}V_t^*}\left(\frac{\omega(h_t)}{c_t(h_t)}\right)^{(1+\gamma)/(1-\gamma)}$$
$$\times\left[\frac{(1-\pi_{t+n-1}(i,1))^2}{\pi_{t+n-1}(i,1)\omega(1)}+\frac{\pi_{t+n-1}(i,2)c_{t+n}(2)}{\omega(2)}+\frac{\pi_{t+n-1}(i,3)c_{t+n}(3)}{\omega(3)}\right]. \tag{A22}$$

Finally, the partial derivative of equation (A17) with respect to $\delta_{t+n-1}(i)$, evaluated at the optimal policy, is

$$\frac{\partial^2 V_t(\Delta^*_{t+n-1}(i),\delta^*_{t+n-1}(i))}{\partial \Delta_{t+n-1}(i)\partial\delta_{t+n-1}(i)} = -\frac{\gamma\pi_t^{n-1}(h_t,i)\pi_{t+n-1}(i,1)\pi_{t+n-1}(i,2)}{\beta^{n/\gamma}R^{n(1+1/\gamma)}V_t^*}\left(\frac{\omega(h_t)}{c_t(h_t)}\right)^{(1+\gamma)/(1-\gamma)}$$
$$\times\left[-\frac{1-\pi_{t+n-1}(i,1)}{\omega(1)}-\frac{(1-\pi_{t+n-1}(i,2))c_{t+n}(2)}{\omega(2)}+\frac{\pi_{t+n-1}(i,3)c_{t+n}(3)}{\omega(3)}\right]. \tag{A23}$$

Substituting the value function (A11) and dividing by V_t^*,

$$\frac{\partial^2 L_t(n)}{\partial \Delta_{t+n-1}(i)^2} = -\frac{\gamma \pi_t^{n-1}(h_t, i)\pi_{t+n-1}(i,2)^2 \omega(h_t)}{\beta^{n/\gamma} R^{n(1+1/\gamma)} c_t(h_t) W_t^2}$$
$$\times \left[\frac{\pi_{t+n-1}(i,1)}{\omega(1)} + \frac{(1-\pi_{t+n-1}(i,2))^2 c_{t+n}(2)}{\pi_{t+n-1}(i,2)\omega(2)} + \frac{\pi_{t+n-1}(i,3) c_{t+n}(3)}{\omega(3)}\right]. \tag{A24}$$

$$\frac{\partial^2 L_t(n)}{\partial \delta_{t+n-1}(i)^2} = -\frac{\gamma \pi_t^{n-1}(h_t, i)\pi_{t+n-1}(i,1)^2 \omega(h_t)}{\beta^{n/\gamma} R^{n(1+1/\gamma)} c_t(h_t) W_t^2}$$
$$\times \left[\frac{(1-\pi_{t+n-1}(i,1))^2}{\pi_{t+n-1}(i,1)\omega(1)} + \frac{\pi_{t+n-1}(i,2) c_{t+n}(2)}{\omega(2)} + \frac{\pi_{t+n-1}(i,3) c_{t+n}(3)}{\omega(3)}\right]. \tag{A25}$$

$$\frac{\partial^2 L_t(n)}{\partial \Delta_{t+n-1}(i) \partial \delta_{t+n-1}(i)} = -\frac{\gamma \pi_t^{n-1}(h_t, i)\pi_{t+n-1}(i,1)\pi_{t+n-1}(i,2) \omega(h_t)}{\beta^{n/\gamma} R^{n(1+1/\gamma)} c_t(h_t) W_t^2}$$
$$\times \left[-\frac{1-\pi_{t+n-1}(i,1)}{\omega(1)} - \frac{(1-\pi_{t+n-1}(i,2)) c_{t+n}(2)}{\omega(2)} + \frac{\pi_{t+n-1}(i,3) c_{t+n}(3)}{\omega(3)}\right]. \tag{A26}$$

Appendix B. Health and Retirement Study

The Health and Retirement Study is a panel survey designed to study the health and wealth dynamics of the elderly in the United States. The data consist of five cohorts: the Study of Assets and Health Dynamics among the Oldest Old (born before 1924), the Children of Depression (born 1924 to 1930), the initial HRS cohort (born 1931 to 1941), the War Baby (born 1942 to 1947), and the Early Baby Boomer (born 1948 to 1953). Many of the variables that we use are from the RAND HRS (Version L), which is produced by the RAND Center for the Study of Aging with funding from the National Institute on Aging and the Social Security Administration. Whenever necessary, we use variables from both the core and exit interviews to supplement the RAND HRS. The data consist of ten waves, covering every two years between 1992 and 2010.

The Health and Retirement Study continues to interview respondents that enter nursing homes. However, any respondent that enters a nursing home receives a zero sampling weight because these weights are based on the non-institutionalized population of the Current Population Survey. Therefore, the use of sampling weights would lead us to underestimate nursing home expenses, which account for a large share of out-of-pocket health expenses for older households. Because nursing home expenses are important for this paper, we do not use sampling weights in any of our analysis.

Since the third wave, the survey asks bracketing questions to solicit a range of values for questions that initially receive a non-response. Based on the range of values implied by the bracketing questions, we use the following methodology to impute missing observations. For each missing observation, we calculate the minimum and maximum values that are implied by the responses to the bracketing questions. For each non-missing observation, we set the minimum and maximum values to be the valid response. We then estimate the mean and standard deviation of the variable in question through interval regression, under the assumption of log-normality. Finally, we fill in each missing observation as the conditional mean of the distribution in the bracketed range.

A. *Out-of-Pocket Health Expenses*

Out-of-pocket health expenses from the RAND HRS consist of the total amount paid for hospitals, nursing homes, doctor visits, dentist visits, outpatient surgery, prescription drugs, home health care, and special facilities. We measure out-of-pocket health expenses at the household level as the sum of these expenses for both the male respondent and his spouse (if married).

Since the third wave, out-of-pocket health expenses at the end of life are available through the exit interviews. Without end-of-life expenses, we would underestimate the true cost of poor health in old age, especially in the upper tail of the distribution (Marshall et al., 2011). Out-of-pocket health expenses from the exit interviews consist of the total amount paid for hospitals, nursing homes, doctor visits, prescription drugs, home health care, other health services, other medical expenses, and other non-medical expenses. For the last core interview prior to death of the primary respondent, we add out-of-pocket health expenses at the end of life from the exit interviews.

We estimate the life-cycle profile of out-of-pocket health expenses, on the subsample of households without long-term care insurance, through a panel regression with household fixed effects. We model the logarithm of real out-of-pocket health expenses as a function of dummies for 65 or older and poor health, a quadratic polynomial in age, log income, and the interaction of the dummies with the age polynomial and log income. The dummy for 65 or older accounts for potential changes in household behavior that arise from eligibility for Social Security and Medicare. We use the estimated regression model to predict out-of-pocket health expenses in the absence of long-term care insurance by age, health, and birth cohort.

B. *Income*

Our measure of income includes labor income, Social Security disability and supplemental security income, Social Security retirement income, and unemployment or workers compensation. It excludes pension and annuity income as well as capital income. We calculate after-tax income by subtracting the federal income tax, estimated through the NBER TAXSIM program (Version 9). Household income is the sum of income for both the male respondent and his spouse (if married).

We estimate the life-cycle profile of income through a panel regression with household fixed effects. We model the logarithm of real after-tax income as a function of a dummy for 65 or older, a quadratic polynomial in age, and the interaction of the dummy with the age polynomial. We use the estimated regression model to predict income by age and birth cohort.

C. Life Insurance

The ownership and the face value of life insurance are from the core interviews. Term life insurance refers to individual and group policies that have only a death benefit. Whole life insurance refers to policies that build cash value, from which the policyholder can borrow or receive cash upon surrender. In the first to third waves, the face value of all policies is the sum of the face value of term and whole life insurance. In the fourth wave, only the face value of all policies, and not the breakdown between term and whole life insurance, is available. In fifth to tenth waves, the face value of term life insurance is the difference between the face values of all policies and whole life insurance.

We estimate the life-cycle profile of the face value of life insurance through a panel regression with household fixed effects. We model the logarithm of the real face value of life insurance as a function of dummies for 65 or older and poor health, a quadratic polynomial in age, log income, and the interaction of the dummies with the age polynomial and log income. We use the estimated regression model to predict the face value of life insurance by age, health, and household fixed effect.

D. Annuities Including Private Pensions

We define the ownership of annuities including private pensions as either participation in a defined-benefit plan at the present employer or positive pension and annuity income.

We estimate the life-cycle profile of pension and annuity income through a panel regression with household fixed effects. We model the logarithm of real pension and annuity income as a function of dummies for 65 or older and poor health, a quadratic polynomial in age, log income, and the interaction of the dummies with the age polynomial and log income. We use the estimated regression model to predict pension and annuity income by age, health, and household fixed effect.

E. Financial and Housing Wealth

Household assets include checking, savings, and money market accounts; CD, government savings bonds, and T-bills; bonds and bond funds; IRA and Keogh accounts; businesses; stocks, mutual funds, and investment trusts; and primary and secondary residence. Household liabilities include all mortgages for primary and secondary residence, other home loans for primary residence, and other debt. Financial and housing wealth is total assets minus total liabilities.

We estimate the life-cycle profile of financial and housing wealth through a panel regression with household fixed effects. We model the logarithm of real financial and housing wealth as a function of dummies for 65 or older and poor health, a quadratic polynomial in age, log income, and the interaction of the dummies with the age polynomial and log income. We use the estimated regression model to predict financial and housing wealth by age, health, and household fixed effect.

Appendix C. Welfare Cost of Household Insurance Choice under Alternative Assumptions

In Table C.1, we assume that life insurance, annuities, and long-term care insurance are more expensive than actuarially fair by calibrating their discount rates to be 0%, while the riskless interest rate is 2%.

In Table C.2, we assume that term life insurance matures at age 65 for households younger than 65. For households older than 65, we continue to assume that term life insurance matures in two years. This alternative assumption lengthens the maturity of term life insurance relative to the baseline calibration in which it always matures in two years.

In Table C.3, we redefine poor health to be a state in which out-of-pocket health expenses are higher than the 75th percentile conditional on age, birth cohort, and the ownership of long-term care insurance. The criterion for the predicted mortality rate remains at higher than the median conditional on age and birth cohort. This alternative definition makes the out-of-pocket health expenses in Panel C of Table III more extreme. Out-of-pocket health expenses for households in poor health are \$4.2k per year at age 51 and \$38.5k per year at age 93.

In each of these cases, we reestimate household preferences and the welfare cost of household insurance choice, following the methodology described in Section IV. In Tables C.1 and C.2, we find that the results are nearly identical to the benchmark case in Table VII. In Table C.3, we find that the lifetime welfare cost is slightly higher under the alternative definition of poor health, which implies that our benchmark results are conservative.

Table C.1: Welfare Cost of Household Insurance Choice under Actuarially Unfair Insurance
This table reports the welfare cost of household insurance choice under an alternative assumption that insurance is more expensive than actuarially fair. The discount rates on life insurance, annuities, and long-term care insurance are calibrated to 0% annually, while the riskless interest rate is 2%. The sample consists of males aged 51 and older in the Health and Retirement Study from 1992 to 2010.

	Age					
	51–57	58–64	65–71	72–78	79–85	86–
Panel A: Per-period welfare cost (median in % of total wealth)						
Total cost	0.03	0.03	0.03	0.02	0.02	0.02
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.15)
Cost due to health delta	0.01	0.01	0.02	0.01	0.01	0.01
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.25)
Cost due to mortality delta	0.01	0.01	0.01	0.01	0.00	0.01
	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.16)
Panel B: Lifetime welfare cost (median in % of total wealth)						
Total cost	3.59	3.24	3.03	1.76	0.79	0.40
	(0.27)	(0.20)	(0.22)	(0.23)	(0.27)	(0.48)
Cost due to health delta	0.53	0.45	0.45	0.25	0.14	0.16
	(0.06)	(0.04)	(0.03)	(0.04)	(0.08)	(0.38)
Cost due to mortality delta	3.99	3.59	3.40	1.92	0.80	0.35
	(0.30)	(0.22)	(0.24)	(0.25)	(0.29)	(0.50)

Table C.2: Welfare Cost of Household Insurance Choice under Term Life Insurance with Longer Maturity

This table reports the welfare cost of household insurance choice under an alternative assumption that term life insurance matures at age 65 for households younger than 65. For households older than 65, term life insurance matures in two years. The sample consists of males aged 51 and older in the Health and Retirement Study from 1992 to 2010.

	Age					
	51–57	58–64	65–71	72–78	79–85	86–
Panel A: Per-period welfare cost (median in % of total wealth)						
Total cost	0.03	0.03	0.02	0.02	0.01	0.02
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.15)
Cost due to health delta	0.01	0.02	0.01	0.01	0.01	0.01
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.25)
Cost due to mortality delta	0.01	0.01	0.01	0.01	0.00	0.01
	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.18)
Panel B: Lifetime welfare cost (median in % of total wealth)						
Total cost	3.22	2.87	2.74	1.60	0.71	0.39
	(0.27)	(0.19)	(0.22)	(0.23)	(0.27)	(0.50)
Cost due to health delta	0.43	0.34	0.34	0.20	0.13	0.16
	(0.07)	(0.04)	(0.04)	(0.06)	(0.09)	(0.40)
Cost due to mortality delta	3.53	3.14	3.03	1.71	0.73	0.33
	(0.30)	(0.21)	(0.24)	(0.24)	(0.28)	(0.51)

Table C.3: Welfare Cost of Household Insurance Choice under an Alternative Definition of Poor Health

This table reports the welfare cost of household insurance choice under an alternative assumption that poor health is a state in which out-of-pocket health expenses are higher than the 75th percentile conditional on age, birth cohort, and the ownership of long-term care insurance. The criterion for the predicted mortality rate remains at higher than the median conditional on age and birth cohort. The sample consists of males aged 51 and older in the Health and Retirement Study from 1992 to 2010.

	Age					
	51–57	58–64	65–71	72–78	79–85	86–
Panel A: Per-period welfare cost (median in % of total wealth)						
Total cost	0.03	0.03	0.03	0.03	0.02	0.06
	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.37)
Cost due to health delta	0.01	0.01	0.02	0.01	0.01	0.03
	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.38)
Cost due to mortality delta	0.01	0.01	0.01	0.01	0.01	0.01
	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.31)
Panel B: Lifetime welfare cost (median in % of total wealth)						
Total cost	4.11	3.84	3.68	2.29	1.11	0.83
	(0.30)	(0.23)	(0.26)	(0.29)	(0.34)	(0.67)
Cost due to health delta	0.44	0.41	0.47	0.33	0.25	0.39
	(0.05)	(0.04)	(0.04)	(0.07)	(0.14)	(0.59)
Cost due to mortality delta	4.03	3.67	3.45	2.03	0.84	0.31
	(0.31)	(0.23)	(0.26)	(0.27)	(0.31)	(0.56)

Appendix D. Transition Probabilities for Insurance Ownership

In Table D.1, we use a probit model to predict the ownership of a given policy at two years from the present interview. The key explanatory variable is whether the household is a present policyholder. Households aged 51 that are present policyholders of term life insurance are 72 percentage points more likely to be a policyholder at the next interview. Similarly, households aged 51 that are present policyholders of whole life insurance are 66 percentage points more likely to be a policyholder at the next interview. Households aged 51 that are present policyholders of annuities including private pensions are 93 percentage points more likely to be a policyholder at the next interview. Finally, households aged 51 that are present policyholders of long-term care insurance are 20 percentage points more likely to be a policyholder at the next interview.

Based on the predicted probabilities from the probit model, we estimate the joint transition matrix for health and insurance ownership. For each household, we then calculate the most likely path of future insurance ownership conditional on realized health. Finally, we calculate the path of future health and mortality delta implied by the path of future insurance ownership (i.e., $\{\Delta_{t+s-1}(i), \delta_{t+s-1}(i)\}_{s=2}^{T-t}$ in Proposition 3).

Table D.1: Predicting Future Insurance Ownership

A probit model is used to predict the ownership of a given policy at two years from the present interview. This table reports the marginal effects on the probability of insurance ownership (in percentage points) with heteroskedasticity-robust t-statistics in parentheses. The omitted cohort is those born prior to 1911. The sample consists of males aged 51 and older in the Health and Retirement Study from 1992 to 2010.

Explanatory variable	Term life insurance		Whole life insurance		Annuities including private pensions		Long-term care insurance	
Policyholder	71.72	(10.34)	66.22	(7.50)	92.75	(49.47)	19.88	(1.99)
65 or older	18.42	(1.42)	-16.82	(-1.31)	15.19	(1.36)	18.81	(2.62)
Poor health	20.74	(2.00)	-12.83	(-1.28)	-13.53	(-1.41)	9.27	(1.43)
(Age – 51)/10	2.75	(0.41)	-5.27	(-0.84)	-15.77	(-3.33)	-0.25	(-0.10)
× Policyholder	6.46	(2.16)	-0.60	(-0.21)	0.39	(0.16)	12.85	(8.21)
× 65 or older	4.13	(0.46)	26.29	(2.96)	13.08	(1.78)	-1.44	(-0.36)
× Poor health	-0.86	(-0.28)	4.34	(1.42)	-0.20	(-0.08)	-1.20	(-0.88)
$(\text{Age} - 51)^2/100$	-4.91	(-1.11)	-1.71	(-0.40)	9.26	(2.97)	1.91	(1.11)
× Policyholder	-1.46	(-1.80)	-0.45	(-0.55)	1.04	(1.56)	-1.71	(-4.02)
× 65 or older	3.84	(0.84)	-3.85	(-0.87)	-10.65	(-3.20)	-1.67	(-0.90)
× Poor health	-0.17	(-0.20)	-1.16	(-1.33)	-0.13	(-0.18)	0.34	(0.83)
Log income	5.91	(6.01)	0.98	(1.22)	3.07	(4.29)	1.32	(3.80)
× Policyholder	-3.26	(-3.40)	0.17	(0.19)	-7.35	(-9.50)	0.35	(0.81)
× 65 or older	-2.69	(-2.90)	-1.03	(-1.17)	-1.18	(-1.49)	-1.21	(-2.88)
× Poor health	-1.85	(-1.93)	0.95	(1.05)	1.17	(1.45)	-0.67	(-1.56)
Birth cohort:								
1911–1915	1.75	(0.39)	-11.10	(-3.33)	0.65	(0.18)	9.60	(2.06)
1916–1920	10.64	(2.71)	-15.02	(-5.25)	-2.87	(-0.79)	8.67	(1.93)
1921–1925	11.88	(3.00)	-16.16	(-5.57)	-5.48	(-1.47)	15.84	(2.98)
1926–1930	13.82	(3.42)	-19.64	(-7.16)	-7.04	(-1.82)	17.68	(3.24)
1931–1935	16.50	(4.04)	-21.15	(-7.09)	-11.72	(-3.00)	17.87	(3.48)
1936–1940	18.22	(4.35)	-25.79	(-8.56)	-16.06	(-4.11)	17.79	(3.65)
1941–1945	20.92	(5.54)	-24.94	(-10.59)	-17.32	(-4.41)	20.87	(3.58)
1946–1950	26.19	(8.18)	-25.81	(-13.22)	-21.48	(-5.57)	24.54	(3.78)
1951–1955	22.85	(6.76)	-25.00	(-16.51)	-26.65	(-7.11)	27.05	(3.84)
Correctly predicted (%):								
Both outcomes	77		85		80		91	
Policyholder only	80		77		81		69	
Non-policyholder only	71		89		78		94	
Observations	18,184		18,432		35,351		34,769	