2. Mind Map: Kinetic energy & Forces in Rolling

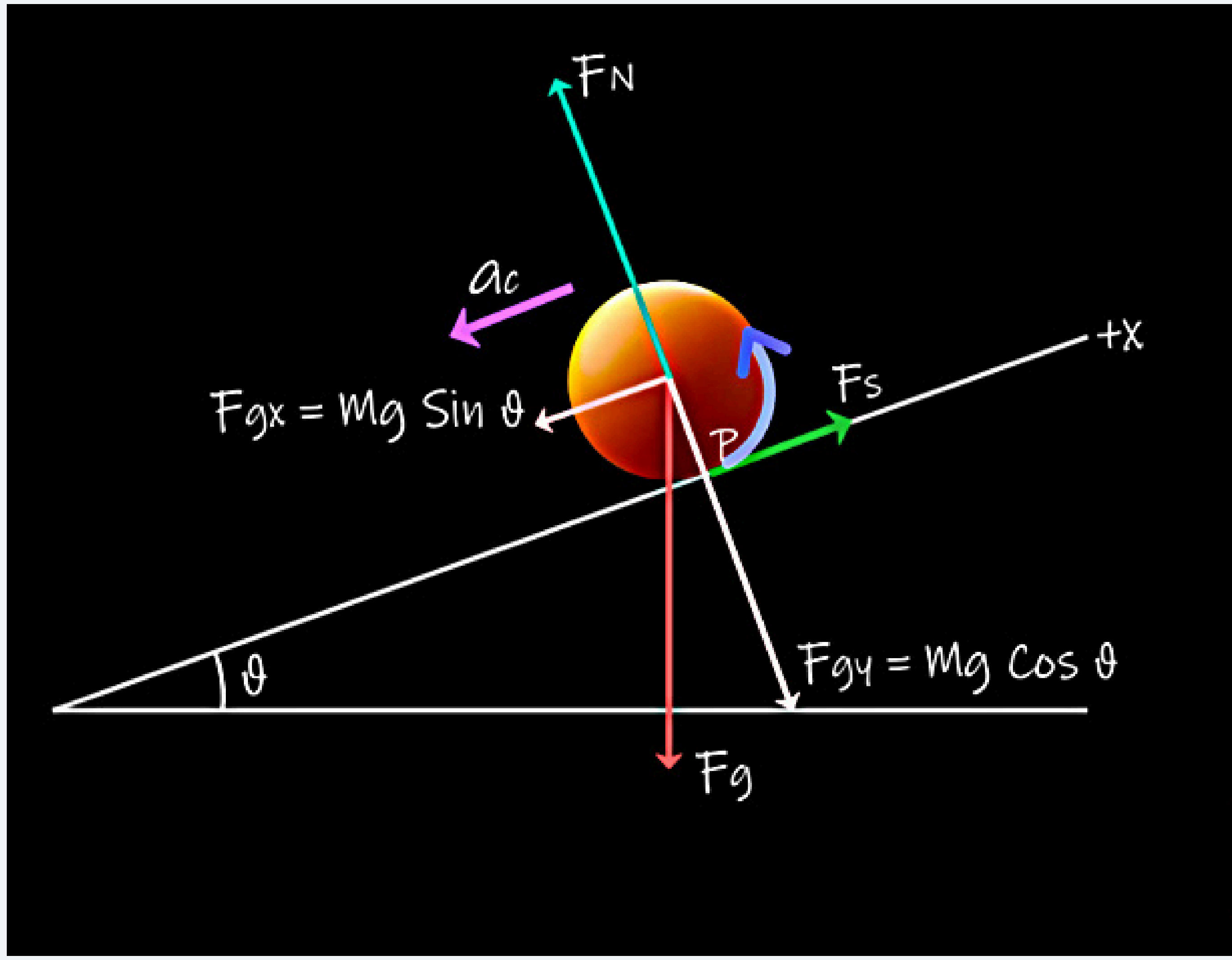
Energy & Forces

- **Kinetic Energies**
  - Rotational Kinetic Energy — Formula: $\frac{1}{2} I \omega^2$
  - Translational Kinetic Energy — Formula: $\frac{1}{2} M v^2$
  - Total Kinetic Energy — $K = \frac{1}{2} M v^2 + \frac{1}{2} I \omega^2$
- **Energy Conservation (example)** — $Mgh = \frac{1}{2} M v^2 + \frac{1}{2} I \omega^2$
  - *(A sphere rolling down an incline from a an initial height h)*
- **Friction and Rolling**
  - Condition for rolling without slipping
    - $V_{com} = \omega R$
    - $a_{com} = R\alpha$
  - Accelerating Wheel
    - Static force of friction kicks in to oppose slip - Motion continues to be rolling without slipping
    - If the force accelerating the wheel creates slip at the wheel that exceeds max. static force of friction - Slippage occurs. Kinetic force of friction kicks in
- **Rolling Sphere**
  - Equations
    - Linear motion: $f_s - Mg \sin \theta = Ma_{com}$
    - Rotational motion: $Rf_s = I_{com}\alpha$
    - Relation: $a_{com} = R\alpha$
  - Linear Acceleration — $a_{com} = -g \sin \theta / (1 + I_{com} / MR^2)$
  - Key Points
    - Gravity drives motion
    - Friction causes rotation

If a rolling object transitions into sliding, some rotational energy converts into heat due to friction. The modified equation would be -
$Mgh = \frac{1}{2} M v^2 + \frac{1}{2} I \omega^2 + E_{thermal}$

Role of Static Friction: *Static friction is crucial for initiating rolling motion and ensuring no slipping occurs. It acts at the point of contact to oppose relative motion between the surface and the rolling object.*
Limiting Static Friction: The force of static friction cannot exceed max F_s or $N\mu_s$. If this limit is reached, slipping occurs.

2. Mind Map: Analysis of a Rolling Sphere



$$F_s - Mg\sin\theta = Ma_c$$

$$\tau = rF_s$$

$$\tau = I_c\,\alpha$$

$$RF_s = I_c\,\alpha$$

$$a_c = R\alpha$$

$$-a_c / R = \alpha$$

$$F_s = -I_c\,(a_c / R^2)$$

$$\boxed{a_c = -g\sin\theta / (1 + I_c / MR^2)}$$

Consider a solid sphere and a hollow sphere, both with the same mass (2 kg) and radius (0.2 m), rolling down a ramp inclined at 30 degrees. Compare their linear accelerations and energies.

Moment of Inertia:

- *Solid sphere:* $I = (2/5) \times m \times R^2$.
- *Hollow sphere:* $I = (2/3) \times m \times R^2$.

Linear Acceleration:

$a = (g \times \sin(\theta)) / (1 + I_c / m R^2)$

1. *For the solid sphere:*

   $a = (g \times \sin(\theta)) / (1 + 2/5) = 3.5\ m/s^2$

2. *For the hollow sphere:*

   $a = (3 \times 9.8 \times 0.5) / 5 = 2.94\ m/s^2$

Key Observations:

- *The solid sphere accelerates faster than the hollow sphere because it has a smaller moment of inertia. This means more energy is converted into translational motion than rotational*
- *The hollow sphere accelerates more slowly because a larger proportion of its energy is used for rotational motion.*

Kinetic Energy Distribution at the bottom of the ramp:

- *For the solid sphere, a greater proportion of its total kinetic energy is in translational motion.*
- *For the hollow sphere, a greater proportion of its total kinetic energy is in rotational motion.*