

習題集 11

(對應 [張旭微積分](#) 極限篇重點十一：夾擠定理)

1. Evaluate $\lim_{x \rightarrow 0} x \cos\left(\frac{1}{x}\right)$ and $\lim_{x \rightarrow \infty} \frac{\sin(\cos(x))}{x}$.
2. Evaluate $\lim_{x \rightarrow 0} \frac{3^x}{3^x - x \cos\left(\frac{3}{x}\right)}$ and $\lim_{x \rightarrow \infty} \frac{3^x + \sin(2x)}{3^x - \cos(3x)}$.
3. Evaluate $\lim_{x \rightarrow \infty} \frac{\frac{1}{\sqrt{x}}}{\cos^2(222x) + \cos(222x) + 1}$.
4. Evaluate $\lim_{x \rightarrow \infty} \frac{x^n}{x^n + x^{n-1} \cos[(n-1)x] + x^{n-2} \cos[(n-2)x] + 1}$.
5. Let $f(x) = \begin{cases} x^2 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases}$. Find $\lim_{x \rightarrow 0} f(x)$.
6. Let $f(x) = \begin{cases} 3x-2 & \text{if } x \in \mathbb{Q} \\ x+3 & \text{if } x \notin \mathbb{Q} \end{cases}$. Find $\lim_{x \rightarrow \frac{5}{2}} f(x)$.
7. Evaluate $\lim_{x \rightarrow 0^+} x \left\lfloor \frac{1}{x} \right\rfloor$.
8. Evaluate $\lim_{x \rightarrow \infty} \frac{1}{x} \left\lfloor \frac{x}{5} \right\rfloor$.
9. Evaluate $\lim_{x \rightarrow \infty} \frac{\sqrt{x}(x - [x])}{3^{\sqrt{x}}}$.
10. Evaluate $\lim_{n \rightarrow \infty} \frac{n! \cdot \log(n!)}{(n!)! 4n^3}$.