Solutions

Linear Equations

Inequations

Simultaneous Equations

Rearranging Formulas

Linear Equations

Q1) a) $3x = 20 - x$

$4x = 20$

$x = 5$

b) $t + 3 = 5 - t$

$2t = 2$

$t = 1$

c) $3 + 5s = 2s + 13$

$3s = 10$

$s = \frac{10}{3}$

d) $5a - 4 = 3a + 6$

$2a = 10$

$a = 5$

e) $3m + 8 = -2m$

$5m = -8$

$m = -\frac{8}{5}$

f) $6y - 11 = 2y + 5$

$4y = 16$

$y = 4$

g) $2b + 7 = 11 - 3b$

$5b = 4$

$b = \frac{4}{5}$

h) $5x - 7 = 3x$

$2x = 7$

$x = \frac{7}{2}$

i) $x = 3x - 2 + 7$

$-5 = 2x$

$-\frac{5}{2} = x$

j) $4a = 3 - 2a - 23$

$6a = -20$

$a = -\frac{20}{6}$

$a = -\frac{10}{3}$

Q2 a) $2(4t+5)=34$

$8t+10=34$

$8t=24$

$t=3$

b) $2(x+3)-5=9$

$2x+6-5=9$

$2x=8$

$x=4$

c) $3r-7(1+r)=12$

$3r-7-7r=12$

$-4r=19$

$r=-\frac{19}{4}$

d) $z(z+2)=z^2+6$

$z^2+2z=z^2+6$

$2z=6$

$z=3$

e) $(x+1)(x-2)=(x+3)^2$

$x^2-x-2=x^2+6x+9$

$-7x=11$

$x=-\frac{11}{7}$

f) $2(x+3)=-2(x+4)$

$2x+6=-2x-8$

$4x=-14$

$x=-\frac{14}{4}$

$x=-\frac{7}{2}$

Q3 a) $\frac{x+1}{4}=5$

$x+1=20$

$x=19$

b) $\frac{x}{2}+\frac{x}{4}=1$

Multiply by 4

$2x+x=4$

$3x=4$

$x=\frac{4}{3}$

c) $\frac{a-1}{2}=\frac{a+1}{4}$

Multiply by 4

$2(a-1)=a+1$

$2a-2=a+1$

$a=3$

d) $\frac{x+1}{2}+\frac{x-1}{3}=4$

$3(x+1)+2(x-1)=4$

$3x+3+2x-2=4$

$5x=3$

$x=\frac{3}{5}$

e) $\frac{x+2}{2}+\frac{x-1}{5}=\frac{1}{20}$

Multiply by 20

$10(x+2)+4(x-1)=1$

$10x+20+4x-4=1$

$14x=-15$

$x=-\frac{15}{14}$

f) $\frac{2}{x}+\frac{1}{3}=5$

Multiply by $3x$

$6+x=15x$

$6=14x$

$x=\frac{14}{6}$

$x=\frac{7}{3}$

Inequations

Q1 a) $3n>9$

$n>3$

b) $t+2<-1$

$t<-3$

c) $b-3\geq -2$

$b\geq 1$

d) $7k>3k-16$

$4k>-16$

$k>-4$

e) $6m-7\leq m$

$5m\leq 7$

$m\leq\frac{7}{5}$

f) $8+2x>3(4-x)$

$8+2x>12-3x$

$5x>4$

$x>\frac{4}{5}$

g) $11-2(4+3x)<39$

$11-8-6x<39$

$-6x<36$

$x>-6$

h) $19+x>15+3(x-2)$

$19+x>15+3x-6$

$-2x>-10$

$x<5$

Simultaneous Equations

Q1) a) $3x - y = 1$ (1)

$x + y = 1$ (2)

(1) + (2) gives

$4x = 2$

$x = \frac{1}{2}$

Substituting $x = \frac{1}{2}$ into (2) gives

$\frac{1}{2} - y = 1$

$y = -\frac{1}{2}$

b) $2x + y = 7$ (1)

$x + y = 4$ (2)

(1) – (2) gives

$x = 3$

Substituting $x = 3$ into (2) gives

$3 + y = 4$

$y = 1$

c) $5x - 2y = 13$ (1)

$3x + 2y = 3$ (2)

(1) + (2) gives

$8x = 16$

$x = 2$

Substituting $x = 2$ into (2) gives

$(3 \times 2) + 2y = 3$

$6 + 2y = 3$

$2y = -3$

$y = -\frac{3}{2}$

d) $2x - 2y = 9$ (1)

$4x - 2y = 16$ (2)

(2) - (1) gives

$2x = 7$

$x = \frac{7}{2}$

Substituting $x = \frac{7}{2}$ into (1) gives

$\left(2\ x\ \frac{7}{2}\right) - 2y = 9$

$7 - 2y = 9$

$-2y = 2$

$y = -1$

Q2) a) $x + 3y = 10$ (1)

$2x + 5y = 18$ (2)

Multiply (1) by 2 to give

$2x + 6y = 20$ (3)

(3) – (2) gives

$y = 2$

Substituting $y = 2$ into (1) gives

$x + (3\ x\ 2) = 10$

$x + 6 = 10$

$x = 4$

b) $2x + y = 10$ (1)

$-x + 2y = 9$ (2)

Multiply (2) by 2 to give

$-2x + 4y = 18$ (3)

(1) + (3) gives

$5y = 28$

$y = 5.6$

Substituting $y = 5.6$ into (1) gives

$2x + 5.6 = 10$

$2x = 4.4$

$x = 2.2$

c) $5x - 4y = 24$ (1)

$2x = y + 9$ (2)

Rearranging (2) gives

$2x - y = 24$ (2)

Multiply (2) by 4 to give

$8x - 4y = 96$ (3)

(3) – (1) gives

$3x = 72$

$x = 24$

Substituting $x = 24$ into (2) gives

$(2\ x\ 24) - y = 24$

$48 - y = 24$

$y = 24$

d) $-3x + 2y = 5$ (1)

$4x + 3y = -1$ (2)

Multiplying (1) by 3 and (2) by 2 gives

$-9x + 6y = 15$ (3)

$8x + 6y = -2$ (4)

(3) – (4) gives

$-17x = 17$

$x = -1$

Substituting $x = -1$ into (1) gives

$3 + 2y = 5$

$y = 1$

Q3) a) $4x + y = 9$

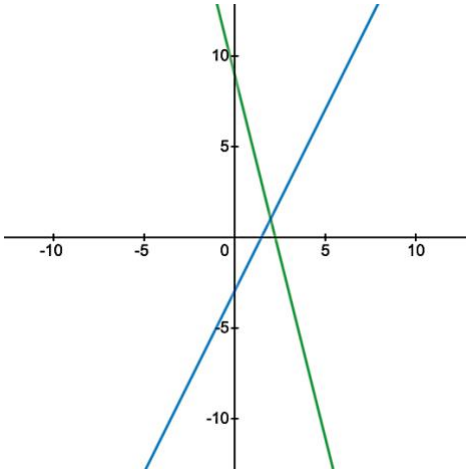
Rearrange

$y = -4x + 9$

$2x - y = 3$

Rearrange

$y = 2x - 3$



Estimated solutions are $x = 2, y = 1$

b) $2x + 3y = 8$

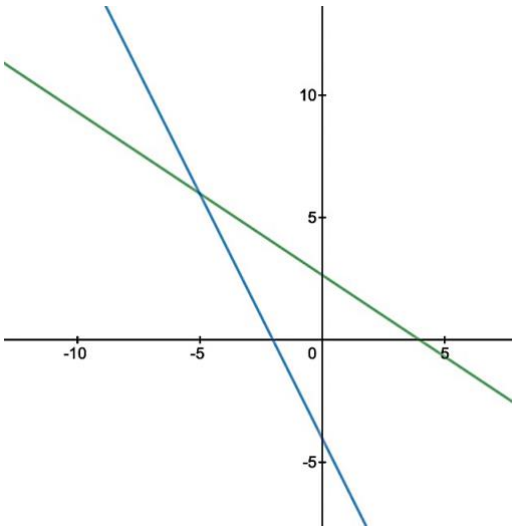
Rearrange

$y = -\frac{2}{3}x + \frac{8}{3}$

$2x + y = -4$

Rearrange

$y = -2x - 4$



Estimated solutions are $x = -5, y = 6$

c) $x - 3y = 8$

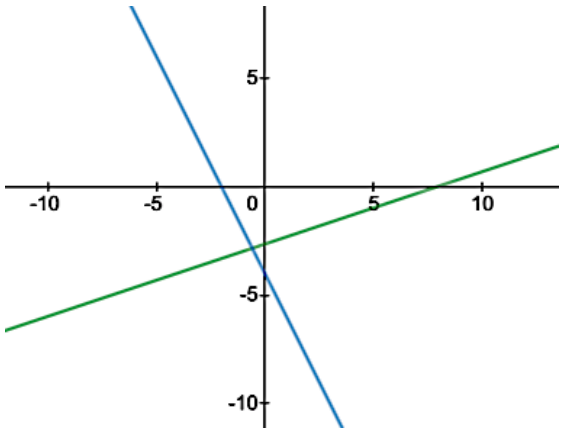
Rearrange

$y = \frac{1}{3}x - \frac{8}{3}$

$2x + y = -4$

Rearrange

$y = -2x - 4$



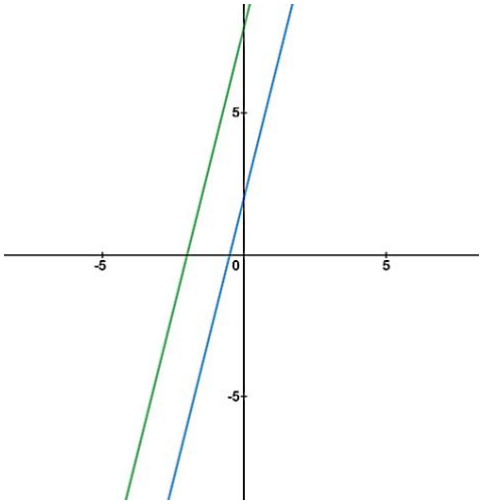
Estimated solutions are $x = -1, y = -3$

d) $y - 4x = 8$

Rearrange

$y = 4x + 8$

$y = 4x + 2$



There are no solutions since the lines are parallel and therefore never intersect.

Note that lines are parallel if they have the same gradient.

Q4) a) Let t be the price of a cup of tea

Let c be the price of a cup of coffee

Forming equations to represent David and Jenny's purchases gives

$2c + 3t = 9.75$ (1)

$c + 4t = 7.75$ (2)

Multiply (2) by 2 to give

$2c + 8t = 15.50$ (3)

(3) – (1) gives

$5t = 5.75$

$t = 1.15$

Substituting $t = 1.15$ into (2) gives

$c + (4\ x\ 1.15) = 7.75$

$c + 4.60 = 7.75$

$c = 3.15$

So, a cup of coffee costs £3.15 and a cup of tea costs £1.15

b) Let p be the cost of a pen

Let c be the cost of a pencil

Forming equations to represent the total costs gives

$9p + 5c = 3.2$ (1)

$7p + 8c = 2.9$ (2)

Multiply (1) by 8 to and (2) by 5 to give

$72p + 40c = 25.6$ (3)

$35p + 40c = 14.5$ (4)

(3) – (4) gives

$37p = 11.1$

$p = 30$

When substituting be careful not to confuse pounds and pence. I'll choose to change £3.20 into 320 pence.

Substituting $p = 30$ into (1) gives

$(9\ x\ 30) + 5c = 320$

$270 + 5c = 320$

$5c = 50$

$c = 10$

So, a pen costs 30p and a pencil costs 10p

c) Let t be the cost of a table and let c be the cost of a chair

Forming two equations to represent the total costs gives

$2t + 3c = 2{,}000$ (1)

$3t + 2c = 2{,}500$ (2)

Multiply (1) by 3 and (2) by 2 to give

$6t + 9c = 6{,}000$ (3)

$6t + 4c = 5{,}000$ (4)

(3) - (4) gives

$5c = 1{,}000$

$c = 200$

Substituting $c = 200$ into (1) gives

$2t + (3\ x\ 200) = 2{,}000$

$2t + 600 = 2{,}000$

$2t = 1{,}400$

$t = 700$

So, a chair costs £200 and a table costs £700

Rearranging Formulas

Q1 a) $s = t + 4$

$t + 4 = s$

$t = s - 4$

b) $s = t - 2$

$t - 2 = s$

$t = s + 2$

c) $s = 3 - t$

$t + s = 3$

$t = 3 - s$

d) $a = 5t$

$5t = a$

$t = \frac{a}{5}$

e) $a = \frac{t}{5}$

$\frac{t}{5} = a$

$t = 5a$

f) $s = \frac{3t}{5}$

$\frac{3t}{5} = s$

$3t = 5s$

$t = \frac{5s}{3}$

Q2 a) $3a - x = a + 2x$

$3a - a = 2x + x$

$2a = 3x$

$a = \frac{3x}{2}$

b) $a + 2 = x(3 + a)$

$a + 2 = 3x + ax$

$a - ax = 3x - 2$

$a(1 - x) = 3x - 2$

$a = \frac{3x - 2}{1 - x}$

c) $z = \frac{a - 3}{5 - a}$

$z(5 - a) = a - 3$

$5z - az = a - 3$

$5z + 3 = a(1 + z)$

$a(1 + z) = 5z + 3$

$a = \frac{5z + 3}{1 + z}$

d) $x(a - 1) = b(a + 2)$

$xa - x = ba + 2b$

$xa - ba = 2b + x$

$a(x - b) = 2b + x$

$a = \frac{2b + x}{x - b}$

Q3 a) $r = t^2$

$t^2 = r$

$t = \sqrt{r}$

b) $r = \sqrt{t}$

$\sqrt{t} = r$

$t = r^2$

c) $r = \frac{\sqrt{t}}{5}$

$\frac{\sqrt{t}}{5} = r$

$\sqrt{t} = 5r$

$t = (5r)^2$

$t = 25r^2$

d) $3t^2 + r = s$

$3t^2 = s - r$

$t^2 = \frac{s-r}{3}$

$t = \sqrt{\frac{s-r}{3}}$

e) $\sqrt{t+3} = s$

$t + 3 = s^2$

$t = s^2 - 3$

f) $\frac{1}{2}\sqrt{2t-4} = s$

$\sqrt{2t-4} = 2s$

$2t - 4 = (2s)^2$

$2t - 4 = 4s^2$

$2t = 4s^2 + 4$

$t = 2s^2 + 2$