CUBE NOTES

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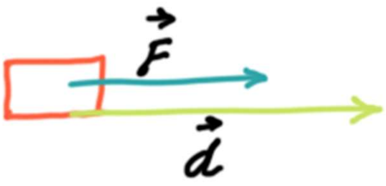
Vectors: Vector Dot Product

When do we use vector dot product

- Take example of a force acting parallel to displacement. Work done is the product of force and displacement. (W = Fd)
- However, the work done by a force acting at an angle is less in magnitude. This is because part of the force F acts vertically, and part horizontally. *Only the horizontal part does work.*
- Using Dot Product: The dot product of vectors is a powerful tool to find solutions in such situations.



Work done = simple product of Force and displacement

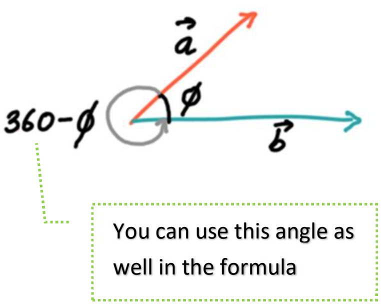
Types of Vector Multiplication

- *Scalar with Vector:* Multiplying a vector with a scalar changes its length but not its direction. If the scalar is negative, the direction also changes (it becomes reverse)
- *Vector with Vector*: Two types of multiplication –
  - *Dot Product*: Yields a scalar value.
  - *Cross Product*: Yields a vector value (covered in next lesson)

Dot Product or Scalar Product

$$\vec{a} \cdot \vec{b} = ab \cos \phi$$

Here *a* and *b* are magnitudes of vectors, and ϕ is the angle betv



Commutative property: $a \cdot b = b \cdot a$

Maximum Value of dot product: When vectors are parallel or antiparallel

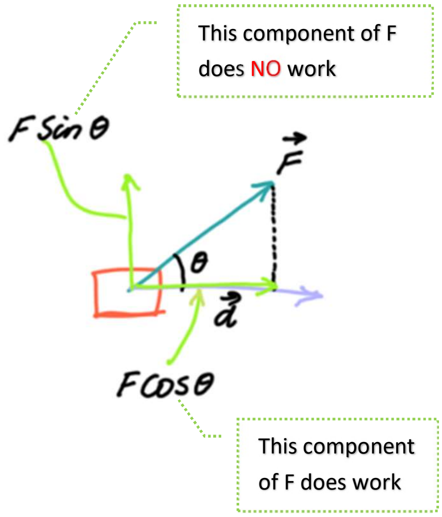
Zero Value of dot product: When the angle between vectors is 90°

Work done using Dot Product

Example: Find work done, using dot product, by a force F that acts at an angle θ with the X axis

$$W = \vec{F} \cdot \vec{d} = Fd\cos\theta$$

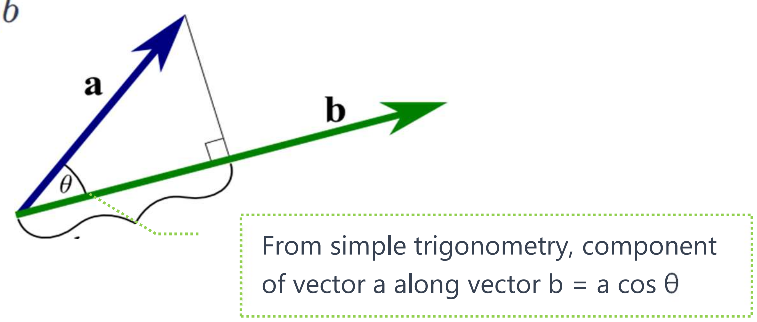
where $\vec{F}$ and $\vec{d}$ are vectors



Visualization of Dot Product

Component along Direction: Dot product can be expressed as the product of the magnitude of one vector and the *component of the other vector along its direction.*

$$\vec{a} \cdot \vec{b} = ab\cos\theta = (a\cos\theta)b$$



Dot Product in Unit Vector Notation

If $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ *and* $\vec{\mathbf{b}} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$

then

$$\vec{a}.\vec{b} = a_x b_x + a_y b_y + a_z b_z$$