



Elastic Collisions & Derivation of Equations



For a collision to be called elastic, 2 conditions are to be fulfilled

• Condition 1: Linear Momentum is conserved

 $m_1V_{1i} = m_1V_{1f} + m_2V_{2f}$

• Condition 2: Kinetic Energy is conserved $(1/2) m_1 v_{1i^2} = (1/2) m_1 v_{1f^2} + (1/2) m_2 v_{2f^2}$

Stationary Target Collisions

Conservation of momentum gives: $m_1v_{1i} = m_1v_{1f} + m_2v_{2f}$ Conservation of kinetic energy gives: $(1/2) m_1v_{1i}^2 = (1/2) m_1v_{1f}^2 + (1/2) m_2v_{2f}^2$

Equations for Final Velocities (from above 2 equations)

For object
$$|: v_{1f} = v_{1i}(m_1 - m_2)/(m_1 + m_2) - For object 2: v_{2f} = v_{1i}(2m_1)/(m_1 + m_2)$$

Always +ve (moves forward)



Special Cases (continuation of previous slide)





Collisions with Moving Targets

Conservation of momentum gives: m1v1i + m2v2i = m1v1f + m2v2f

Conservation of KE gives: $(1/2) m_1 v_1 i^2 + (1/2) m_2 v_2 i^2 = (1/2) m_1 v_1 f^2 + (1/2) m_2 v_2 f^2$

Equations for Final Velocities (from the above 2 equations)

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For object 1:

v_{1f} = v_{1i} (m_1 - m_2) / (m_1 + m_2) + V_{2i} (2m_2) / (m_1 + m_2)

For object 2:

v_{2f} = v_{1i} (2m_1) / (m_1 + m_2) + (m_2 - m_1) / v_{2i} (m_1 + m_2)
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If v2i is set = 0, you get the same equations as in slide 3



Elastic Collisions: 2D



Conservation of momentum in component form:

- 1 $1 = m_1 v_{1f} \cos \theta_1 + m_2 v_{2f} \cos \theta_2$
- 2 γ -axis: 0 = -m₁v_{1f} sin θ_1 + m₂v_{2f} sin θ_2

Conservation of kinetic energy in 2D

$$3 (1/2) m_1 v_{1i^2} = (1/2) m_1 v_{1f^2} + (1/2) m_2 v_{2f^2}$$

There are 7 variables in equations 1,2 & 3. If 4 are known, the 3 equations can be used to find the other 3

