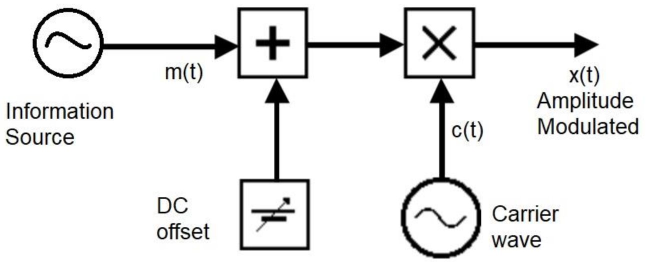
Amplitude Modulation

Time Domain Representation

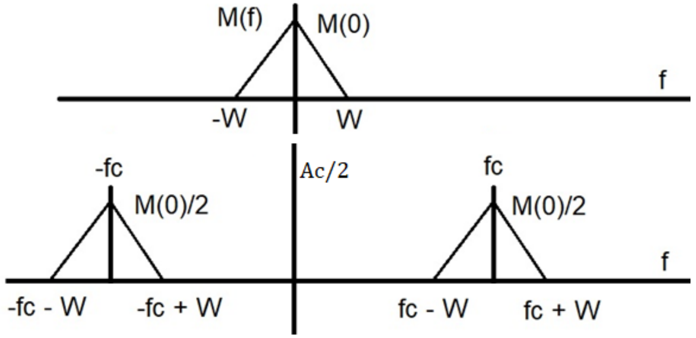


$$x_{AM}(t) = (A_c + m(t))c(t)$$
$$x_{AM}(t) = (A_c + m(t))A_c \cos(2\pi f_c t)$$

$$x_{AM}(t) = A_c^2\left(1 + \frac{m(t)}{A_c}\right)\cos(2\pi f_c t)$$
$$x_{AM}(t) = A'_c\left(1 + \frac{A_m}{A_m}\frac{m(t)}{A_c}\right)\cos(2\pi f_c t)$$
$$x_{AM}(t) = A'_c\left(1 + \frac{A_m}{A_c}\frac{m(t)}{A_m}\right)\cos(2\pi f_c t)$$
$$x_{AM}(t) = A'_c(1 + a m_n(t))\cos(2\pi f_c t)$$

Modulation index $a = \frac{A_m}{A_c}$ Normalized message $m_n(t) = \frac{m(t)}{A_m}$

Frequency Domain Representation



$\frac{A_c}{2}(\delta(f - f_c) + \delta(f + f_c))$ will exist at f_c & $-f_c$

$$m(t) \leftrightarrow M(f)$$
$$x_{AM}(t) = (A_c + m(t))\cos(2\pi f_c t)$$
$$x_{AM}(t) = A_c\cos(2\pi f_c t) + m(t)\cos(2\pi f_c t)$$

$$\cos(2\pi f_0 t + \theta) \leftrightarrow \frac{1}{2}\left(e^{j\theta}\delta(f - f_c) + e^{-j\theta}\delta(f + f_c)\right)$$

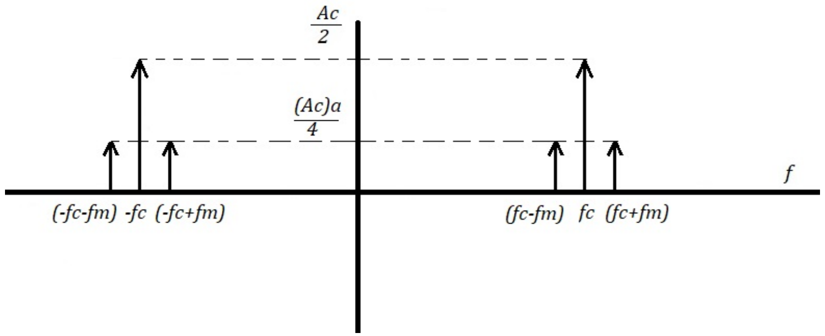
$$A_c\cos(2\pi f_c t) \leftrightarrow \frac{A_c}{2}(\delta(f - f_c) + \delta(f + f_c))$$

$$x(t)\cos(2\pi f_0 t) \leftrightarrow \frac{1}{2}(X(f - f_0) + X(f + f_0))$$

$$m(t)\cos(2\pi f_c t) \leftrightarrow \frac{1}{2}(M(f - f_c) + M(f + f_c$$

$$X_{AM}(f) = \frac{A_c}{2}(\delta(f - f_c) + \delta(f + f_c)) + \frac{1}{2}(M(f - f_c) + M(f + f_c))$$

Single-tone AM



$$m(t) = \cos(2\pi f_m t) \leftrightarrow \frac{1}{2}(\delta(f - f_m) + \delta(f + f_m))$$
$$c(t) = A_c\cos(2\pi f_c t) \leftrightarrow \frac{A_c}{2}(\delta(f - f_c) + \delta(f + f_c))$$

$$x_{AM}(t) = A_c[1 + a\cos(2\pi f_m t)]\cos(2\pi f_c t)$$
$$= A_c\cos(2\pi f_c t) + A_c a\cos(2\pi f_m t)\cos(2\pi f_c t)$$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
$\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2\cos\alpha\cos\beta$
$\cos\alpha\cos\beta = \frac{\cos(\alpha + \beta) + \cos(\alpha - \beta)}{2}$

$$= A_c\cos(2\pi f_c t) + \frac{A_c a}{2}[\cos(2\pi(f_c - f_m)t + \cos(2\pi(f_c + f_m)t)$$
$$x_{AM}(t) = A_c\cos(2\pi f_c t) + \frac{A_c a}{2}\cos(2\pi(f_c - f_m)t + \frac{A_c a}{2}\cos(2\pi(f_c + f_m)t$$

$$X_{AM}(f) = \frac{A_c}{2}(\delta(f - f_c) + \delta(f + f_c)) + \frac{A_c a}{2}\left[\frac{(\delta(f - (f_c - f_m)) + \delta(f + (f_c - f_m)))}{2}\right] + \frac{A_c a}{2}\left[\frac{(\delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m)))}{2}\right]$$