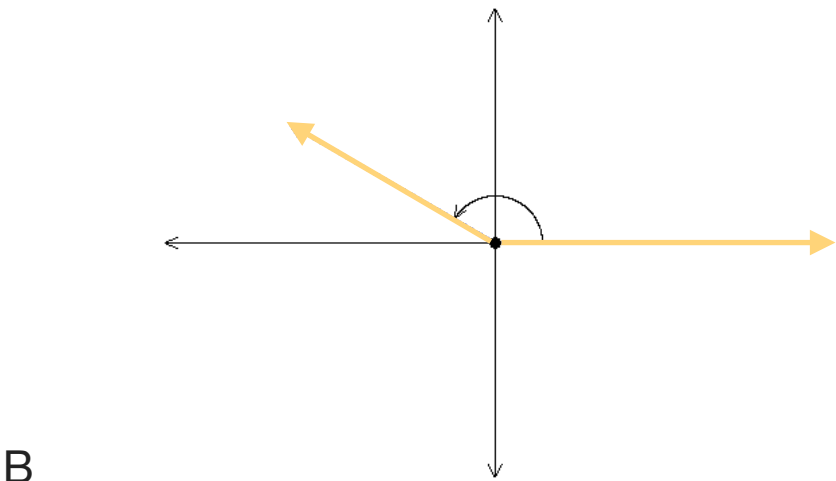
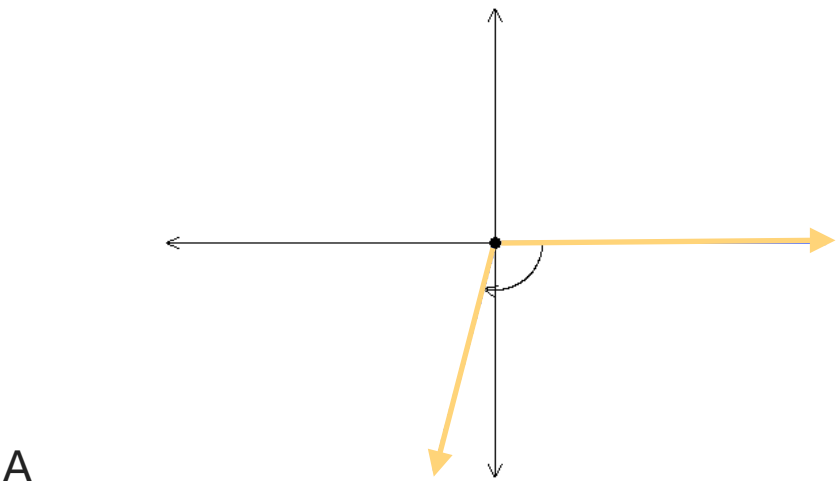


Topic: Sketching angles in standard position

Question: Which of the following is a drawing of the given angle in standard position?

-210°

Answer choices:



Solution: D

An angle of -210° is negative, so its terminal side is reached from its initial side by making a rotation of 210° in the negative (clockwise) direction about the origin. Note that

$$\frac{-210^\circ}{-360^\circ} = \frac{7}{12}$$

Therefore,

$$-210^\circ = -\frac{7}{12}(360^\circ)$$

Also,

$$-\frac{3}{4}(360^\circ) = -\frac{9}{12}(360^\circ) < -\frac{7}{12}(360^\circ) < -\frac{6}{12}(360^\circ) = -\frac{1}{2}(360^\circ)$$

The terminal side of an angle of $-(3/4)360^\circ$ is on the positive vertical axis, and the terminal side of an angle of $-(1/2)360^\circ$ is on the negative horizontal axis. Since

$$-\frac{3}{4}(360^\circ) < -\frac{7}{12}(360^\circ) < -\frac{1}{2}(360^\circ)$$

the terminal side of an angle of $-(7/12)360^\circ$ is between the positive vertical axis and the negative horizontal axis. Combined with the fact that the terminal side of a negative angle is reached from its initial side by making a negative (clockwise) rotation about the origin, we see that D is the correct answer.

Topic: Sketching angles in standard position

Question: Think about the terminal side of the angle.

How many full turns about the origin, and what additional rotation of absolute value less than 2π (if any) about the origin, are made to reach the terminal side of an angle of $(27/4)\pi$ radians (in standard position) from its initial side?

Answer choices:

- A Three full turns about the origin and an additional rotation of $(3/4)\pi$ radians about the origin, both in the counterclockwise direction
- B Three full turns about the origin in the clockwise direction, and an additional rotation of $\pi/4$ radians about the origin in the counterclockwise direction
- C Two full turns about the origin and an additional rotation of $5\pi/4$ radians about the origin, both in the counterclockwise direction
- D Three full turns about the origin and an additional rotation of $3\pi/4$ radians about the origin, both in the clockwise direction

Solution: A

Since $(27/4)\pi$ is a positive angle, we reach its terminal side from its initial side by making some number of full turns about the origin in the positive (counterclockwise) direction, and possibly an additional rotation (of absolute value less than 2π radians) about the origin in the positive direction.

Note that

$$\frac{27}{4}\pi = \frac{27}{4} \left(\frac{2}{2} \right) \pi = \frac{27}{8}(2\pi) = \frac{24+3}{8}(2\pi) = \frac{24}{8}(2\pi) + \frac{3}{8}(2\pi) = 3(2\pi) + \frac{3}{4}\pi$$

Also,

$$\frac{3}{4}\pi = \frac{3}{8}(2\pi) < 2\pi$$

Therefore, we first make three full turns ($3(2\pi)$ radians) about the origin in the counterclockwise direction. Then “what's left over” is $(3/4)\pi$, so we make an additional rotation of $(3/4)\pi$ radians about the origin in the counterclockwise direction.