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A Rational Approach to Pricing of Catastrophe Insurance

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Abstract

A methodology for rational pricing of catastrophe insurance is described. The methodology has two components: a solvency- and stability-based pricing framework, and an engine to quantify the loss variability that drives solvency and stability. Generalization to account for contagious effects of catastrophes and multiple occurrence of peril is presented in detail.

Key words: Insurance, pricing, catastrophe, multiple perils, capacity, risk load

Catastrophes due to natural or man-made causes have three characteristics that distinguish them from other events of property and casualty losses. They occur infrequently and unpredictably, but can exact high costs due to their large footprint. For insurers, the large loss and footprint represent a good market opportunity, on the one hand, but great risk on the other. However, infrequent occurrence drives volatility, which is exacerbated by the absence of norms or precedence; catastrophes do not happen often enough to establish a track record in the actuarial sense.

Pricing of catastrophe insurance must take these unique characteristics into account. In this article, we describe how capacity-based pricing models are used as the starting point for a rational approach to pricing. A key element of the solvency and stability model is the loss exceedance probability. The loss exceedance probability of a portfolio determines the capacity inherent in the portfolio, and capacity is a commodity that drives the pricing structure along with other economic considerations such as profit, investment return, and market condition. Quantification of the exceedance probability involves knowing the correlation between any two losses, or the covariance loss matrix. This matrix is difficult to quantify for most insurance applications, but modern computerized techniques such as IRAS[1] have become available for that purpose. IRAS can also be used to compute the loss exceedance probability of a portfolio for complex scenarios, including multi-occurrence of various perils such as earthquakes, hurricanes and floods, which then becomes the foundation for premium determination and risk management.

Details of the methodology are presented in the following sections along with illustrations.

1. Solvency, stability and pricing

One can go into great detail in discussing the financial operation of an insurance business (e.g., see Pentikainen et al., 1989), such as losses on policies, unallocated loss adjustment expenses, inflation, taxes, operating expenses, commission, reinsurance costs, etc. on the debit side, premium income and investment income from the credit side, the competitive environment, capital markets, overall books of business, and regulatory and geographic constraints. However, we shall focus on the fundamentals, because a basic model of the insurance business suffices for the present purpose.

Succinctly stated, the goal of the insurance business is to maximize its return on capital while maintaining survival and stability (see also the excellent exposition by Stone, 1973). *Survival (or solvency)* is usually expressed in terms of probability. If probability of ruin per year is defined as 1 − probability of survival per year = P_1, then probability of survival equals $1 - P_1$. For example, P_1 could be 1 in 100,000. Thus, probability of ruin per year is 1 in 100,000, and the probability of survival per year is 0.99999. For *stability*, the probability that the combined loss and expense ratio in any year will exceed its target by X percentage points (e.g., 4%) must be less than P_2, e.g., 1 in 100. If L denotes loss, E the expenses, P the premium income, and C the capital, these constraints can be expressed as,

$$\Pr[(L + E) \geq (P + C)] < P_1 \tag{1}$$

and

$$\Pr[(L + E)/P - \text{target} \geq X] < P_2 \tag{2}$$

The insolvency probability, P_1, and the stability parameters P_2 and X are set formally or implicitly by management. For large insurance companies stability rather than survival is likely to be at issue when a new commitment is being considered.

The profit objective can be expressed as $\max[P - (L + E)]$ if other economic and political considerations (such as investment income from capital) are ignored for simplicity. However, a more common practice is to set a target rate of return in the form of a combined loss and expense ratio (e.g., a ratio of 0.96 means a 4% target rate of return; if the expense ratio is 0.36, then the maximum target loss ratio is 0.60).

Any solvency-stability constraint set (P_1, P_2, X) corresponds to a maximum portfolio *loss exceedance probability* (LEP). A LEP is depicted in figure 1, and a point on the curve defines the probability corresponding to a loss threshold, i.e., the probability that the dollar loss to the portfolio due to some combination of events is equal to or greater than

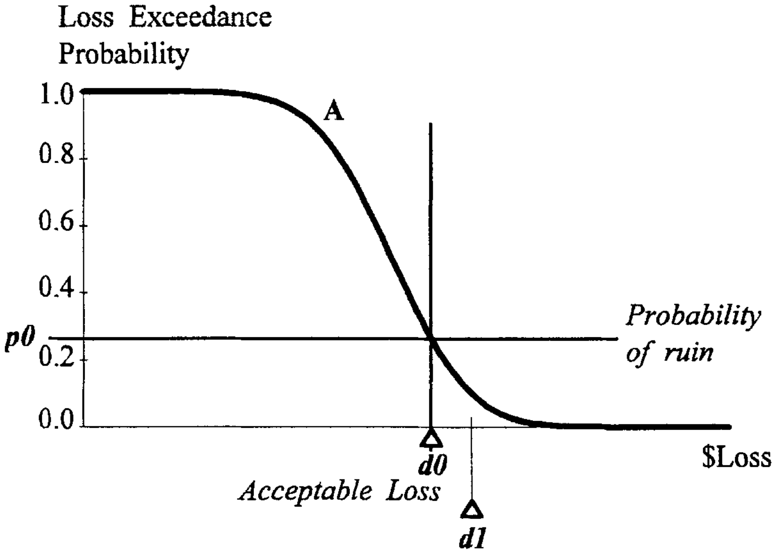

Figure 1. Loss exceedance probability, acceptable loss and probability of ruin.

the threshold. Assuming that the capital structure of the company is such that the maximum loss it can sustain is d_0, then the probability of ruin is p_0, as indicated in the figure. Alternately, if the acceptable probability of ruin is p_0, then the maximum loss sustainable is d_0.

However, if the company reserve can cover a loss higher than d_0, say d_1, as indicated in the figure, the leeway between the maximum LEP and the actual defines the *capacity*. An insurance company with a given capital structure can enhance its stability or lower its insolvency probability by swapping out less attractive policies in the portfolio. Alternately, excess capacity can be traded for income just like a commodity; less attractive policies can be swapped in return for higher premiums, as depicted in figure 2.

Capacity can also be discussed in terms of stability. For that purpose, the loss probability density function given in figure 3 is more illustrative than the exceedance probability even though they contain the same information.[2] With reference to the figure, variability in the loss estimate corresponds to the girth of the bell curve, which depends on probability moments such as the standard deviation, σ, etc.: The wider the girth, the higher the probability that the loss will exceed income. Hence, girth portends variability and destability. Of the two cases denoted by curves A and B in the figure, portfolio B is less attractive because, for a given income level, denoted by point I, the probability of negative income (i.e., the area under the density curve beyond I) is higher for B than for A. Alternately, B can be made acceptable if the premium income is increased sufficiently so that the probability of negative income is now the same as for A.

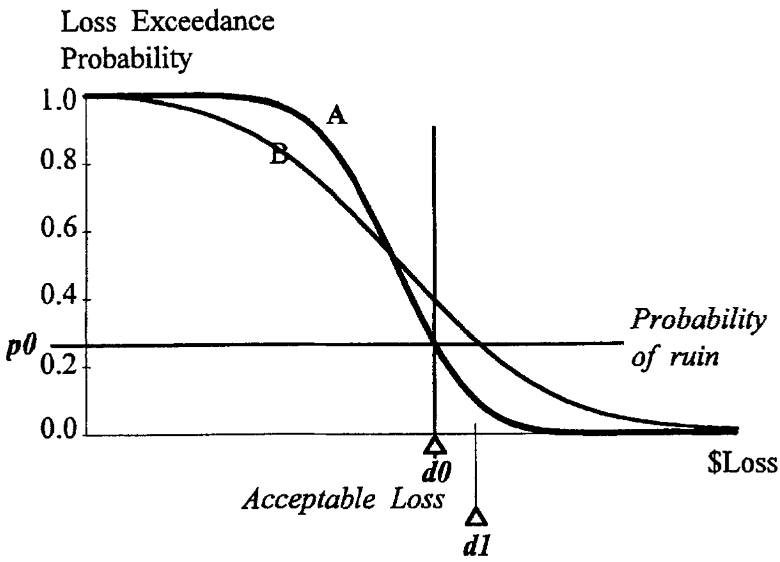

Figure 2. Using excess capital (*d1* − *d0*) to accommodate less attractive portfolios in exchange for higher premium income. Acceptable probability of ruin is not exceeded.

Although the girth of the bell curve depends on σ and higher (probability) moments, σ is often used as the measure of the girth for simplicity. In fact, the ratio of σ/μ, where μ is the mean loss, is called the *exposure ratio* and has been identified as the major measure of stability (see Stone, 1973).

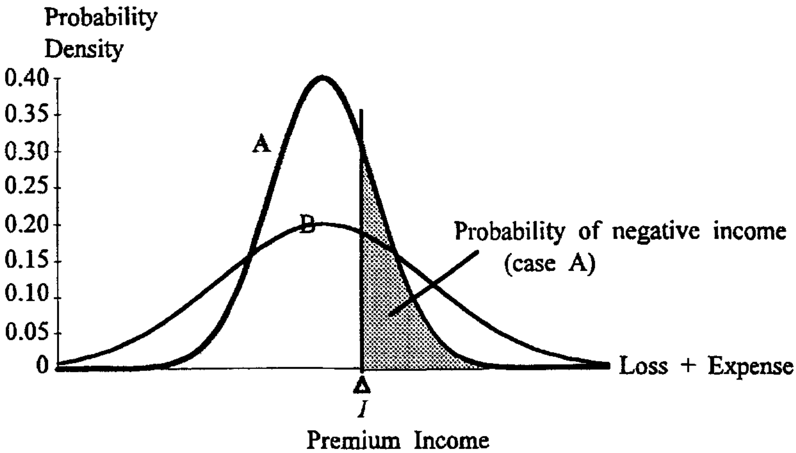

Figure 3. Comparison of two portfolios. B is more unstable than A because its probability of negative income is higher.

Finally, for a given portfolio, the capital structure can be adjusted either in part or in whole, as depicted in figure 4, to realize risk pass-through while income is maintained at the same level. We shall show how the *IRAS* computation engine makes these and other management decisions possible by providing a rational, integrated framework in which the exposure ratio, probability density function, or loss exceedance probability for treaties, policies, and portfolios are computed and studied.

2. Event loss uncertainty matrix

Several sources of uncertainties are involved in loss estimation, including event occurrence uncertainty, uncertainty in the hazard given an event has occurred, building performance (asset damage) uncertainty given a hazard, and incomplete knowledge of the portfolio in general. All contribute to the variability in the loss distribution, and all are considered in IRAS. We shall use event uncertainty to illustrate how IRAS is used to support rational pricing. Hence, although only event uncertainty is referenced explicitly, it is understood that losses computed in IRAS include the effects of the other uncertainties by default.

The end result of an IRAS application is an *event loss uncertainty matrix* such as depicted in table 1. The matrix gives the mean annual loss due to any event of interest, and, more importantly, the standard deviation of the loss. The latter is a measure of the variability in the loss estimate, but both are important in pricing.

The events include *basic events and compound events,* and the distinction is made solely for clarity of presentation. First, a list of all reasonable events (earthquakes, hurricanes, tornadoes, floods, etc.) that may affect the book of business is made in column 1. Each peril is assumed to have an annual probability of occurrence, column 2; each such occurrence is considered a basic event. Note that no reference is made to occurrences of other perils (or reoccurrence of the same peril within the year), and damage due to the

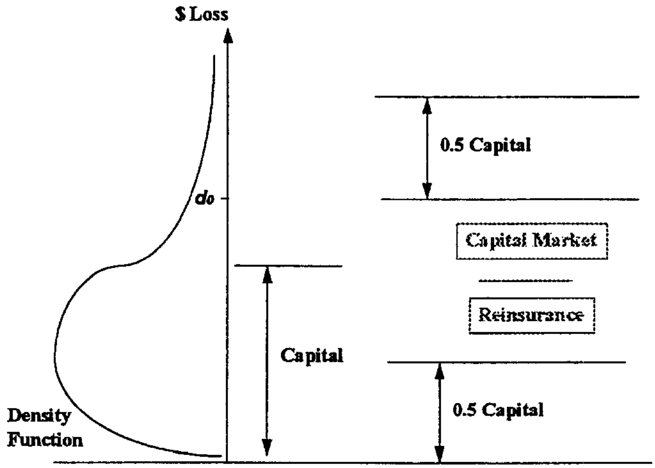

Figure 4. Capital restructuring to enhance survivability and stability.

Table 1. Sample event loss uncertainty matrix

Event (1)	Annual probability (2)	Mean annual loss (3)	Standard deviation σ (4)
Total		Mean annual loss =	Standard deviation=

event is computed independently of other damages. Compound events are combinations of the basic events, i.e., more than one event occurs within the year and their losses are compounded. We shall see that compound events or multi-occurrences of peril are important to the variability in loss estimates.

Column 3 as presented in the table is symbolic only. Entries in column 3 denote loss from any insurance contract, be it primary, quota share, surplus share, etc., or facultatives at the policy or portfolio level. In fact, any loss computed in IRAS can be entered; the methodology remains the same. The loss can also be ground-up loss, gross loss (before reinsurance) or net loss. Column 4 contains the standard deviations associated with the mean losses in column 3; they constitute the product of the extended IRAS methodology.

The event loss uncertainty matrix then contains all the information on loss that is required to compute the LEP. We show how the matrix is developed for basic (singly-occurring) events and compound (multiply-occurring) events. Multi-occurrences of the same or different kinds of peril within a given time span are less probable than single occurrences of the perils. Nevertheless, *multi-occurrences are of interest because the losses can be much higher; in the worst case, they may be the cause of ruin.*

When the number of basic events is small, combination logic can be used to exhaust all permutations of multi-occurrences, and the corresponding exceedance probabilities computed exactly. As the number of events increases, combination explosion rules out analytic calculation, and numerical methods must be used. We show how stochastic stimulation techniques can give a more accurate estimate of the exceedance probability (and, hence, expected loss) at the expense of computation time, when such accuracy is required. We also show that a simple assumption regarding the correlation of events significantly reduces the simulation time, so that the simulation approach offers an attractive compromise between accuracy and expediency.

3. Loss exceedance probability (LEP)

Consider three probable events such as A, B and C defined below.

$$
\begin{aligned}
&P(A) = 0.05; \quad && P(\bar{A}) = (1.0 - 0.05) = 0.95; \quad && L(A) = 30 \\
&P(B) = 0.10; \quad && P(\bar{B}) = (1.0 - 0.10) = 0.90; \quad && L(B) = 20 \\
&P(C) = 0.15; \quad && P(\bar{C}) = (1.0 - 0.15) = 0.85; \quad && L(C) = 10
\end{aligned} \tag{3}
$$

Each event has an *annual probability of occurrence* associated with it, denoted by $P(A)$, $P(B)$, etc. Following common convention, $P(\bar{E})$ denotes the probability that the event E does not occur. Hence, $P(\bar{E}) = 1 - P(E)$. The amount of loss that an event will incur, or *the single-event loss*[3], will be denoted by $L()$. Note that the same assets may be affected by more than one event, or the different events may involve distinct groups of assets. That is not an important point because the assumption is made here that all assets will have been repaired before the next event, if any, occurs. Cumulative damage, i.e., amplification of damage due to preexisting damaged conditions of the asset, is not considered in this discussion. In other words, the effect of multi-occurrence on loss is caused exclusively by the addition of the single-event losses based on pristine asset condition in each case.

The sample data are collected in table 2 below. From these basic data, it is easy to compute the probabilities and losses associated with multiple events. For three events A, B and C, there are a total of eight combinations of multiple events, as shown in table 3, where, following convention, $AB\bar{C}$ stands for the occurrences of event A and B, but not C, and so on. For example, consider the first row. If as denoted by ABC all three events occur, the total loss is $30 + 20 + 10 = \$60$, and the probability of this compound event[4] is, assuming the events are independent, $P(A)*P(B)*P(C) = 0.05*0.10*0.15 = 0.00075$. Entries in the other rows are obtained in similar fashion. The process can be visualized most clearly in the form of a Venn diagram such as shown in figure 5. The loss and probability associated with the eight compound events are as noted in the figure.

The expected loss for each compound event is computed according to the formula (event loss)*(event probability), and shown as column 4. The *total expected loss* is then the sum of the column, or \$5 in this case.

It is interesting to note that the same total expected loss can be obtained directly from table 2. As is done for table 3, for each of the events A, B and C shown in table 2, we multiply the loss column by the probability column to obtain the expected loss, which is then appended to the table. The complete new table is shown as table 2a, and the total expected loss is \$5, the same as obtained previously by considering all possible combinations of the three events A, B and C in table 3. Hence, the total expected loss computed based seemingly on merely individual events in table 2a includes the contribution of all multi-events! That this must be so also becomes clear when reference is made to the Venn diagram introduced previously in figure 5. For example, consider the circle that denotes all events involving A. Each of the two double-overlap portions of the circle is the sum of two parts:

$$\frac{\$50}{0.00425} = \frac{\$30}{0.00425} + \frac{\$20}{0.00425}$$

$$\frac{\$40}{0.00675} = \frac{\$30}{0.00675} + \frac{\$10}{0.00675}$$

where the first part can be allocated to A and the second part to B and C for the first and second line, respectively. Similarly, the triple-overlap portion can be broken into:

Table 2. Sample data for three events

Event (1)	Loss (in \$) (2)	Probability (3)
A	30	0.05
B	20	0.10
C	10	0.15

$$\frac{\$60}{0.00075} = \frac{\$30}{0.00075} + \frac{\$20}{0.00075} + \frac{\$10}{0.00075}.$$

The three parts go to A, B and C, respectively. Summing all parts that revert to A, we have:

$$\frac{\$30}{0.03825} + \frac{\$30}{0.00425} + \frac{\$30}{0.00675} + \frac{\$30}{0.00075},$$

or \$30 / 0.05, which is also the loss due to event A multiplied by the probability of event A. Other pieces in the diagram can be decomposed and allocated in similar fashion, and the operation of table 3 is shown to be equivalent to that of table 2a.

Hence, if one is interested only in the total expected loss from all combinations of multi-events and if constituent data such as probabilities and losses from the basic events are available, the simple operations illustrated in table 2a suffice. In other words, the effect of multi-occurrences is already included in table 2 (or 2a), and there is no need to go to the expanded table 3. However, table 3 contains detailed information that is not available in table 2 per se, but is important in loss estimates. In particular, if one is interested in the LEP which gives the probability of the loss exceeding a certain level, such information can be readily extracted from table 3 but not from table 2 per se. With reference to table 3, column 2 shows there are seven loss levels (\$0, 10, 20, 30, 40, 50 and 60). Their corresponding exceedance probabilities can be constructed from column 3 as follows. Only one compound event ABC can attain the highest loss level of \$60, with a probability of 0.00075. Next, we see that two compound events can exceed a loss level of \$50, viz, ABC and $AB\bar{C}$; hence, the composite probability for that loss level is 0.00075 + 0.00425.

Table 3. The expected loss based on combination of events

Event (1)	Loss (in \$) (2)	Probability (3)	Expected loss (4) = (2) × (3)
ABC	60	0.05 × 0.10 × 0.15 = 0.00075	0.0450
$AB\bar{C}$	50	0.05 × 0.10 × 0.85 = 0.00425	0.2125
$A\bar{B}C$	40	0.05 × 0.90 × 0.15 = 0.00675	0.2700
$A\bar{B}\bar{C}$	30	0.05 × 0.90 × 0.85 = 0.03825	1.1475
$\bar{A}BC$	30	0.95 × 0.10 × 0.15 = 0.01425	0.4275
$\bar{A}B\bar{C}$	20	0.95 × 0.10 × 0.85 = 0.08075	1.6150
$\bar{A}\bar{B}C$	10	0.95 × 0.90 × 0.15 = 0.12825	1.2825
$\bar{A}\bar{B}\bar{C}$	0	0.95 × 0.90 × 0.85 = 0.72675	0.0000
Total		1.00000	5.0000

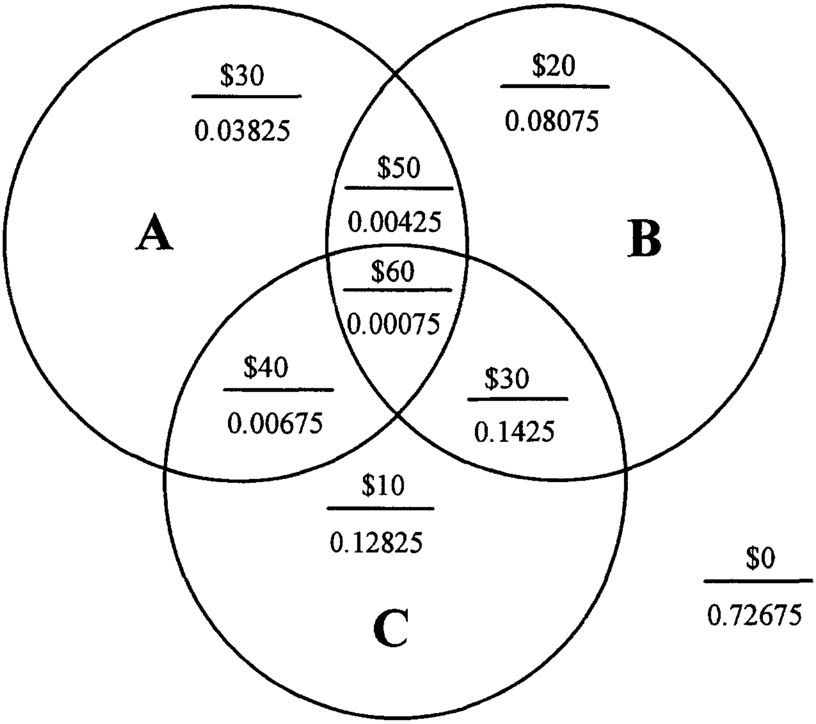

Figure 5. Venn diagram for example.

This process is continued for other levels of loss, and the result is plotted in figure 6. The exceedance probability curve is monotonically decreasing, starting with 1 for a loss of zero and approaching zero as the loss level increases.

To reiterate, we see that, as opposed to the total expected loss, the exceedance probability curve cannot be obtained directly from table 2, but only through its expanded counterpart, table 3, because all possible loss levels from various combinations of events are directly available from the expanded table.

When generalized to N events, the basic table will have N entries, but the expanded table will have 2^N entries. Hence, the size of the tables can become large very quickly as N increases. For this and other reasons, numerical methods are favored. In subsequent

Table 2a. Expected loss based on annual rates of events

Event (1)	Loss (in \$) (2)	Probability (3)	Expected Loss (in \$) (4) = (2) × (3)
A	30	0.05	1.5
B	20	0.10	2.0
C	10	0.15	1.5
Total			5.0

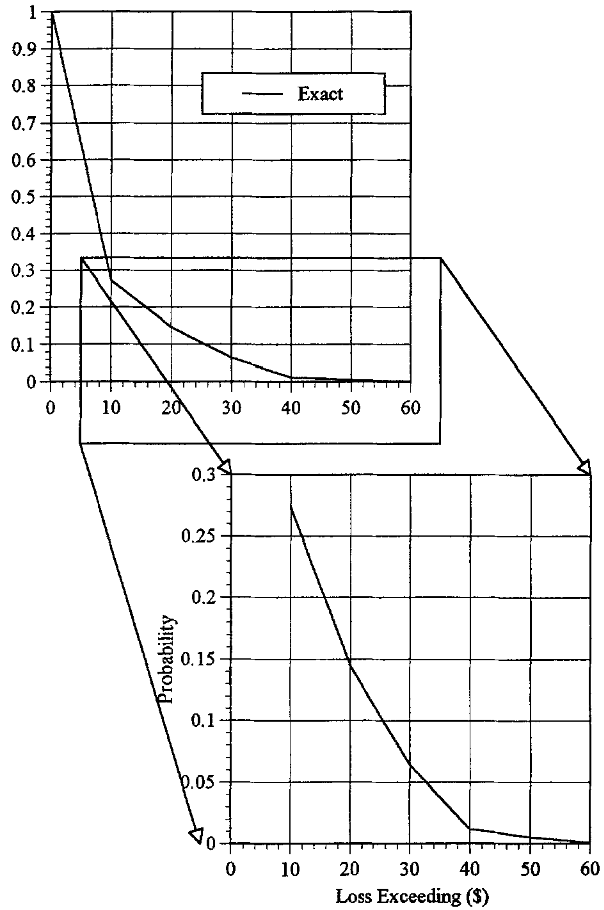

Figure 6. Exceeding probability for various loss levels.

sections, we describe current IRAS methodology in relation to multi-occurrences of perils, and how stochastic simulation is used to compute the expected losses under those circumstances.

4. Loss exceedance probability from stochastic simulation

LEPs that consider the effects of multi-occurrences of perils are difficult to obtain analytically except for the simplest cases involving a few events. Even then, the portfolio structure presents complications which are likely to rule out analytic procedures. This suggests that numerical methods be used. We now describe some results obtained with stochastic simulation techniques.

The probability of exceedance for any loss x can be computed by integrating the probability density function for loss from x to ∞. The probability density function for loss can be estimated by simulation as follows:

- For each event generate a random number between 0–1. If the generated random number is smaller than the probability of the event, the event will occur. Repeat for all events of interest to identify the events that are "occurring" from those that are not.
- Find the total loss of all occurring events.
- Increment the counter of the total loss by one, representing another round of event on-off selection, and repeat the first two steps.
- Repeat the above steps a large number of times (>1000).
- Divide the loss counters from all simulations by the total number of simulations to get an estimate of the probability density function.

We illustrate the procedure with an example which consists of five events as given in table 4. Figure 7 compares the exact probability of exceedance (solid line) with that obtained using simulations (dotted lines). It is seen also that the accuracy of the probability of exceedance obtained by simulation increases as the number of simulations increases. At 1000 simulations the simulated probability of exceedance is close to the exact value. The curve marked with single-occurrence denotes the probability of exceedance that would have been obtained if the effects of multi-occurrence are ignored.[5] Note that neglecting the effects of multi-occurrence can lead to significant error in the LEP.

5. Capacity-based pricing

Assume for the moment that pricing is based solely on the rate of return on capital, i.e., without regard to survival and stability constraints. The corresponding premium, called the *economic premium*, is simply the loss ratio:

$$P = \frac{\bar{L} + \bar{E}}{(1 - r)} \tag{4}$$

or

Table 4. Data for example

Event number	Loss ($)	Probability
1	60	0.1
2	50	0.2
3	40	0.2
4	30	0.3
5	20	0.2

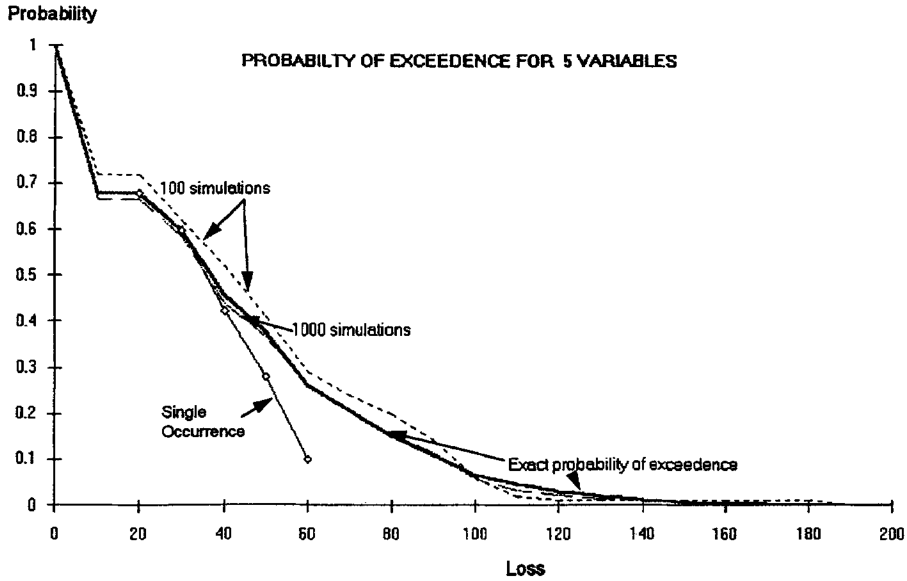

Figure 7. Probability of exceeding for example.

$$P = \frac{\bar{L}}{(1 - r - e)} \tag{5}$$

where $\bar{L}$ is the long-term annual loss expectation and $\bar{E}$ is the expected expense incurred by underwriting the risk, r the underwriting profit ratio, and e the expense ratio. For instance, for a risk with average \$100 per year in loss, an expense ratio of 35%, and a required profit ratio of 5%, the economic premium based on (5) is $100/(1 - 0.35 - 0.05) = \$167$.

However, in an uncertain insurance environment, a portfolio of a given size and expected return that is within the stability and survival constraints is worth more than a portfolio of identical size having the same expected return that is not within the constraints, everything else being the same. A portfolio with substantial excess capacity is even more valuable. *Survivability and stability are positively priced commodities, and risks that generate capacity are more valuable to an insurer than capacity consumers* (see Stone, 1973).

Hence, if the two classes of risk are available only at their economic premiums, insurers would show a distinct preference for the capacity generators, which will then be bid below the economic premium. Conversely, capacity risks will be underwritten above their economic premiums. Hence, pricing must be based on capacity effect as well as the expected loss, and a first effort may be as follows:

$$P = \frac{\bar{L}}{1 - r - e - d} \tag{6}$$

where d is the differential based on capacity (stability) considerations.

Equation 6 is the basis for capacity/stability-based pricing. The magnitude of d in any specific case will depend on the exposure ratio of the risk, the size of the risk, the size and composition of the portfolio, the constraints observed by the insuring company, and competitive factors relating to the capacity and exposure of the insurance industry as a whole. Generally speaking, d could be negative for those risks which add substantial capacity to the portfolio, while d is likely to be highly positive for most of the capacity risks. The greater the uncertainty of the risk characteristics, the greater would be the additional differential.

Capacity-based pricing formulae currently in use include:[6]

- Standard Deviation Principle $P = E(L) + \alpha\sigma(L)$
- Variance Principle: $P = E(L) + \beta\sigma^2(L)$

where α and β are parameters, $E(L)$ is the expected loss, $\sigma(L)$ the standard deviation of the loss.

The inclusion of the parameters α (or β) in the capacity-based formulae denotes the insurer's reluctance to take on the risk measured by $\sigma(L)$ (or σ^2 (L)). For this reason, the product $\alpha\sigma(L)$ (or $\beta\sigma^2(L)$) is also called the *risk load,* in the sense that it is the extra cost associated with the assumed risk.

For instance, Kreps (1990) has shown that

$$\alpha = \frac{yz\,(2SC + \sigma)}{1 + y\,(S' + S)} \tag{7}$$

where S and S' are the standard deviations of the expected loss for the existing and new book, respectively, C the correlation of the new contract with the existing book, z the level of stability required, and y the yield rate in the capital markets. When $C = 1$, and $S >> \sigma$, (7) reduces to $\alpha = yz/(1 + y)$ and the risk load is independent of σ. Meyers (1994) has derived a formula for risk load that is of the form:

$$R = \bar{\lambda}\{\sigma^2(L) + 2\sum_{i=1}^{n} Cov(\bar{x}_i, L)\} \tag{8}$$

where $\bar{x}_i$ are the expected losses of existing policies in the portfolio, and $Cov(.,.)$ denotes the covariance. $\bar{\lambda}$ is called the risk load multiplier, and is a function of the marginal rate of return and marginal cost of capital.

We use an illustration to show how IRAS can be used to support capacity-based pricing. Consider the following table as an example of an existing book of business. Event *A1* is associated with fault *A*, *B1* with fault *B* and so on.[7] The events are assumed independent for now.

Event *A1* is assumed to have an annual probability p_i of 0.1 or 10%. If it occurs, it will incur a net loss x_i of 3.0. Hence, the mean annual loss $E[x_i]$, where $E[.]$ is the expectation operation, is then $p_i{*}x_i = 3.0{*}0.1 = 0.3$. The variance of the loss for this case is 0.81.[8]

Entries in the other rows of the table have similar meanings, and no further elaboration is needed.

Hence, the sum of the mean annual loss from the six events is 1.40, and the sum of the variance is 3.025.[9] For uncorrelated events, the variance of the book is the sum of the variances of the contracts in the book. Hence, the standard deviation for the book is $\sqrt{3.025} = 1.74$, and the cofficient of variation for the book, *cv*, is 1.74/1.4 = 1.24.

Suppose two new contracts, *X1* and *Y1*, are added to the book, and that they are affected by faults *X* and *Y* which are uncorrelated with faults *A–F*, i.e., their losses are not correlated with the *A, B, .., F* events. Data for the new contracts are given in table 6. The sum of the mean losses for the new contracts is 0.45, the sum of their variances is 2.1375, the standard deviation is 1.46, and the *cv* is 1.46/0.45 = 3.24. Hence, a book consisting of only these two contracts has low stability.

We examine how adding the new contracts will affect the book of business. The new book, denoted by New Book 1, is given by table 7. New Book 1 has a total mean loss of 1.85 and a sum of variance of 5.1625. The *cv* is then $\sqrt{5.1625}/1.85 = 1.23$, which is much smaller than that of the two contracts X1 and Y1 by themselves (viz., 3.24), and even smaller than that of the original book (viz., 1.24). *Hence, adding contracts which are independent of existing contracts in a book can decrease the volatility of the book even if the new contracts are themselves more volatile.*

Suppose the contracts added are not independent, and indeed the losses are affected by an event on fault *D*. To denote this dependency, we denote *X1* by *D2* and *Y1* by *D3*, and the new book by New Book 2. Hence, data for New Book 2 are as given in table 8. An event on fault *D* will thus lead to losses in rows *D1, D2,* and *D3*. The impact of this correlation is that the variance of the book is increased by 2*1.09*1.31=2.86 for correlation between D1 and D2, by 2*1.09*0.65=1.42 for *D1* and *D3*, and by 2*1.31*0.65 = 1.70 for *D2* and *D3*.[10] The variance of the New Book 2 is 5.1625 + 2.86 + 1.42 + 1.70 = 11.14, and the *cv* is $\sqrt{11.14}/1.85 = 1.80$. While still lower compared with that of the new contracts (3.24), the volatility of New Book 2 is higher than the original book (1.24) and New Book 1 (1.23). *Hence, adding contracts which are correlated with existing contracts in an book increases the volatility of the book.*

Table 5. Sample book of business

Event, i (1)	Annual probability, p_i (2)	Net loss, x_i (3)	Mean annual loss, $E[x_i]$ (4)	$E[x_i^2]$ (5)	Variance $= \sigma^2[x_i]$ (6)	$\sigma[x_i]$ (7)
A1	0.10	3.0	0.30	0.900	0.81	0.90
B1	0.20	1.5	0.30	0.450	0.36	0.60
C1	0.30	0.5	0.15	0.075	0.0525	0.23
D1	0.05	5.0	0.25	1.250	1.1875	1.09
E1	0.10	2.5	0.25	0.625	0.5625	0.75
F1	0.30	0.5	0.15	0.075	0.0525	0.23
Total			1.40	3.375	3.025	1.74

Table 6. Data for sample new contracts

Event (1)	Annual probability (2)	Net loss (3)	Mean annual loss (4)	Variance (6)	Standard deviation, σ (5)
X1	0.05	6.0	0.30	1.71	1.31
Y1	0.05	3.0	0.15	0.4275	0.65
Total			0.45	2.1375	1.46

The quantitative impact of correlation depends on the degree of correlation and the standard deviations of the events involved. For instance, a cursory look at column 7 of New Book 2 above indicates that the destabilizing impact will be smaller if the new contracts are correlated with *F1* instead of *D1* since the standard deviation of *F1* is only 0.23/1.09 or 21% of *D1*. A smaller degree of correlation, represented by the value of the correlation coefficient, has a similar effect. In the present context, the coefficient has a maximum value of 1 and a minimum value of 0. The latter case corresponds to the independent events examined in New Book 1.

6. Summary

Catastrophe risk is very different from other more common and less devastating risks such as auto and fire. Losses from catastrophes are large, highly unpredictable, and contagious. Furthermore, the uncertainty associated with the occurrence of catastrophes is large because they do not happen very often; the actuarial database from which cause/effect information may be gathered is small. For earthquakes, the time scale is of the order of hundreds of years, and even when loss data could be recorded as in recent events, the data are fragmentary and uncertain. More important, such loss experiences are unlikely to be representative of modern society due to constant changes in the built environment, technology, business infrastructure, asset valuation and demographic distribution.

Table 7. New Book 1.

Event, i (1)	Annual probability, p_i (2)	Net loss, x_i (3)	Mean annual loss, $E[x_i]$ (4)	$E[x_i^2]$ (5)	Variance $=\sigma^2[x_i]$ (6)	$\sigma\ [x_i]$ (7)
A1	0.10	3.0	0.30	0.900	0.81	0.90
B1	0.20	1.5	0.30	0.450	0.36	0.60
C1	0.30	0.5	0.15	0.075	0.0525	0.23
D1	0.05	5.0	0.25	1.250	1.1875	1.09
E1	0.10	2.5	0.25	0.625	0.5625	0.75
F1	0.30	0.5	0.15	0.075	0.0525	0.23
X1	0.05	6.0	0.30	1.8	1.71	1.31
Y1	0.05	3.0	0.15	0.45	0.4275	0.65
Total			1.85	5.625	5.1625	2.27

Table 8. New Book 2

Event, i (1)	Annual probability, p_i (2)	Net loss, x_i (3)	Mean annual loss, $E[x_i]$ (4)	$E[x_i^2]$ (5)	Variance $=\sigma^2[x_i]$ (6)	$\sigma[x_i]$ (7)
A1	0.10	3.0	0.30	0.900	0.81	0.90
B1	0.20	1.5	0.30	0.450	0.36	0.60
C1	0.30	0.5	0.15	0.075	0.0525	0.23
D1	0.05	5.0	0.25	1.250	1.1875	1.09
E1	0.10	2.5	0.25	0.625	0.5625	0.75
F1	0.30	0.5	0.15	0.075	0.0525	0.23
D2	0.05	6.0	0.30	1.8	1.71	1.31
D3	0.05	3.0	0.15	0.45	0.4275	0.65

Because the effects of catastrophes are felt by a much larger region than, say, isolated auto accidents or fires, geographic diversifications takes on new scale and meaning.[11] Domino effects in catastrophes are also prominent. A damaged asset will in turn enhance the damage to another asset, either directly, such as when debris from a collapsed building creates havoc on its neighbors (called collocation or collateral damage), or indirectly, such as when loss of power exacerbates communication functions and recovery efforts (called functional or dependency damage). Losses from catastrophe are said to be spatially and functionally correlated.

A rational approach for pricing that takes these unique characteristics into account has been described. Capacity-based pricing models are used as a starting point; the models account for the important interaction between catastrophe loss, survival and stability. A key pricing parameter in these models is the exposure ratio, viz., the ratio of the standard deviation of the loss to the mean loss. The exposure ratio of the risk (policy) being contemplated denotes its destabilizing potential, and the aggregate ratio indicates whether the risk is a capacity generator (stabilizing) or a capacity consumer (destabilizing). Hence, capacity is a commodity and should be recognized in the pricing structure along with economic considerations such as profit, investment return, market condition, and regulatory/political constraints. It is shown how computerized loss estimation systems such as IRAS can be used to quantify the exposure ratio of each policy, the aggregate exposure ratio of a portfolio, and the loss exceedance probability for credible scenarios.

Paramount in any rational pricing paradigm is the accounting of the contagious effects of catastrophes and the effects due to multiple occurrence of peril. Quantification of contagious effects requires knowing the correlation relation between any two losses, or the covariance matrix. This matrix is difficult to quantify for most applications except by simulation techniques such as IRAS. Losses due to multi-occurrences of the same or different kinds of peril can be much larger than that due to a single peril, and these "excess" losses are important to solvency even though they have very small probability of occurring. We have shown how they can be quantified using the IRAS framework. In short, all quantitative information on potential losses needed for rational pricing is provided.

In summary, IRAS can be viewed as the engine that powers the pricing vehicle. Quantitative estimates of losses and their variability constitute the "drive train". The IRAS finance module, details of which will be deferred to another article, is the "transmission" that converts loss information at the treaty, policy or portfolio level and management preferences into a pricing structure.

The same IRAS framework can be used for many elements of insurance planning other than pricing. For example, treaty screening and reinsurance strategy can be addressed, as is obvious from the previous discussion.

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Notes

1. *Investment and Insurance Risk Assessment System,* an application software by Risk Management Solutions, Inc.
2. Recall that the relation between LEP and density function is $LEP(L) = \int_l^\infty p(x)dx$, where $p(x)$ denotes the loss probability density function and L the loss level of interest.
3. In IRAS terminology, this loss can be the maximum loss, the mean loss, etc., given that the event occurs. The uncertainty in loss is caused by uncertainties in hazards such as attenuated ground shock and building performance. We denote it simply as loss; the uncertainty hereinafter is caused by uncertainty of the event, and the probability of exceedance is caused by the probability of occurrence of the event.
4. To eliminate confusion, we shall henceforth refer to events that occur singly as the basic events, and events that occur together as compound events.
5. When the effects of multi-occurrence are ignored, the LEP can be derived readily by using the range probability, i.e., the probability that the loss is within certain ranges. Details are not included herein as they are not germane to the discussion.
6. The formulae have been greatly simplified as they do not include the many other factors that enter into pricing. This is done to highlight the role of IRAS.
7. Faults stand for some source of hazards, which can be hurricanes, floods, etc.
8. The variance of x, denoted by $\mathrm{Var}[x]$ is $\mathrm{Var}[x] = E[x^2] - E[x]*E[x]$, and the standard deviation σ is related to the variance by $\sigma[x]*\sigma[x] = \mathrm{Var}[x]$.
9. In general, not equal to $\Sigma E[x_i^2] - (\Sigma E[x_i])^2$ or 1.415.
10. When two losses are correlated, the increment in variance is given by $2p_{ij}\sigma_i\sigma_j$, *where* p_{ij} is the correlation coefficient, σ_i the standard deviation of event i, and σ_j that of event j. The correlation coefficient has been assumed to be 1, the maximum value possible, for simplicity.
11. For example, consider the impact of the Northridge earthquake on small, local insurance companies whose portfolios are mainly in the Southern California area.

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