

Test – Polynomial Functions, Equations & Inequalities

<u>Syllabus content on test</u>: • polynomial functions • quadratic formula and discriminant • factor and factor theorems • polynomial division • sum and product of roots of a polynomial equation • rational functions • solving equations involving radicals, absolute value and in quadratic form • solving inequalities

total marks on test: 60

Part I: No calculator – questions 1-7 [39 marks]

- 1. The polynomial $x^3 + 3x^2 + ax + b$ has the same remainder when divided by x 2 as when divided by x + 1. Find the value of a. [5 marks]
- 2. The function g is defined as $g(x) = \frac{4x+8}{2-x}$, $x \in \mathbb{R}$, $x \neq 2$. Sketch the graph of y = g(x), clearly indicating and stating the equations of any asymptotes and the coordinates of any axes intercepts. [5 marks]
- 3. Given that 2-i is a root of the equation $x^3 6x^2 + 13x 10 = 0$ find the other two roots. [5 marks]
- **4.** x-2 and x+2 are factors of $x^3 + ax^2 + bx + c$, and it leaves a remainder of 10 when divided by x-3. Find the values of a, b and c. [6 marks]
- 5. Given $f(x) = x^2 + x(k-1) + k^2$, find the range of values of k so that f(x) > 0 for all real values of x. [5 marks]
- 6. If α and β are the roots of the equation $2x^2 + 6x 5 = 0$, find a quadratic equation with integer coefficients whose roots are:
 - (a) 2α , 2β

(b) $\frac{1}{\alpha+1}, \frac{1}{\beta+1}$

[8 marks – 4 marks each]

7. The polynomial $h(x) = (px+q)^3$ leaves a remainder of -8 when divided by (x+1), and a remainder of -27 when divided by (x+3). Find the value of p and the value of q. [5 marks]

Part II: calculator allowed – questions 8-12 [21 marks]

- 8. Find the values of x for which $\frac{x^2+3}{x-1} > 7$ [4 marks]
- 9. Find a polynomial of lowest degree with integer coefficients that has zeros $x = \frac{3}{2}$ and x = 1 2i.

[4 marks]

10. Solve the inequality $x+3 > \sqrt{7x+9}$

[4 marks]

11. Find the value(s) of m such that the following equation has two distinct real solutions.

$$mx^2 - 2x - m - 2 = 0$$
 [4 marks]

12. Find the roots of the quadratic equation $x^2 - 6x + k = 0$ given that one root is three times the other root and that k is a constant. [5 marks]

Bonus question: The polynomial $P(x) = x^3 + mx^2 + nx - 8$ is divisible by (x+1+i). Find the value of m and the value of n.