

Vectors – Equations of Lines

No calculator allowed on all exercises

[answers on next page]

♦ All of the exercises are suitable for both Maths SL & Maths HL. ♦

- Find a vector equation, in the form $\vec{r} = \vec{a} + \lambda\vec{b}$, of the line passing through the points $A(2, -3)$ and $B(-5, 2)$.
 - Does the point $C(-12, 7)$ lie on the line AB ? Explain.
- A line passes through the point $(-1, 4, 0)$ and is parallel to the vector $2\mathbf{i} - 6\mathbf{j} + \mathbf{k}$. The point P with coordinates $(2, a, b)$ lies on the line. Find the value of a and the value of b .
- A line passes through the point $(-3, 5)$ and its direction is perpendicular to the vector $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$. Find the equation of the line in the form $ax + by = c$ where a , b and c are integers to be determined.

4. Consider the two lines L_1 and L_2 given as follows:

$$L_1: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -1 \\ -7 \end{pmatrix} \qquad L_2: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \\ -6 \end{pmatrix} + \mu \begin{pmatrix} -6 \\ 2 \\ -4 \end{pmatrix}$$

(a) P is the point on L_1 when $\lambda = 1$. Find the position vector of P .

(b) Show that P is also on L_2 .

(c) A third line, L_3 , has a direction vector of $\begin{pmatrix} a \\ 3 \\ c \end{pmatrix}$. If L_1 and L_3 are parallel, find the value of a and the value of c .

5. Position vectors \vec{OP} and \vec{OR} are $2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $6\mathbf{i} + 5\mathbf{j} + \mathbf{k}$ respectively. Show that the line $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} + 2\mathbf{k} + t(5\mathbf{i} - 3\mathbf{j} + \mathbf{k})$ is a perpendicular bisector of the line segment $[PR]$.

6. The two lines $\vec{r}_1 = \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 4 \\ 0 \end{pmatrix}$ and $\vec{r}_2 = \begin{pmatrix} 3 \\ -10 \\ -6 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -5 \\ -4 \end{pmatrix}$ intersect at point A . Find the coordinates of A .

7. A , B and C are points with position vectors $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$, $2\mathbf{j} + 2\mathbf{k}$ and $3\mathbf{i} - \mathbf{j} + \mathbf{k}$ respectively. Find a vector equation of each of the following lines:

- line through the origin parallel to \vec{AB} ;
- line through C parallel to \vec{AB} ;
- line through B and C .

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ANSWERS

$$1. \quad (a) \quad \vec{r} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ -5 \end{pmatrix}$$

other answers possible; position vector of known pt could be $\begin{pmatrix} -5 \\ 2 \end{pmatrix}$ & direction vector could be $\begin{pmatrix} -7 \\ 5 \end{pmatrix}$

$$(b) \text{ yes, when } \lambda = -2, \begin{pmatrix} 2 \\ -3 \end{pmatrix} - 2 \begin{pmatrix} 7 \\ -5 \end{pmatrix} = \begin{pmatrix} 2-14 \\ -3+10 \end{pmatrix} = \begin{pmatrix} -12 \\ 7 \end{pmatrix}$$

$$2. \quad a = -5, b = \frac{3}{2}$$

$$3. \quad 2x - y = -11 \quad [\text{ or } -2x + y = 11]$$

$$4. \quad (a) \quad \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$$

$$(b) \quad a = -15, c = 21$$

$$6. \quad A(0, 5, 6)$$

$$7. \quad (a) \quad \vec{r} = t \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$(b) \quad \vec{r} = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$(c) \quad \vec{r} = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -3 \\ -1 \end{pmatrix}$$