

Vectors – Equations of Lines

No calculator allowed on all exercises

- ♦ All of the exercises are suitable for both Maths SL & Maths HL. ♦
- **1.** (a) Find a vector equation, in the form $\vec{r} = \vec{a} + \lambda \vec{b}$, of the line passing through the points A(2, -3) and B(-5, 2).
 - (b) Does the point C(-12, 7) lie on the line AB? Explain.
- 2. A line passes through the point (-1, 4, 0) and is parallel to the vector $2\mathbf{i} 6\mathbf{j} + \mathbf{k}$. The point *P* with coordinates (2, a, b) lies on the line. Find the value of *a* and the value of *b*.
- **3.** A line passes through the point (-3, 5) and its direction is perpendicular to the vector $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$. Find the equation of the line in the form ax + by = c where *a*, *b* and *c* are integers to be determined.
- **4.** Consider the two lines L_1 and L_2 given as follows:

$$L_1: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -1 \\ -7 \end{pmatrix} \qquad \qquad L_2: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \\ -6 \end{pmatrix} + \mu \begin{pmatrix} -6 \\ 2 \\ -4 \end{pmatrix}$$

- (a) P is the point on L_1 when $\lambda = 1$. Find the position vector of P.
- (b) Show that P is also on L_2 .

(c) A third line,
$$L_3$$
, has a direction vector of $\begin{pmatrix} a \\ 3 \\ c \end{pmatrix}$. If L_1 and L_3 are parallel, find the value of a

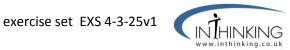
and the value of c.

- 5. Position vectors \vec{OP} and \vec{OR} are $2\mathbf{i} \mathbf{j} + 3\mathbf{k}$ and $6\mathbf{i} + 5\mathbf{j} + \mathbf{k}$ respectively. Show that the line $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} + 2\mathbf{k} + t(5\mathbf{i} 3\mathbf{j} + \mathbf{k})$ is a perpendicular bisector of the line segment [PR].
- 6. The two lines $\vec{r}_1 = \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 4 \\ 0 \end{pmatrix}$ and $\vec{r}_2 = \begin{pmatrix} 3 \\ -10 \\ -6 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -5 \\ -4 \end{pmatrix}$ intersect at point A. Find the

coordinates of A.

- **7.** *A*, *B* and *C* are points with position vectors $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$, $2\mathbf{j} + 2\mathbf{k}$ and $3\mathbf{i} \mathbf{j} + \mathbf{k}$ respectively. Find a vector equation of each of the following lines:
 - (a) line through the origin parallel to \vec{AB} ;
 - (b) line through C parallel to \vec{AB} ;
 - (c) line through B and C.

```
© InThinking – IB Maths HL & SL
```



Vectors – Equations of Lines

ANSWERS

1. (a)
$$\vec{r} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ -5 \end{pmatrix}$$

other answers possible; position vector of known pt could be $\begin{pmatrix} -5\\2 \end{pmatrix}$ & direction vector could be $\begin{pmatrix} -7\\5 \end{pmatrix}$

(b) yes, when
$$\lambda = -2$$
, $\begin{pmatrix} 2 \\ -3 \end{pmatrix} - 2 \begin{pmatrix} 7 \\ -5 \end{pmatrix} = \begin{pmatrix} 2 - 14 \\ -3 + 10 \end{pmatrix} = \begin{pmatrix} -12 \\ 7 \end{pmatrix}$

2. $a = -5, b = \frac{3}{2}$

3.
$$2x - y = -11$$
 [or $-2x + y = 11$]

$$4. (a) \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$$

(b)
$$a = -15, c = 21$$

A(0, 5, 6)6.

7. (a)
$$\vec{r} = t \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

(b)
$$\vec{r} = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

(c) $\vec{r} = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -3 \\ -1 \end{pmatrix}$