Solutions

Domain & Range of a Function

Q1) a) Restriction at $x = 6$

b) Restriction at $x = -9$

c) Restrictions are all $x > 12$

d) Restrictions are all $x < 8$

e) Let $(x + 2)(x - 6) = 0$

Giving $x = -2, x = 6$

So, restrictions occur at $x = -2, x = 6$

f) Let $x^2 - 3x + 2 = 0$

$(x - 2)(x - 1) = 0$

$x = 1, x = 2$

So, restrictions at $x = 1, x = 2$

Also, for $\sqrt{x}$, $x \geq 0$

So, restrictions are all $x < 0, x = 1, x$

Q2) a) Let $x^2 - 2x - 15 \geq 0$

$(x - 5)(x + 3) \geq 0$

$5 \leq x \leq -3$

Check elsewhere to learn how to solve quadratic inequations

So, the domain of f is all x; $5 \leq x \leq -3$

b) Let $x^2 - 7x + 12 = 0$

$(x - 3)(x - 4) = 0$

$x = 3, x = 4$

So, the domain of g is all $x; x \neq 3, x \neq 4$

c) Let $x^2 + 6x - 16 = 0$

$(x + 8)(x - 2) = 0$

$x = -8, x = 2$

So, the domain of f is all $x; x \neq -8, x \neq 2$

d) Let $3x - 2x^2 = 0$

$x(3 - 2x) = 0$

$x = 0, x = \frac{3}{2}$

So, the domain of h is all $x; x \neq 0, x \neq \frac{3}{2}$

Composite Functions

Q1) $f(x) = x - 2,\ g(x) = 4x - 4$

$f(g(x)) = f(4x - 4)$

$= 4x - 4 - 2 = 4x - 6$

Q2) $f(x) = x^2 - 1,\ g(x) = 3x - 2$

$f(g(x)) = f(3x - 2)$

$= (3x - 2)^2 - 1$

$= 9x^2 - 12x + 4 - 1$

$= 9x^2 - 12x + 3$

Q3) $f(x) = 3x + 4,\ g(x) = x^2$

$f(g(x)) = f(x^2)$

$= 3x^2 + 4$

Q4) $f(x) = 1 - 3x^2,\ g(x) = x + 1$

$f(g(x)) = f(x + 1)$

$= 1 - 3(x + 1)^2$

$= 1 - 3(x^2 + 2x + 1)$

$= 1 - 3x^2 - 6x - 3$

$= -3x^2 - 6x - 2$

Q5) $f(x) = 2x^2 - 4x + 5, g(x) = 3 - x$

$f(g(x)) = f(3 - x)$

$= 2(3 - x)^2 - 4(3 - x) + 5$

$= 2(9 - 6x + x^2) - 12 + 4x + 5$

$= 18 - 12x + 2x^2 - 12 + 4x + 5$

$= 2x^2 - 8x + 11$

Q6) $f(x) = x^2 + 1, g(x) = 3x - 4$

$f(g(x)) = f(3x - 4)$

$= (3x - 4)^2 + 1$

$= 9x^2 - 24x + 16 + 1$

$= 9x^2 - 24x + 17$

Inverse Functions

Q1) $f(x) = 6x - 2$

Let $y = 6x - 2$

$6x = y + 2$

$x = \frac{y + 2}{6}$

$f^{-1}(x) = \frac{x + 2}{6}$

Q2) $g(x) = 5 - 3x$

Let $y = 5 - 3x$

$3x = 5 - y$

$x = \frac{5 - y}{3}$

$g^{-1}(x) = \frac{5 - x}{3}$

Q3) $h(x) = \frac{1}{4}(x + 3)$

Let $y = \frac{1}{4}(x + 3)$

$4y = x + 3$

$x = 4y - 3$

$h^{-1} = (4x - 3)$

Q4) $f(x) = \frac{1}{4}(x^2 - 1)$

Let $y = \frac{1}{4}(x^2 - 1)$

$4y = x^2 - 1$

$x^2 = 4y + 1$

$x = \sqrt{4y + 1}$

$f^{-1}(x) = \sqrt{4x + 1}$

Q5) $h(x) = 4x^3 - 5$

Let $y = 4x^3 - 5$

$4x^3 = y + 5$

$x^3 = \frac{y+5}{4}$

$x = \sqrt[3]{\frac{y+5}{4}}$

$h^{-1}(x) = \sqrt[3]{\frac{x+5}{4}}$

Q6) $g(x) = \frac{\sqrt{2x+14}}{2}$

Let $y = \frac{\sqrt{2x+14}}{2}$

$2y = \sqrt{2x+14}$

$4y^2 = 2x + 14$

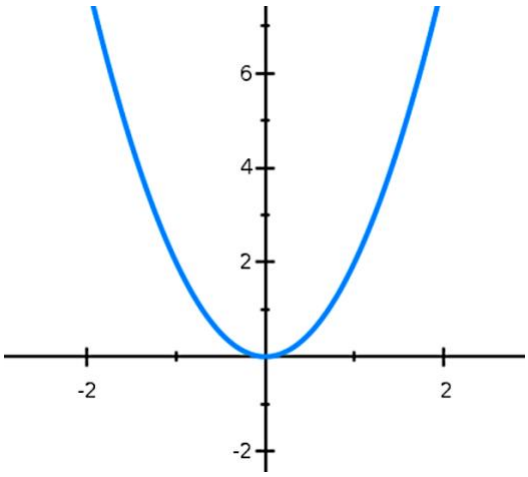
$2x = 4y^2 - 14$

$x = 2y^2 - 7$

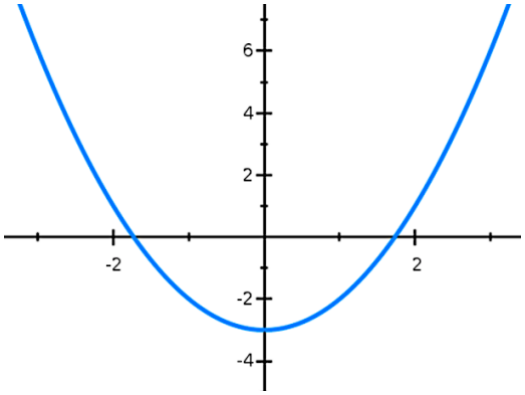
$g^{-1}(x) = 2x^2 - 7$

Graph Transformations

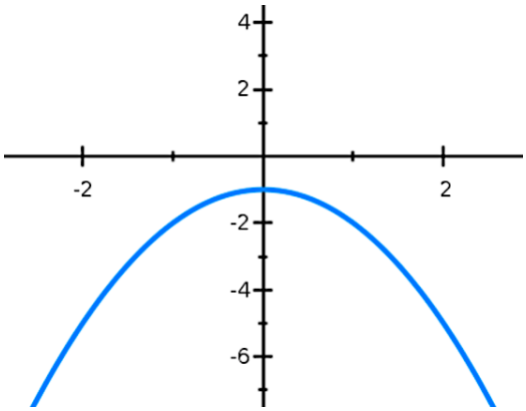
Q1) a) $y = 2f(x)$



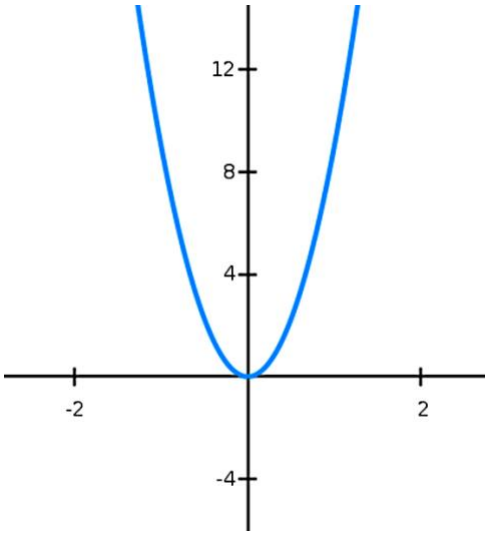
b) $y = f(x) - 3$



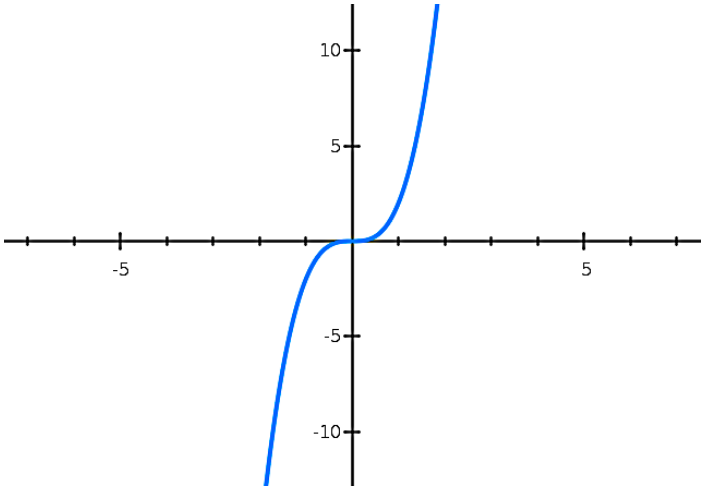
c) $y = -f(x) + 1$



d) $y = f(3x)$



Q2) a) $y = 2f(x)$



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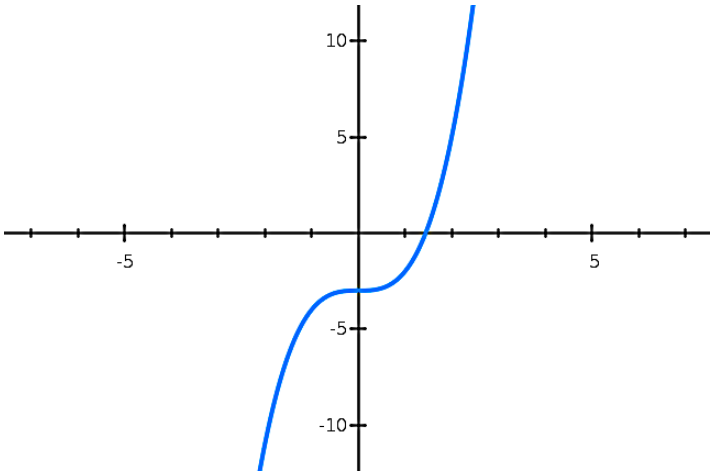

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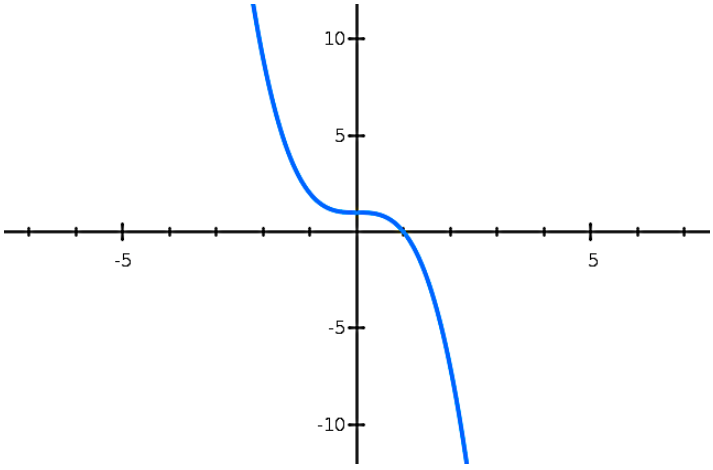

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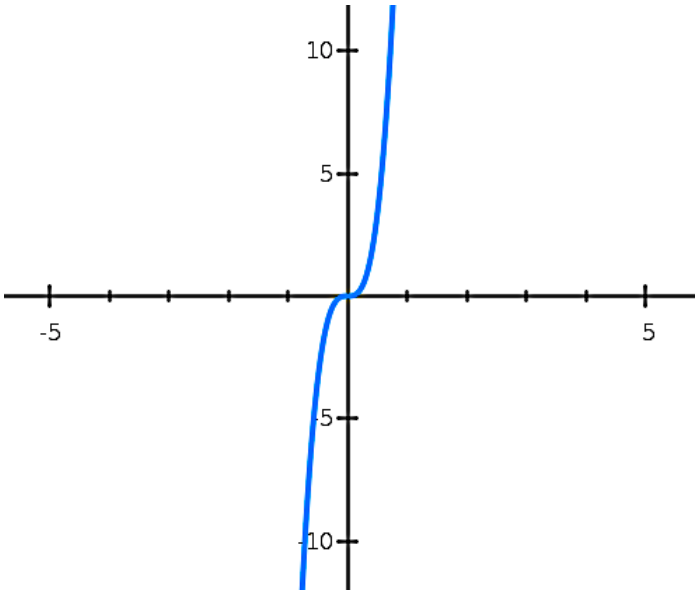
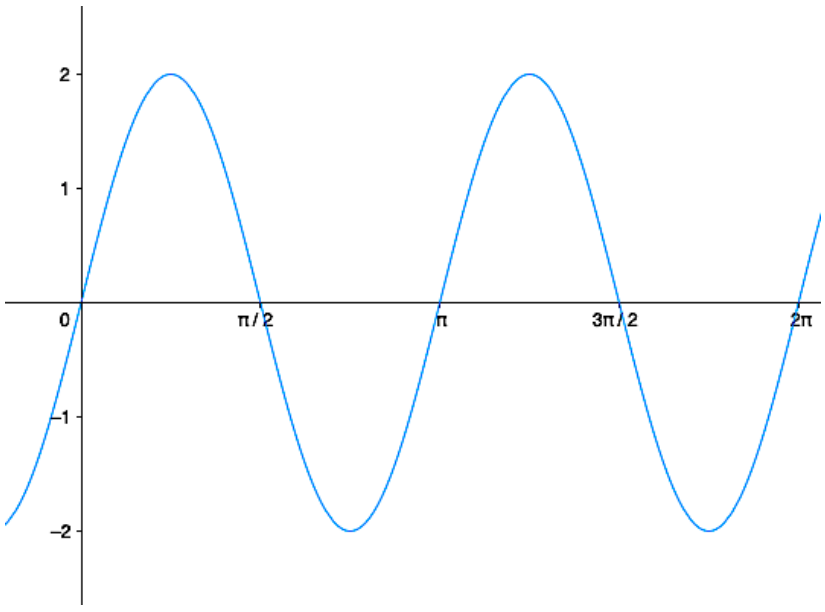
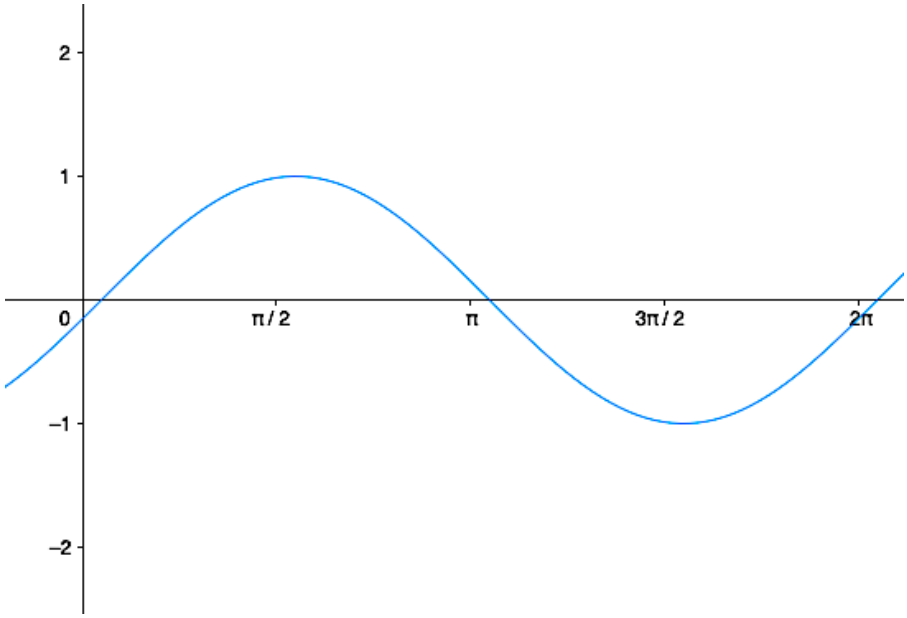


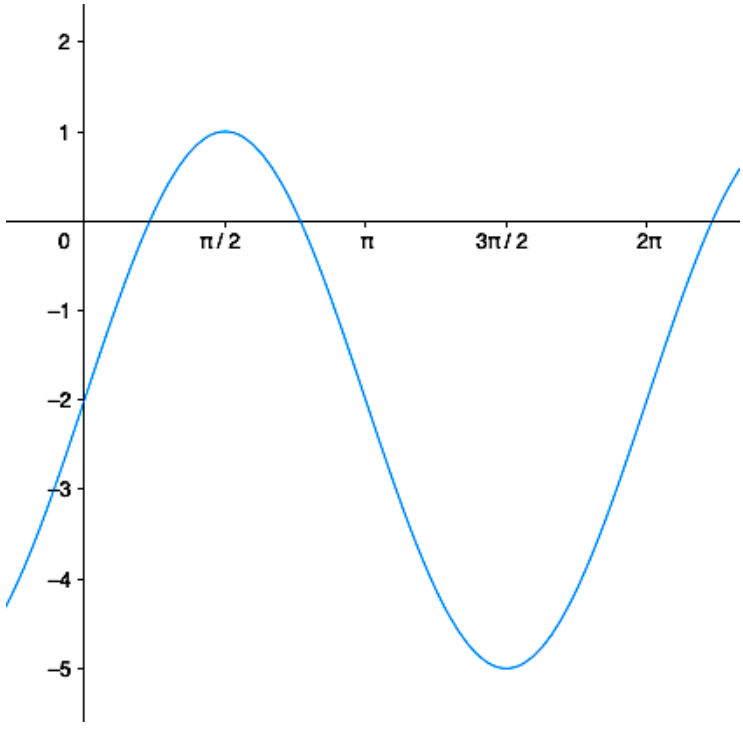
Q3) $y = 2\ sin\ 2x$



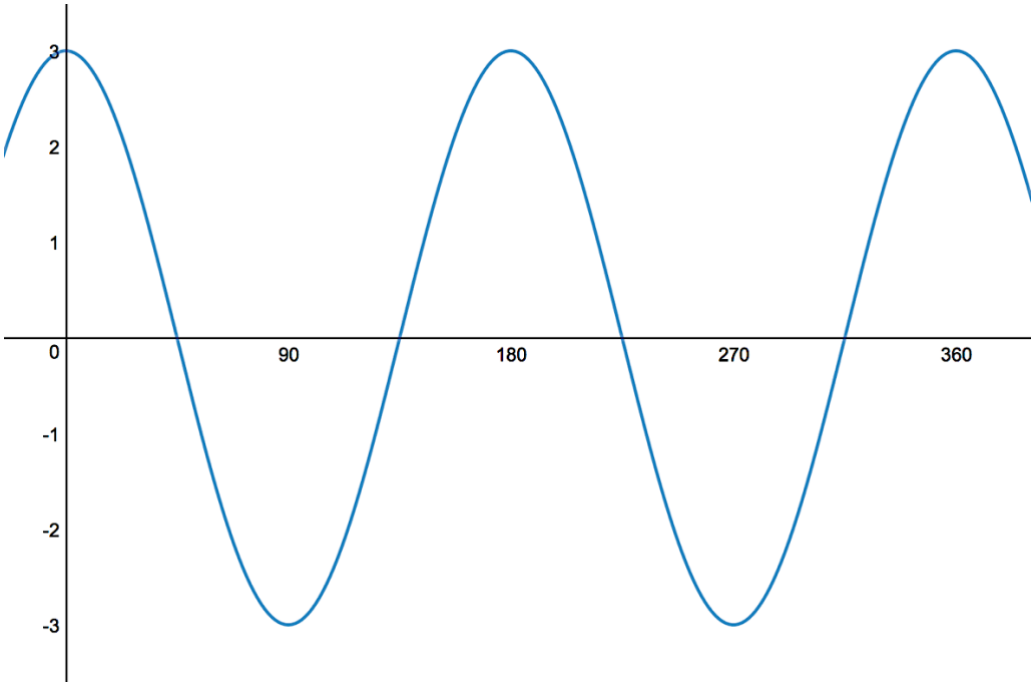
Q4) $y = -\cos(x - 30)°$



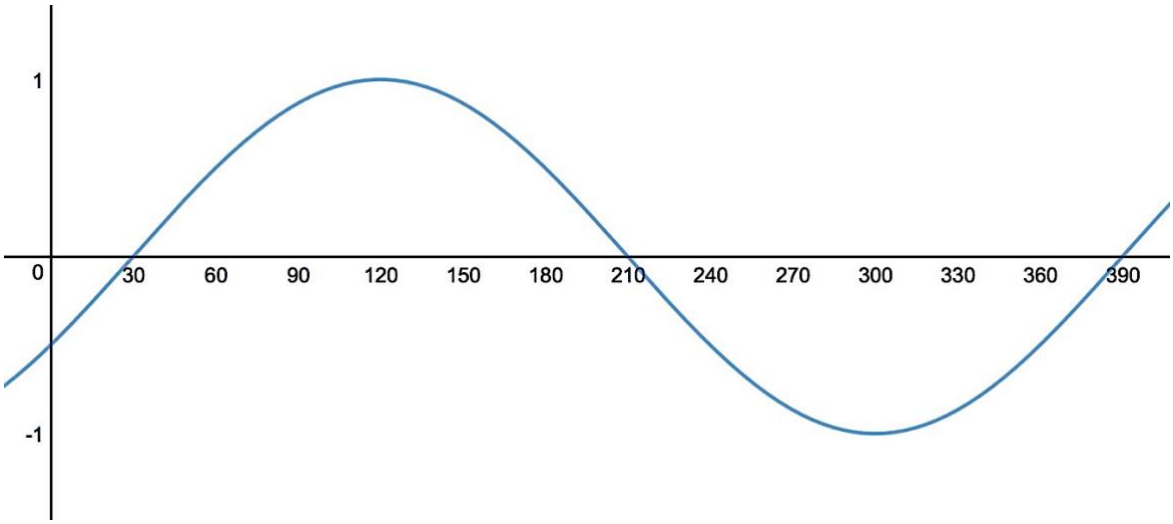
Q5) $y = 3\ sin\ x - 2$



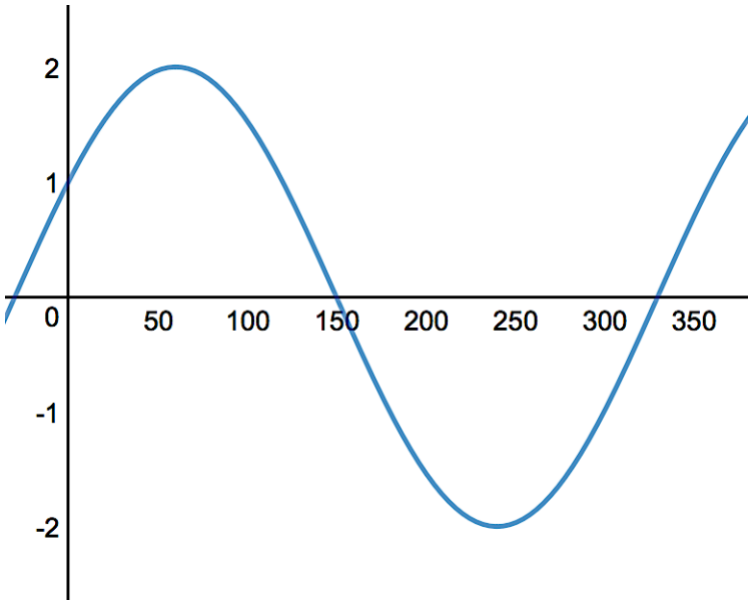
Q6) $a = 3, b = 2$, giving $y = 3\cos 2x$



Q7) $a = 30$ giving $y = \sin(x - 30)°$



Q8) $a = 2, b = 60$ giving $y = 2\cos(x - 60)°$



Q9) $a = 3, b = 1$ giving $y = 3\sin x + 1$