# NEW JERSEY CENTER
## FOR TEACHING & LEARNING

# PMI®

## Progressive Mathematics Initiative®

We, at the New Jersey Education Association, are proud founders and supporters of NJCTL, an independent non-profit organization with the mission of empowering teachers to lead school improvement for the benefit of all students.

**njea**

# 7th Grade

# Ratios and Proportions

**2017-09-26**

**www.njctl.org**

# Table of Contents

*Click on the topic to go to that section*

**Teacher Notes**

# Writing Ratios

# Ratios

What do you know about **ratios**?

When have you seen or used ratios?

# Ratios

**Ratio** - A comparison of two numbers by division

Ratios can be written three different ways:

$$a \; to \; b \qquad a:b \qquad \frac{a}{b}$$

Each is read, "the ratio of a to b."
Each ratio should be in simplest form.

Find the ratio of boys to girls in this class

# Ratios Video

Click for a ratios video

# Writing Ratios

There are 48 animals in the field. Twenty are cows and the rest are horses.

Write the ratio in three ways:

a. The number of cows to the number of horses

b. The number of horses to the number of animals in the field

Remember to write your ratios in simplest form!

1 There are 27 cupcakes. Nine are chocolate, 7 are vanilla and the rest are strawberry. What is the ratio of vanilla cupcakes to strawberry cupcakes?

○ A

$7:9$

○ B $\dfrac{7}{27}$

$\dfrac{7}{11}$

○ C

$1:3$

○ D

Remember to write your ratios in simplest form!

2   There are 27 cupcakes. Nine are chocolate, 7 are vanilla and the rest are strawberry. What is the ratio of chocolate & strawberry cupcakes to vanilla & chocolate cupcakes?

○ A

○ B   $\frac{20}{16}$
      $\frac{11}{7}$

○ C   $\frac{5}{4}$
      $\frac{16}{20}$

○ D

Remember to write your ratios in simplest form!

3   There are 27 cupcakes. Nine are chocolate, 7 are vanilla and the rest are strawberry. What is the ratio of chocolate cupcakes to total cupcakes?

○ A

            $\frac{7}{9}$

○ B   $\frac{7}{27}$

○ C   $\frac{9}{27}$

            $\frac{1}{3}$

○ D

Answer

4   There are 27 cupcakes. Nine are chocolate, 7 are vanilla and the rest are strawberry. What is the ratio of total cupcakes to vanilla cupcakes?

○ A   27 to 9

○ B   7 to 27

○ C   27 to 7

○ D   11 to 27

# Equivalent Ratios

# Equivalent Ratios

**Equivalent ratios** have the same value.

3 : 2  is equivalent to  6 : 4

1 to 3 is equivalent to 9 to 27

$\frac{5}{6}$  is equivalent to  $\frac{35}{42}$

# Equivalent Ratios

There are two ways to determine if ratios are equivalent.

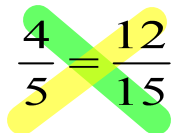1. Common Factor

$$\frac{4}{5} = \frac{12}{15}$$

x 3

$$\frac{4}{5} = \frac{12}{15}$$

x 3

Since the numerator and denominator were m ultiplied by the same value, the ratios are equivalent.

# Equivalent Ratios

2. Cross Products

$$\frac{4}{5} = \frac{12}{15}$$

Since the cross products are equal, the ratios are equivalent.

$$4 \times 15 = 5 \times 12$$

$$60 = 60$$

5

$\frac{4}{9}$ is equivalent to $\frac{8}{18}$

○ True

○ False

6

$\frac{5}{9}$ is equivalent to $\frac{30}{54}$

○ True

○ False

**7**

$18:12$ is equivalent to $\frac{9}{6}$, which is equivalent to $\frac{36}{24}$

○ True

○ False

8

$1 : 7$ is equivalent to $\frac{10}{70}$ , which is equivalent to $5$ to $65$

○ True

○ False

# Rates

# Rates Video

Click for video

# Rates

**Rate**: a ratio of two quantities measured in different units

Examples of rates:

4 participants/2 teams

5 gallons/3 rooms

8 burgers/2 tomatoes

# Unit Rates

**Unit rate**: Rate with a denominator of one
Often expressed with the word "per"

Examples of unit rates:

34 miles/gallon

2 cookies per person

62 words/minute

# Finding a Unit Rate

Six friends have pizza together. The bill is $63.
What is the cost per person?

Hint: Since the question asks for cost per person,
the cost should be first, or in the numerator.

click

Since unit rates always have a denominator of one,
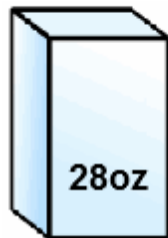rewrite the rate so that the denominator is one.

click

# Click for Practice

## Unit Price Game

*Are you getting **Value For Money**?*

The "Unit Price" tells you the cost per liter, per kilogram, per pound, etc, of what you want to buy.

It is a good way of comparing costs. To help you become expert at calculating Unit Prices we have this game for you:

**28oz**

**$1.95**

$1.57 per oz

$0.14 per oz

$2.07 per oz

$0.07 per oz

Score

0

9  Sixty cupcakes are at a party for twenty children. How many cupcakes per person?

10  John's car can travel 94.5 miles on 3 gallons of gas. How many miles per gallon can the car travel?

11 The snake can slither 240 feet in half a day. How many feet can the snake move in an hour?

12 There are five chaperones at the dance of 100 students. How many students per chaperone are there?

13  The recipe calls for 6 cups of flour for every four eggs. How many cups of flour are needed for one egg?

14 Sarah rode her bike $14\frac{1}{4}$ miles in $\frac{3}{4}$ hour.

What is Sarah's unit rate in miles per hour?

15  An airplane's altitude changed -378 feet over 7 minutes. What was the mean change of altitude in feet per minute?

Answer

16  A $4\frac{1}{2}$ -ounce hamburger patty has $25\frac{1}{2}$ grams of protein, and 6 ounces of fish has 32 grams of protein. Determine the grams of protein per ounce for each type of food.

A hamburger patty has approximately _____ grams of protein per ounce.  The fish has approximately _____ grams of protein per ounce.

- ☐ 0.2
- ☐ 4.5
- ☐ 5.7
- ☐ 21.0
- ☐ 25.5

- ☐ 0.2
- ☐ 5.3
- ☐ 6.0
- ☐ 26.0
- ☐ 32.0

Answer

17  Rosy waxes $\frac{2}{3}$ of her car with $\frac{1}{4}$ bottle of car wax.

At this rate, what fraction of the bottle of car wax will Rosy use to wax her entire car?

**Answer**

# Compare Rates

We often use unit rates to easily compare rates.

Example:
Sebastian and Alexandra both work during the summer. Sebastian worked 26 hours one week and earned $188.50 before taxes. Alexandra worked 19 hours and earned $128.25 before taxes. Who earns more per hour at their job?

Sebastian                                    Alexandra

click

# Compare Rates

Jim traveled 480 miles on a full tank of gas.  His gas tank holds 15 gallons.

Tara traveled 540 miles on a full tank of gas.  Her gas tank holds 18 gallons.

Which person's car gets better gas mileage?

Jim                                             Tara

click

18  Tahira and Brendan going running at the track.  Tahira
runs 3.5 miles in 28 minutes and Brendan runs 4 miles
in 36 minutes.  Who runs at a faster pace (miles per
hour)?

Show your work!

○ A  Tahira

○ B  Brendan

19  Red apples cost $3.40 for ten.
Green apples cost $2.46 for six.
Which type of apple is cheaper per apple?

Show your work!

⭕A  Tahira

⭕B  Brendan

20  Fruity Oats is $2.40 for a 12 oz. box.
Snappy Rice is $3.52 for a 16 oz. box.
Which cereal is cheaper per ounce?

Show your work!

○ A  Fruity Oats

○ B  Snappy Rice

21 Two families drive to their vacation spot. The Jones family drives 432 miles and used 16 gallons of gas. The Alverez family drives 319 miles and uses 11 gallons of gas. Which family got more miles per gallon of gas?

Show your work!

○ A  Jones Family

○ B  Alverez Family

22  Mariella typed 123 words in 3 minutes.
Enrique typed 155 words in 5 minutes.
Who typed more words per minute?

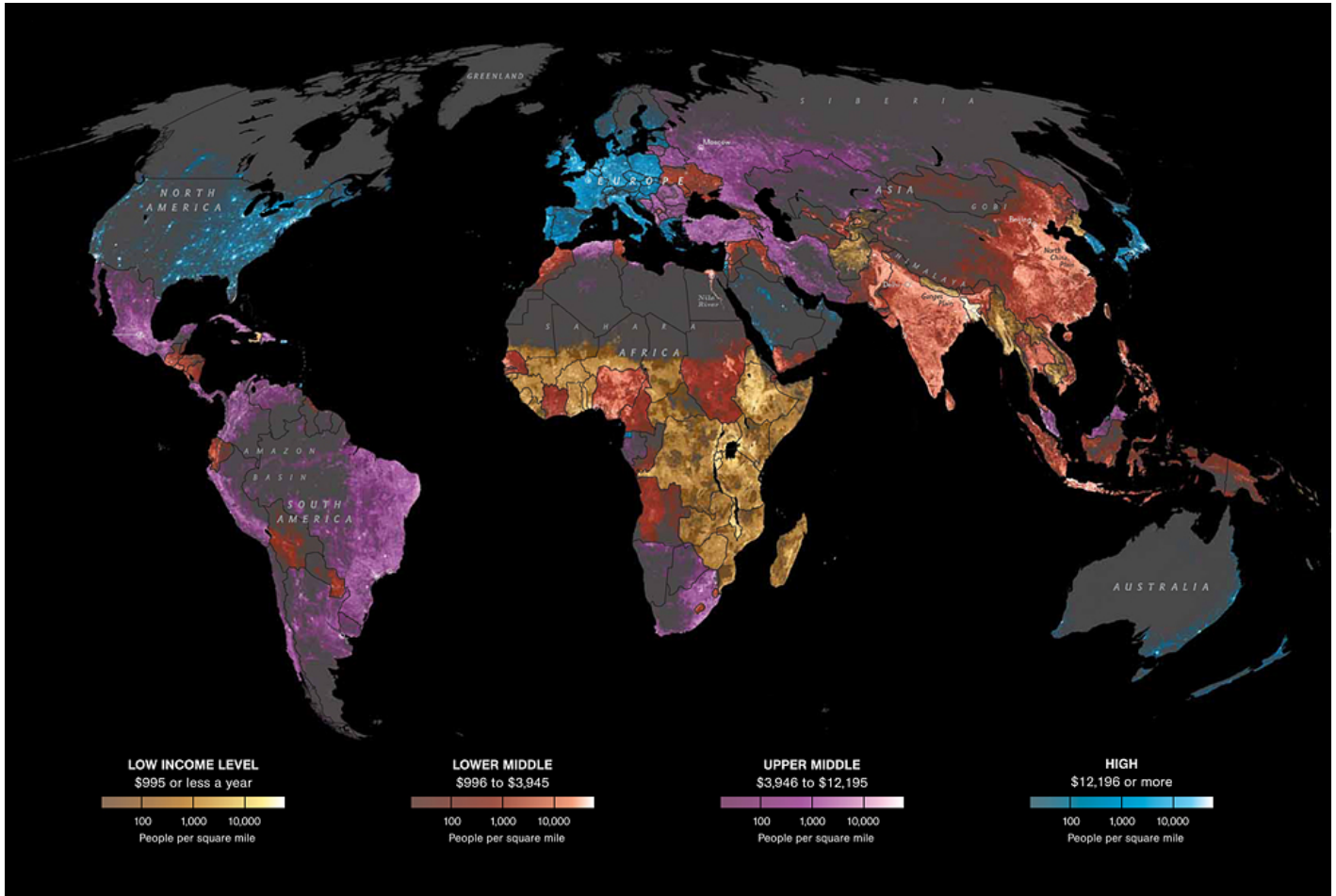Show your work!

○A  Mariella

○B  Enrique

# Population Density

**Population Density**:  A unit rate of people per square mile

This data is compiled by the US Census Bureau every 10 years and is used when determining the number of Representatives each state gets in the House of Representatives.

# Population Density



Legend:
- 0-20
- 20-50
- 50-100
- 100-200
- 200-500
- 500-1000
- 1000+

Population per square mile by state. 2000 census figures.
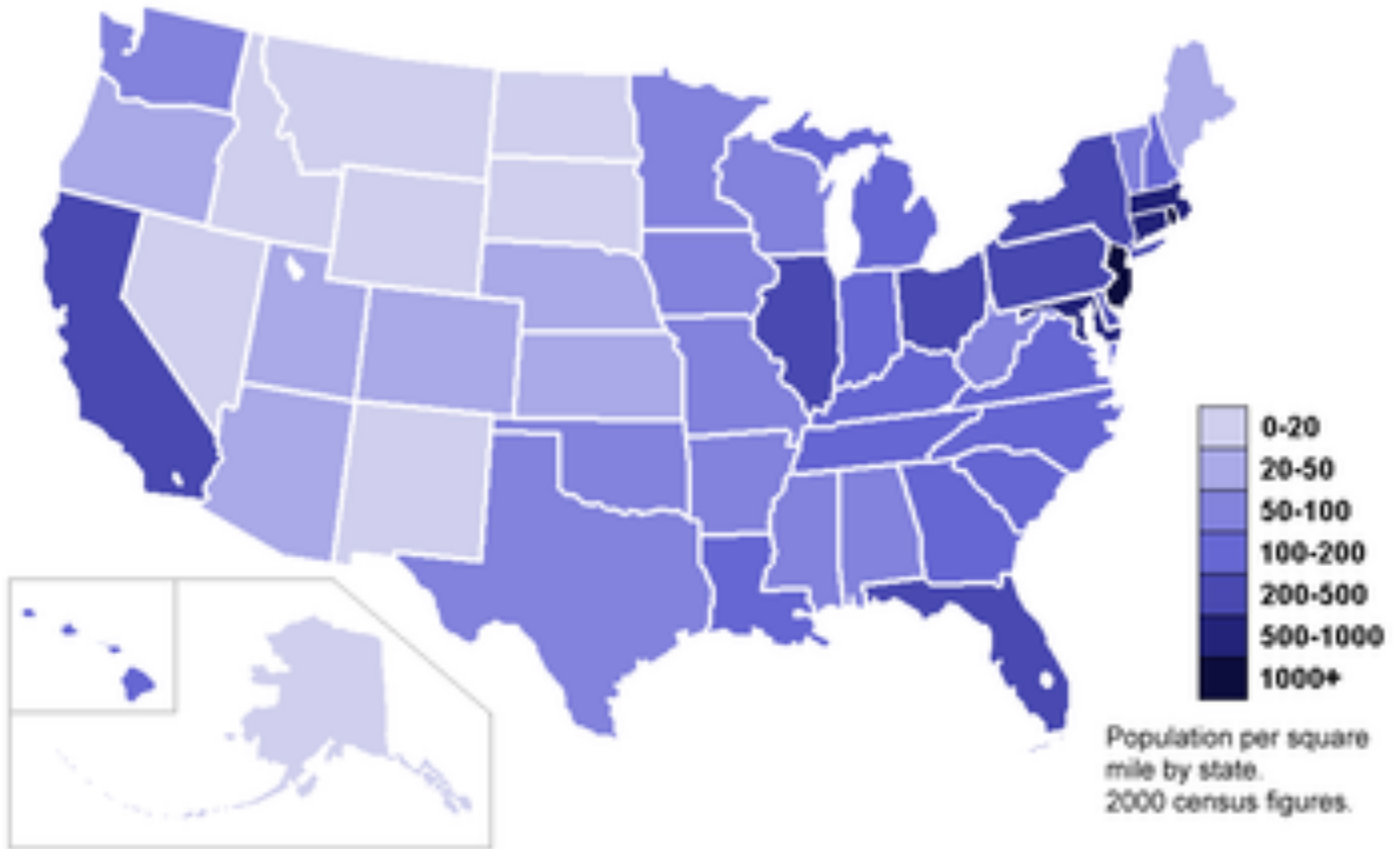
# Click for National Geographic Web Site



LOW INCOME LEVEL
$995 or less a year

100    1,000    10,000
People per square mile

LOWER MIDDLE
$996 to $3,945

100    1,000    10,000
People per square mile

UPPER MIDDLE
$3,946 to $12,195

100    1,000    10,000
People per square mile

HIGH
$12,196 or more

100    1,000    10,000
People per square mile

# Population Density

To calculate population density:

·   Find the population of the state.
 NJ = 8,791,894 people

·   Find the area of the state.
 NJ = 7,790 square miles

·   Divide

$$\frac{Population}{Area} \quad = \quad \frac{8,791,894}{7,790} \quad = \quad 1,129 \; people \; per \; square \; mile$$

# Population Density

We know that New Jersey has a population density of 1,129 people per square mile. Use the links below to compare this data with two other states.

Population Density =

Click the map for population data

$$\frac{\text{Population}}{\text{Area}}$$

Click the table for area data



| Rank | State Name | Area (Sq Miles) |
|------|------------|-----------------|
| 1 | Alaska | 587,878 |
| 2 | Texas | 266,874 |
| 3 | California | 158,648 |
| 4 | Montana | 147,047 |
| 5 | New Mexico | 121,599 |
| 6 | Arizona | 114,007 |
| 7 | Nevada | 110,567 |
| 8 | Colorado | 104,100 |
| 9 | Wyoming | 97,818 |
| 10 | Oregon | 97,052 |
| 11 | Utah | 84,905 |
| 12 | Minnesota | 84,397 |
| 13 | Idaho | 83,574 |
| 14 | Kansas | 82,282 |
| 15 | Nebraska | 77,359 |
| 16 | South Dakota | 77,122 |

23  The population of Newark, NJ is 278,980 people in 24.14 square miles. What is its population density?



Newark, NJ

24  The population of Moorestown, NJ is 19,509 people in 15 square miles.  What is its population density?



**Moorestown, NJ**

25 The population of Waco, TX is 124,009 people in 75.8 square miles. What is its population density?

Texas



Waco

26  The population of Argentina is 40,091,359 people and Argentina is 1,042,476 square miles. What is the population density?

27 The population of Argentina is 40,091,359 people and Argentina is 1,042,476 square miles. What is the population density?



San Luis, Argentina

# Proportions

# Proportions

A **proportion** is an equation that states that two ratios are equivalent.

Example:

$$\frac{2}{3} = \frac{12}{18}$$

$$\frac{5}{9} = \frac{15}{27}$$

# Proportions

If one of the numbers in a proportion is unknown, mental math can be used to find an equivalent ratio.

Example 1:

$$\frac{2}{3} = \frac{6}{x}$$

x 3

$$\frac{2}{3} = \frac{6}{x}$$

$$\frac{2}{3} = \frac{6}{9}$$
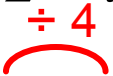
Hint: To find the value of x, multiply 3 by 3 also.

x 3

# **Proportions**

If one of the numbers in a proportion is unknown, mental math can be used to find an equivalent ratio.

Example:

$$\frac{28}{32} = \frac{7}{x}$$

$\div 4$

$$\frac{28}{32} = \frac{7}{x}$$

$$\frac{28}{32} = \frac{7}{8}$$

$\div 4$

Hint: To find the value of x, divide 32 by 4 also.

28  Solve the proportion using equivalent ratios.

$$\frac{2}{5} = \frac{8}{x}$$

29  Solve the proportion using equivalent ratios.

$$\frac{4}{9} = \frac{x}{36}$$

30  Solve the proportion using equivalent ratios.

$$\frac{7}{2} = \frac{35}{x}$$

31  Solve the proportion using equivalent ratios.

$$\frac{x}{60} = \frac{4}{12}$$

32  Solve the proportion using equivalent ratios.

$$\frac{3}{x} = \frac{21}{28}$$

# Proportion

In a proportion, the cross products are equal.

$$\frac{5}{2} \diagdown \frac{30}{12}$$

$$5 \cdot 12 - 2 \cdot 30$$

$$60 - 60$$

# Cross Products

Proportions can also be solved using cross products.

$$\frac{4}{5} \diagdown\diagup \frac{12}{x}$$

Cross multiply

$$4x = 5 \cdot 12$$

$$4x = 60$$

Solve for x

$$x = 15$$

# Cross Products

Example 2

$$\frac{7}{8} \diagup \frac{x}{48}$$

Cross multiply

$$7 \cdot 48 = 8x$$

$$336 = 8x$$

Solve for x

$$42 = x$$

33  Use cross products to solve the proportion.

$$\frac{9}{51} = \frac{x}{17}$$

34  Use cross products to solve the proportion.

$$\frac{x}{12} = \frac{56}{96}$$

35  Use cross products to solve the proportion.

$$\frac{45}{18} = \frac{x}{6}$$

36  Use cross products to solve the proportion.

$$\frac{2}{15} = \frac{x}{60}$$

37 Use cross products to solve the proportion.

$$\frac{7}{x} = \frac{3}{21}$$

38  Today, Joelle walked 20 minutes at a rate of 3 miles per hour, and she ran 15 minutes at a rate of 6 miles per hour.

Part A

How many total miles did Joelle travel while walking and running?

Answer

39  (Continued from previous slide.)

Part B

Tomorrow, Joelle wants to travel a total of 4 miles by walking and running. She plans to run for 20 minutes at a rate of 6 miles per hour.

How many minutes should she walk at a rate of 3 miles per hour to finish traveling the 4 miles?

**Answer**

40  The directions on a bottle of vinegar say, "mix 1 cup of vinegar with 1 gallon of water to make a cleaning solution." The ratio of vinegar to water is 1 to 16.

Part A

How many cups of water should be mixed with $\frac{1}{4}$ cup of vinegar to make the cleaning solution?

**Answer**

41  (Continued from previous slide.)

Part B

How many fluid ounces of vinegar should be mixed with 80 ounces of water to make the cleaning solution?

**From PARCC Sample Test - Calculator**

42  (Continued from previous slide.)

Part C

The bottle contains 1 quart of vinegar. What is the total number of quarts of cleaning solution that can be made using the entire bottle of vinegar?

**Answer**

43 (Continued from previous slide.)

Part D

A spray bottle holds up to 1 cup of the cleaning solution.

When the spray bottle is full, what fraction of the cleaning solution is vinegar?

**Answer**

# Direct & Indirect Relationships in Tables & Graphs

# Proportional Relationships

You can determine if a relationship is proportional by looking at a table of values or the graph.

How?

<u>Table</u>
If all the ratios of numbers in the table are equivalent, the relationship is proportional.

<u>Graph</u>
If the graph of the numbers forms a straight line through the origin (0,0), the relationship is proportional.

# Tables & Proportions

On a field trip, every chaperone is assigned 12 students.  Is the student to chaperone ratio proportional?

If you use a table to demonstrate, you would need several ratios to start.

| Chaperones | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Students | Click | Click | Click | Click | Click |

Next, find the simplified ratios and compare them.  Are they the same?

click to
reveal

# Tables & Proportions

Try this:

The local pizza place sells a plain pie for $10.  Each topping costs an additional $1.50.  Is the cost of pizza proportional to the number of toppings purchased?

| Toppings | 1 | 2 | 3 | 4 |
|----------|-----|-----|-----|-----|
| Cost | 11.50 | 13.00 | 14.50 | 16.00 |

click to reveal

44  Is the relationship shown in the table proportional?

○ Yes

○ No

| Year | 1 | 2 | 4 | 5 |
|---|---|---|---|---|
| Income | $22,000 | $44,000 | $88,000 | $110,000 |

45  Is the relationship shown in the table proportional?

○ Yes

○ No

| x | 2 | 5 | 6 | 9 |
|---|---|---|---|---|
| y | 7 | 17.5 | 21 | 34.5 |

46  Is the relationship shown in the table proportional?

○ Yes

○ No

| x | 1 | 2 | 6 | 9 |
|---|---|---|---|---|
| y | 5 | 11 | 31 | 46 |

47  Is the relationship shown in the table proportional?

○ Yes

○ No

| x | 1 | 2 | 4 | 7 |
|---|---|---|---|---|
| y | 4 | 8 | 16 | 35 |

48  Is the relationship shown in the table proportional?

○ Yes

○ No

| x | 2 | 4 | 6 | 8 |
|---|---|---|---|---|
| y | -3 | -10 | -15 | -20 |

# Proportional Relationships

Remember:

Table
If all the ratios of numbers in the table are equivalent, the relationship is proportional.


Graph
If the graph of the numbers forms a straight line through the origin (0,0), the relationship is proportional.

# Graphs & Proportions

On a field trip, every chaperone is assigned 12 students. Is the student to chaperone ratio proportional?

| Chaperones | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Students | 12 | 24 | 36 | 48 | 60 |

Connected points form a straight line

Line crosses through the origin

Since the graph is a straight line through the origin, the relationship is proportional.
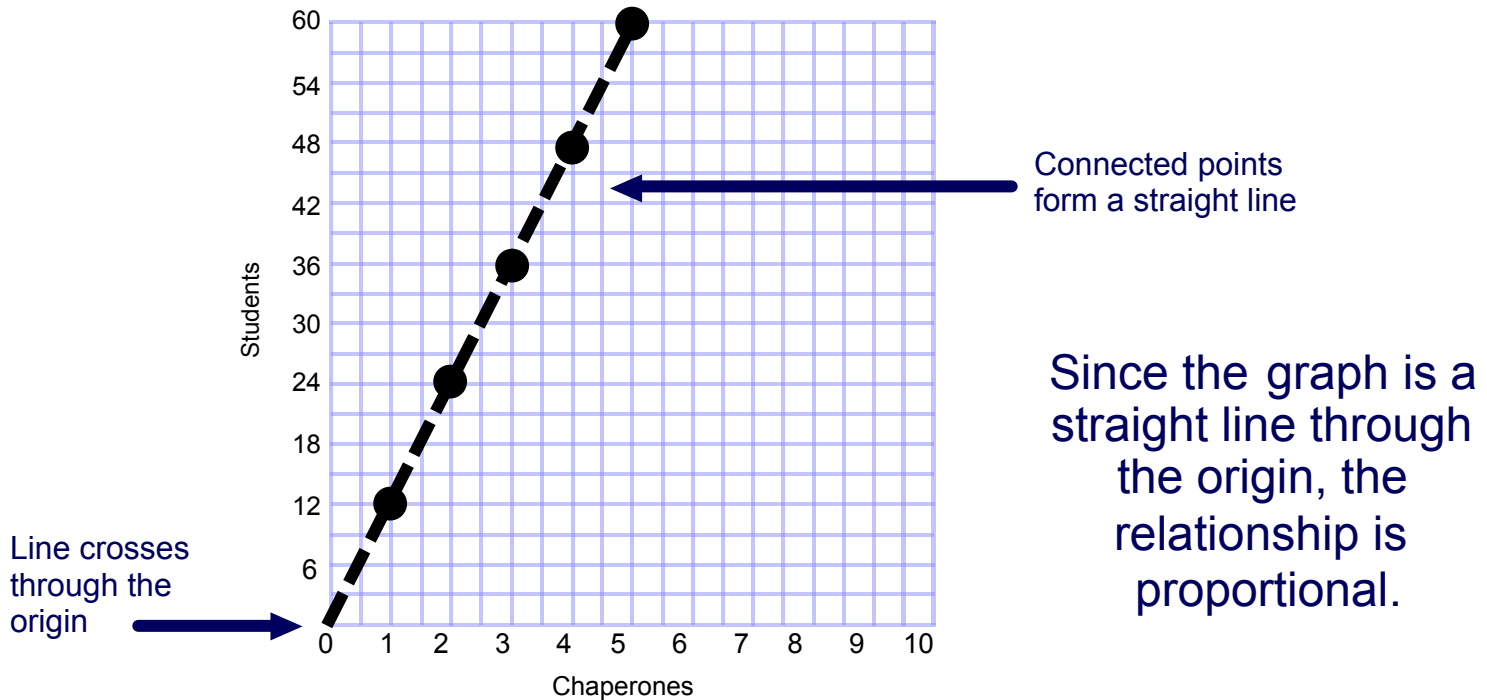
# Graphs & Proportions

Draw a graph to represent the relationship.  Is the relationship proportional?

Click for answer

| X | Y |
|---|---|
| 1 | 5.5 |
| 2 | 7 |
| 3 | 8.5 |
| 4 | 10 |
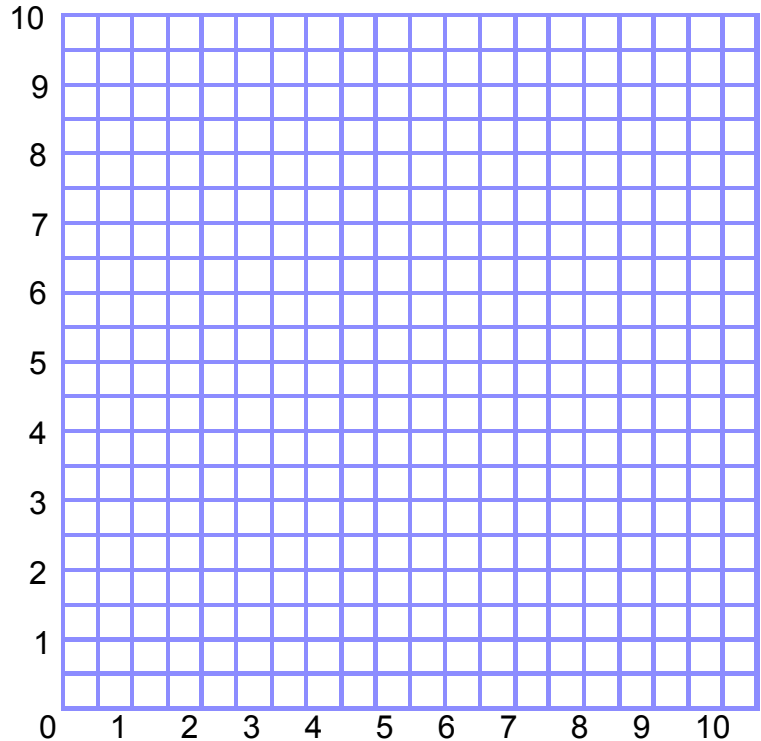
**49** Is the relationship shown on the graph proportional?

○ Yes

○ No
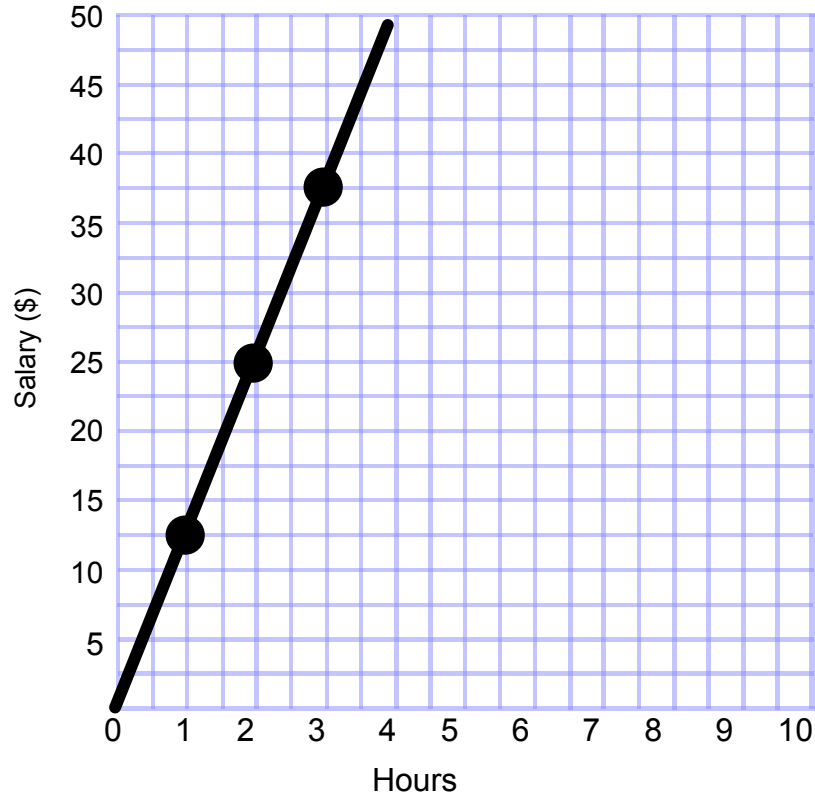
# 50 Is the relationship shown on the graph proportional?

○ Yes

○ No
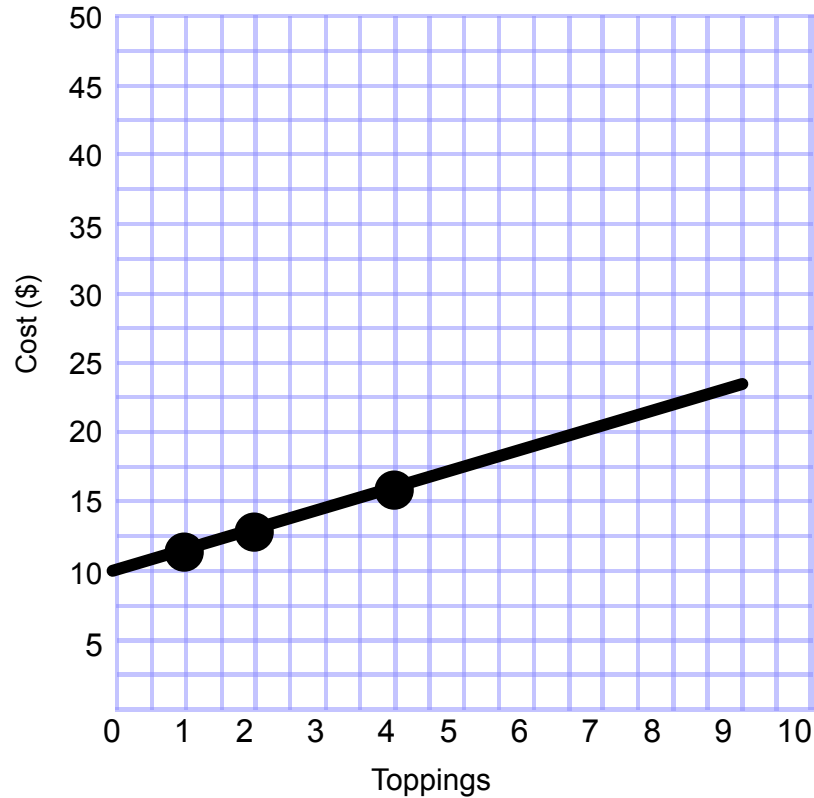
51  Is the relationship shown on the graph proportional?

○ Yes

○ No
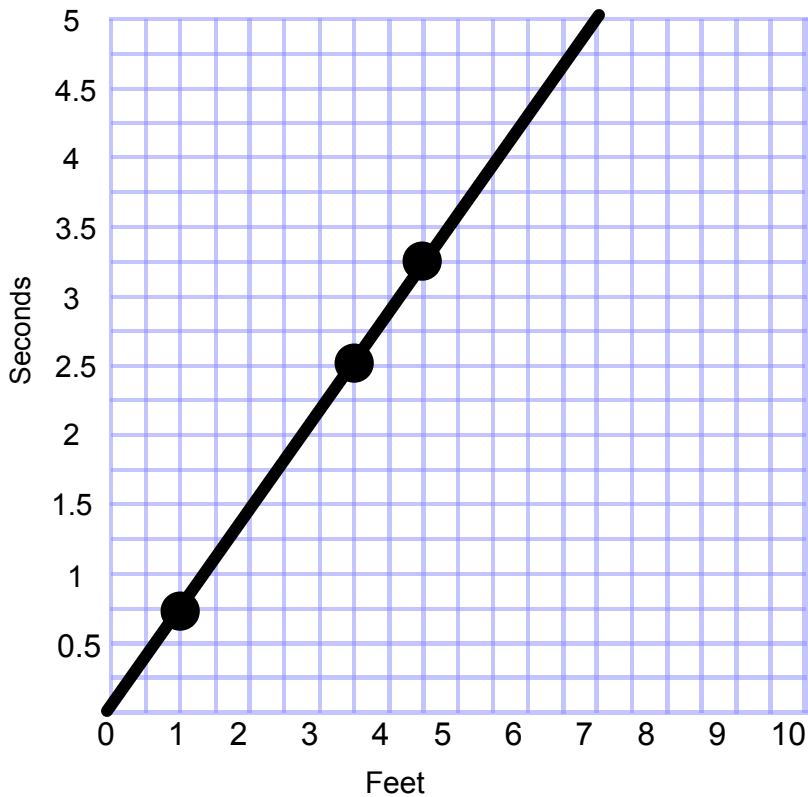
52  Is the relationship shown on the graph proportional?

○ Yes

○ No
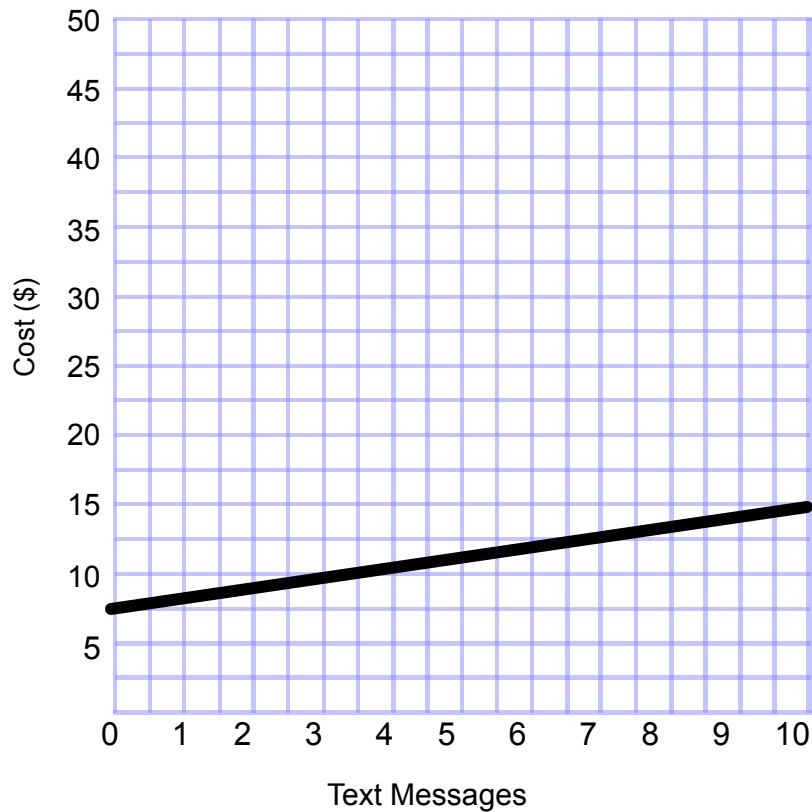
## 53 Is the relationship shown on the graph proportional?

○ Yes

○ No
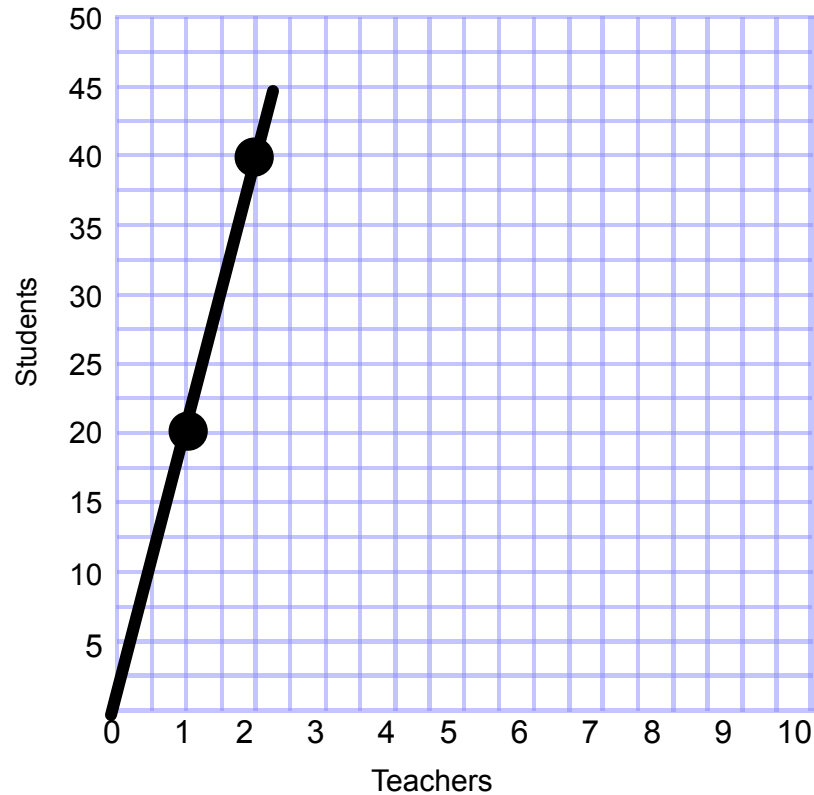
**54** The graph shows the distance in miles, *d,* a car travels in *t* hours.

**Car Travels**
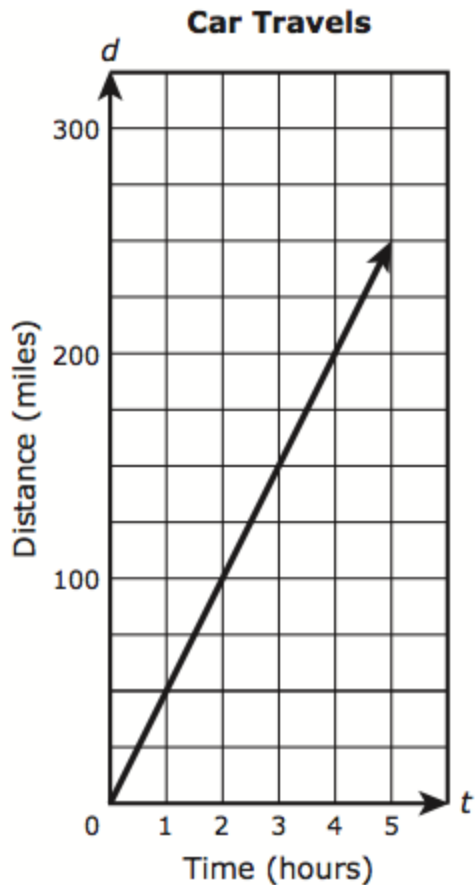


Part A

Explain why the graph does or does not represent a proportional relationship between the variables *d* and *t.*

Answer

**55** (Continued from previous slide.)

Part B

Two cars leave from the same city at the same time and drive in the same direction. The table shows the distances traveled by each car.

**Two Cars Travel**

| Hours of Travel | Miles Traveled by Red Car | Miles Traveled by White Car |
|---|---|---|
| 1 | 77 | 55 |
| 2 | 122 | 110 |
| 3 | 167 | 165 |
| 4 | 212 | 220 |
| 5 | 257 | 275 |

Determine whether the relationship between the number of hours traveled and the number of miles traveled is proportional for each car.  (Use the table to explain how you determined your answers. Describe how the graph of the distance traveled by each car would support your answers.)

**From PARCC Sample Test - Calculator**

# Constant of Proportionality

# Constant of Proportionality

The **constant of proportionality** is a constant ratio (unit rate) in any proportional relationship.

We use the letter $k$ to represent the constant of proportionality.

Equations:

$$y = kx \quad \text{or} \quad k = \frac{y}{x}$$

# Constant of Proportionality

We can find the constant of proportionality from a table of values, equation and a graph.

In a table, simplify any one of the ratios.

| Chaperones | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Students | 12 | 24 | 36 | 48 | 60 |

$$k = \frac{y}{x} = \frac{36}{3} = 12$$

# Constant of Proportionality

Find the constant of proportionality:

| Apples (lbs) | 2 | 2.5 | 3 | 3.5 | 4 |
|:---:|:---:|:---:|:---:|:---:|:---:|
| Cost ($) | 3.96 | 4.95 | 5.94 | 6.93 | 7.92 |

Click

# Constant of Proportionality

Find the constant of proportionality:

| X | Y |
|---|---|
| 3 | 4.5 |
| 4 | 6 |
| 5 | 7.5 |
| 8 | 12 |
| 9 | 13.5 |

Click

56  Find the constant of proportionality.

| X | Y |
|---|---|
| 2 | 1.5 |
| 5 | 3.75 |
| 10 | 7.5 |
| 12 | 9 |

57  Find the constant of proportionality.

| X | Y |
|---|---|
| 2 | 2.5 |
| 3 | 3.75 |
| 4 | 5 |
| 9 | 11.25 |

58 Find the constant of proportionality.

| X | Y |
|-----|-----|
| 50 | 3 |
| 75 | 4.5 |
| 100 | 6 |
| 140 | 8.4 |

59 This table shows a proportional relationship between x and y.

| x | y |
|---|---|
| 2 | 1.25 |
| 4 | 2.5 |
| 6 | 3.75 |
| 10 | 6.25 |

What is the constant of proportionality between x and y? Type your answer as a decimal.

Answer

**From PARCC Sample Test - Non-calculator**

# Constant of Proportionality

In an equation, write the equation in the form y = kx.

Examples:

$$y = 5$$

Click

$$y = \frac{1}{4}x$$

Click

$$y = 3.5x$$

Click

# Constant of Proportionality

Find the constant of proportionality:

$$y = 3.6x$$

Click

$$y = 53x$$

Click

$$y = 0.38x$$

Click

$$y = \frac{3}{8}x$$

Click

$$y = \frac{2}{3}x$$

Click

$$y = 1.85x$$

Click

60 Find the constant of proportionality.

$$y = \frac{1}{9}x$$

61 Find the constant of proportionality.

$$y = 12.9x$$

62  Find the constant of proportionality.

$y = 0.45x$

63  Which equation has a constant of proportionality equal to 4?

A  $4y = 4x$

B  $4y = 12x$

C  $3y = 4x$

D  $3y = 12x$

**Answer**

64  A worker has to drive her car as part of her job. She receives money from her company to pay for the gas she uses. The table shows a proportional relationship between y, the amount of money that the worker received, and x, the number of work-related miles driven.

**Mileage Rates**

| Distance Driven, x (miles) | Amount of Money Received, y (dollars) |
|---|---|
| 25 | 12.75 |
| 35 | 17.85 |
| 40 | 20.40 |
| 50 | 25.50 |

Part A

Explain how to compute the amount of money the worker receives for any number of work-related miles. Based on your explanation, write an equation that can be used to determine the total amount of money, y, the worker received for driving x work-related miles.

**From PARCC Sample Test - Calculator**

65  (Continued from previous slide.)

Part B

On Monday, the worker drove a total of 134 work-related and personal miles, She received $32.13 for the work-related miles she drove on Monday. What percent of her total miles driven were work-related on Monday? Show or explain your work.

**Answer**

# Constant of Proportionality

In a graph, choose a point (x, y) to find and simplify the ratio.

(2, 24)

$$k = \frac{y}{x} = \frac{24}{2} = 12$$

Students (y-axis)

Chaperones (x-axis)
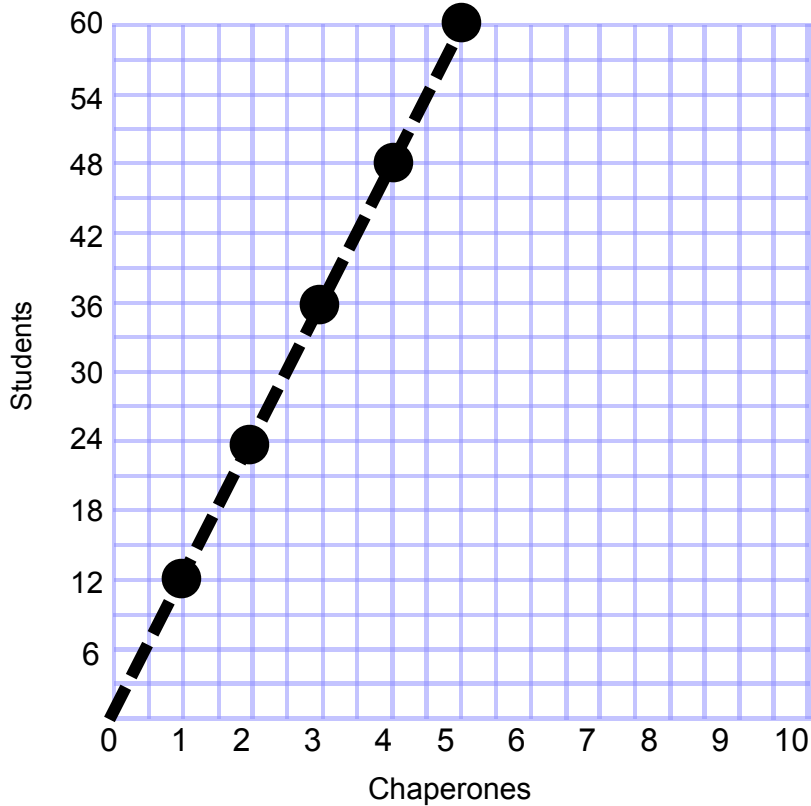
# Constant of Proportionality

Find the constant of proportionality.
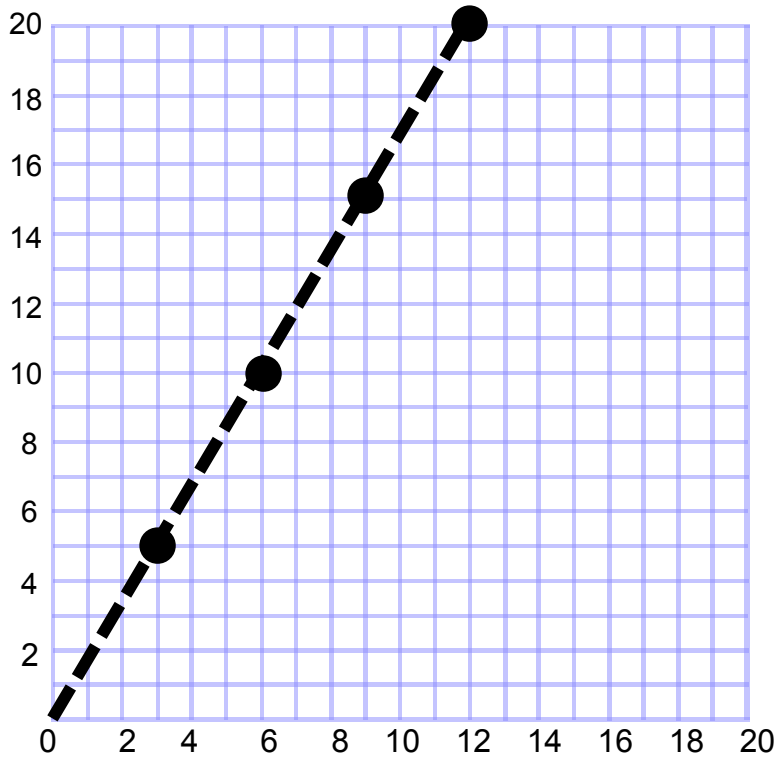


Click

**66** Find the constant of proportionality.
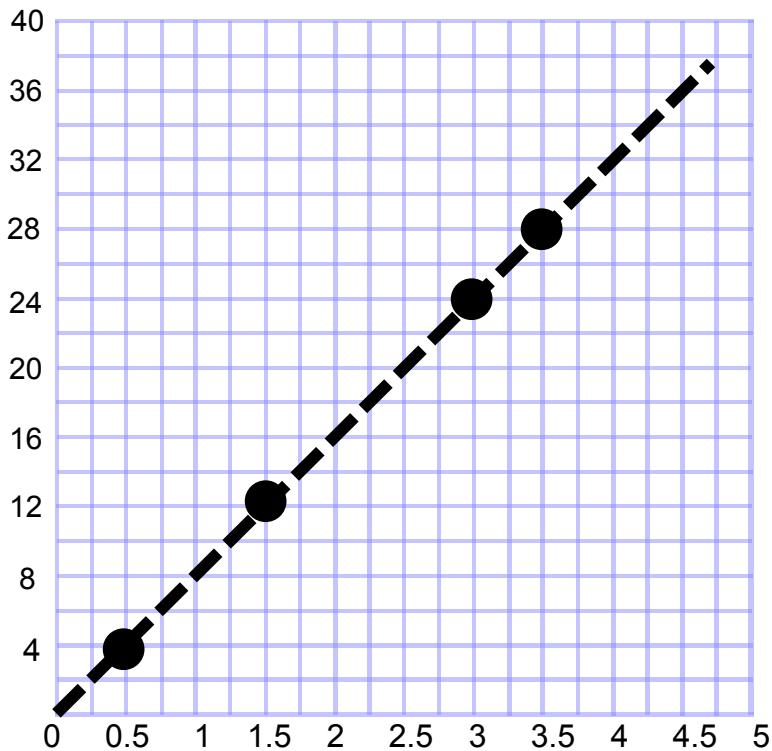
67  Find the constant of proportionality.
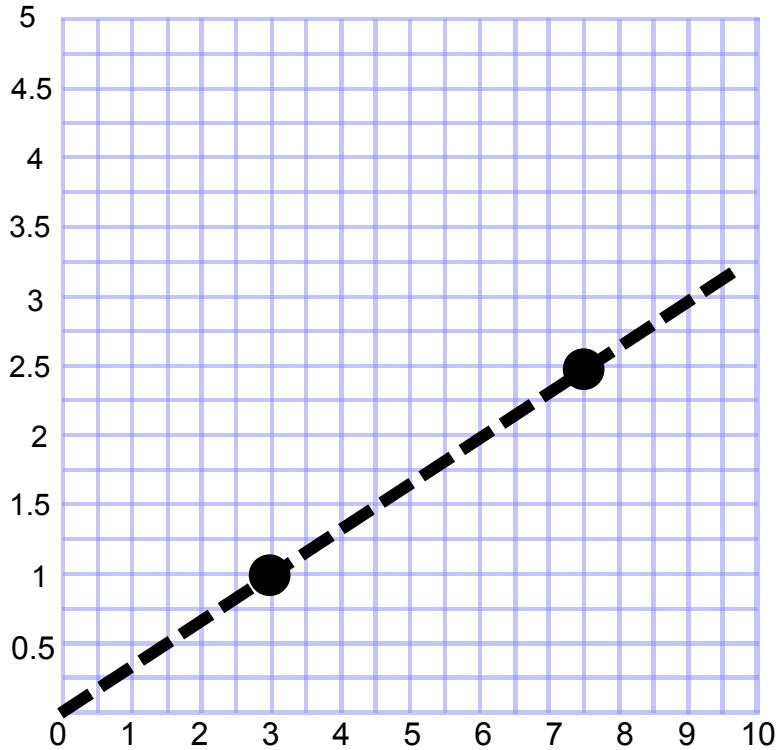


Answer

68  Find the constant of proportionality.
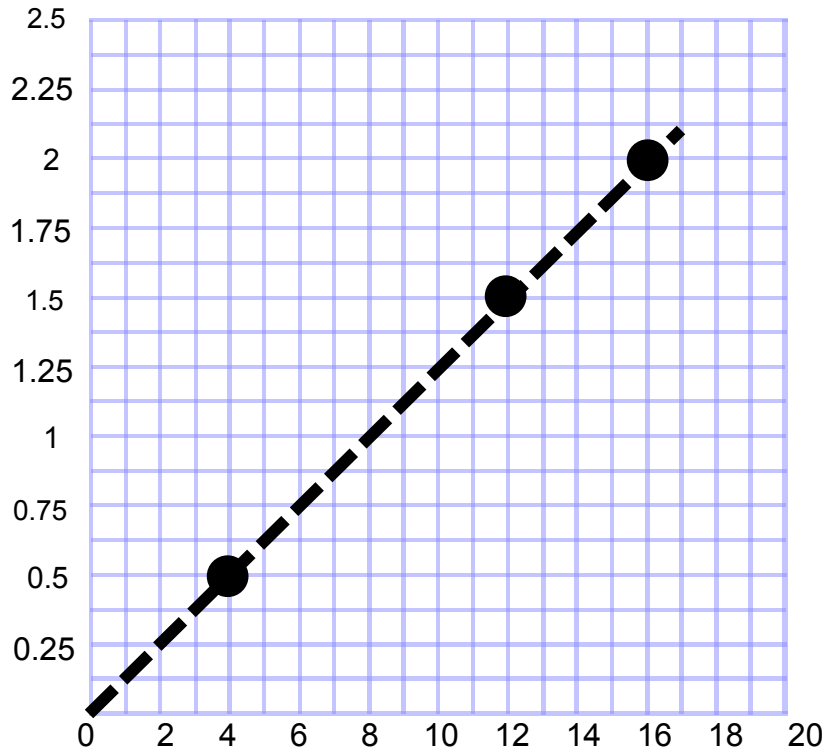
**69** Which relationships have the same constant of proportionality between y and x as in the equation $y = \dfrac{1}{3}x$ ? Select *each* correct answer.

A 
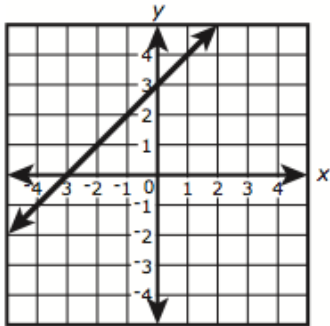
C 
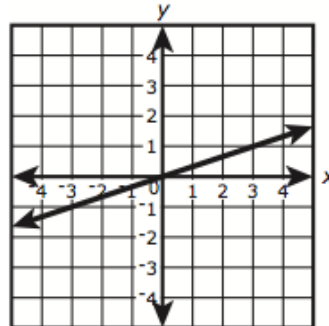
B 
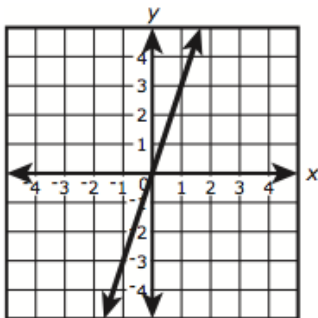
D

| x | −1.5 | 0 | 1.6 | 9.7 |
|---|------|---|-----|------|
| y | −4.5 | 0 | 4.8 | 29.1 |

E

| x | −5.4 | −2.7 | 1.5 | 2.4 |
|---|------|------|-----|-----|
| y | −1.8 | −0.9 | 0.5 | 0.8 |

**From PARCC Sample Test - Non-calculator**

# Writing Equations
# For Proportions

# Writing Equations

The constant of proportionality and the unit rate are equivalent.

We can use the constant of proportionality to help write equations using proportional relationships.

By transforming the equation from: $k = \dfrac{y}{x}$ to $y = kx$, we can

write an equation that can be applied to various situations.

*Remember: x is the independent variable and y is the dependent variable. This means that a change in x will effect y.

# Writing Equations

EXAMPLE

You are buying Jersey Tomatoes for a cost of 2 pounds for $3.98. Write an equation to represent the proportional relationship.

· Let c = cost
     p = pounds

· Determine the unit rate:
    click

· Write an equation to relate the two quantities:
    click

# Writing Equations

At the candy store, you purchase 5 lbs for $22.45.  Write an equation to represent the proportional relationship.

· Let c = cost
      p = pounds

· Determine the unit rate:

   click

· Write an equation to relate the two quantities:
   click

# Writing Equations

Write an equation to represent the proportional relationship shown in the table.

| Gallons | 10 | 15 | 20 | 25 |
|---------|-----|-------|-----|-------|
| Miles | 247 | 370.5 | 494 | 617.5 |

$$k = \frac{m}{g} = \frac{247}{10} = \frac{24.7}{1}$$

Let $g$ = gallons

$m$ = miles

click

70 Write an equation that represents the proportional relationship.

The total cost (c) of grapes for $1.40 per pound(p)

○ A  $c = 1.4p$

○ B  $p = 1.4c$

71 Write an equation that represents the proportional relationship.

| Shirts | 5 | 15 | 25 | 35 |
|---|---|---|---|---|
| Cost | $57.50 | $172.50 | $287.50 | $402.50 |

○ A  $s = 11.5c$

○ B  $c = 11.5s$

○ C  $c = 0.09s$

○ D  $s = 0.09c$

Answer

72  Write an equation that represents the
    proportional relationship.

○A  $y = \dfrac{1}{3}x$

○B  $y = 3x$

○C  $y = 2.5x$

○D  $y = 7.5x$
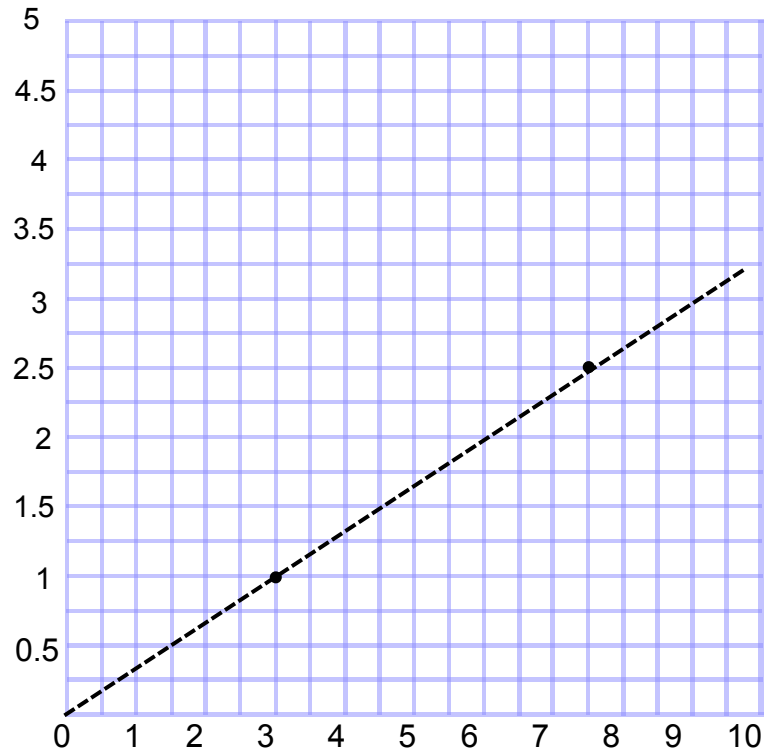
73  Write an equation that represents the proportional relationship.

You are ordering new menus for your restaurant.  You pay $362.50 for 50 menus.

◯ A  $c = 0.14m$

◯ B  $m = 7.25c$

◯ C  $m = 0.14c$

◯ D  $c = 7.25m$

74 Write an equation that represents the proportional relationship.

| Days, d | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| Hours, h | 17 | 25.5 | 34 | 42.5 |

A $d = 8.5h$

B $d = \dfrac{2}{17}h$

C $h = \dfrac{2}{17}d$

D $h = 8.5d$

75  The amount of money Jamie earns is proportional to the number of hours she works. Jamie earns $62.50 working 5 hours.

Create an equation that models the relationship between m, the amount of money Jamie earns, in dollars, and h, the number of hours she works.

Drag and drop the appropriate number and variables into each box.

| 12.05 | 12.50 | 57.50 | m | h |

$$\boxed{\phantom{xxx}} = \boxed{\phantom{xxx}} \quad \boxed{\phantom{xxx}}$$

**From PARCC Sample Test - Non-calculator**

76  The number of parts produced by three different machines are shown in the table.

**Numbers of Machine Parts**

| Minutes | Machine Q | Machine R | Machine S |
|---------|-----------|-----------|-----------|
| 1 | 9 | 8 | 6 |
| 3 | 18 | 24 | 18 |
| 9 | 27 | 72 | 52 |

Only one of the machines produces parts at a constant rate. Write an equation that can be used to represent y, the number of parts produced in x minutes, for that machine.

**From PARCC Sample Test - Non-calculator**

77  Hayden mixed 6 cups of blue paint with 8 cups of yellow paint to make green paint.

Write an equation that shows the relationship between the number of cups of blue paint, *b,* and the number of cups of yellow paint, *y,* that are needed to create the same shade of green paint. The equation should be in the form $b = ky$.

**Answer**

# Understanding Graphs of Proportions

# Graphs of Proportions

Remember, you can use a graph to determine if a relationship is proportional.  How?

If the graph is a straight line going through the origin (0, 0).

Once you determine that the relationship is proportional, you can calculate $k$, the constant of proportionality.  Then, write  an equation to represent the relationship.

What do these equations mean?  Once we have determined the equation, we can understand what the graph was showing us visually.

# Graphs of Proportions

The jitneys in Atlantic City charge passengers for rides. What amount do they charge per ride?

· Find a point on the graph

click

· Use the point to find the unit rate

click

· What does the unit rate represent?

click

· What coordinate pair represents the unit rate?
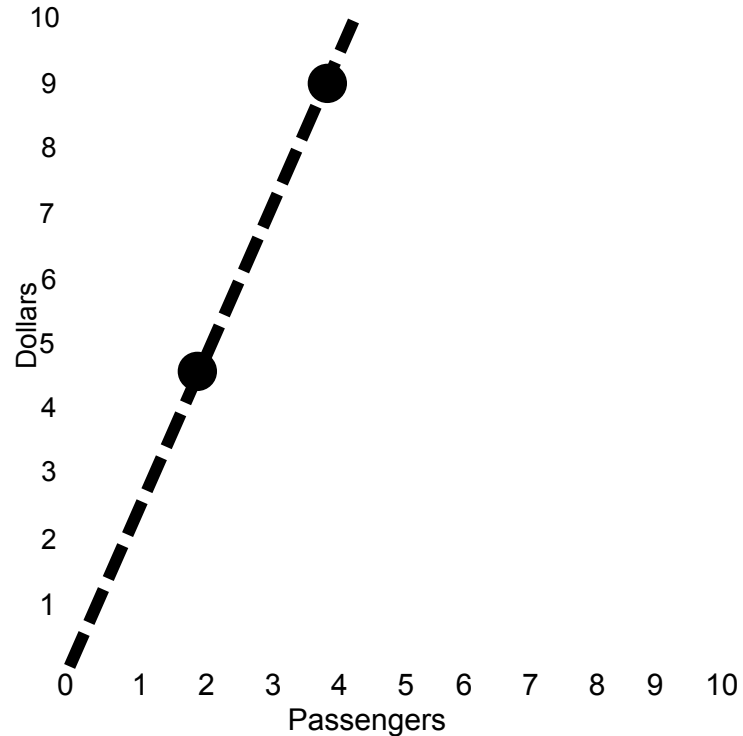
click

· Does the line run through the unit rate?

click

# Graphs of Proportions

Mark drives to work each day. His gas mileage is shown in the graph. What is the unit rate? What does it represent?

· Find a point on the graph

click

· Use the point to find the unit rate

click

· What does the unit rate represent?

click

· What coordinate pair represents the unit rate?
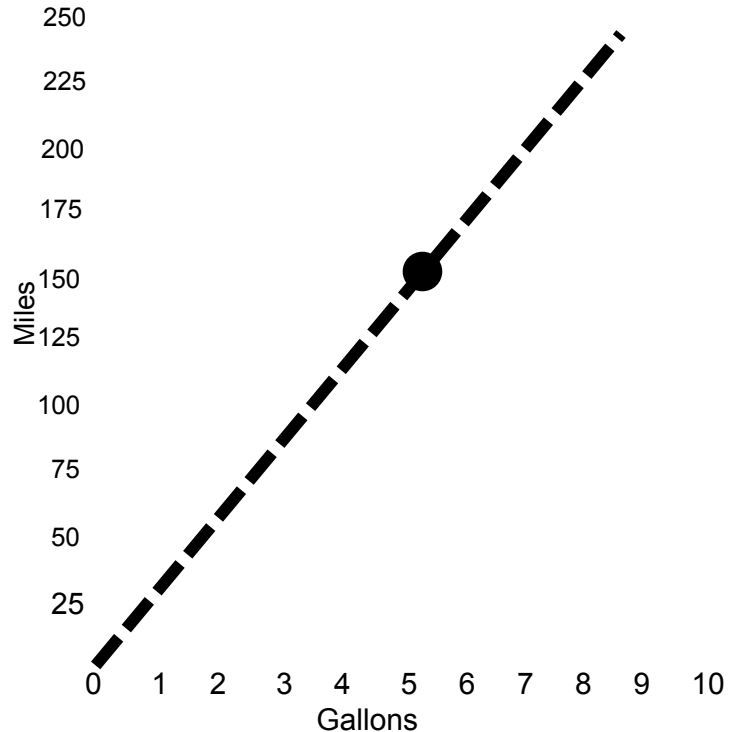
click



· Does the line run through the unit rate?

click

# Graphs of Proportions

Try This:

Jasmine gets paid for every dog that she walks according to the graph at the right. What does she earn per dog?

· Find a point on the graph

click

· Use the point to find the unit rate

click

· What does the unit rate represent?

click

· What coordinate pair represents the unit rate?
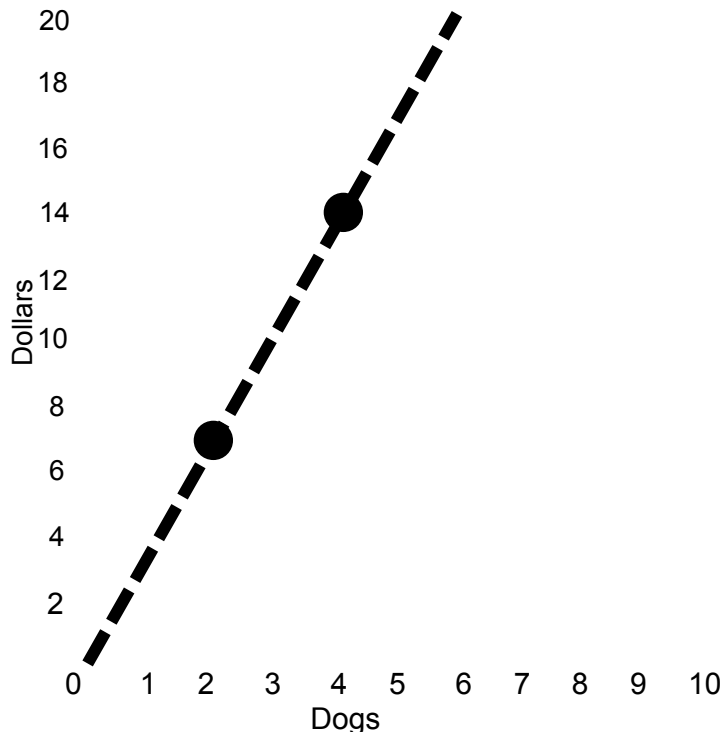
click



· Does the line run through the unit rate?

click

# Graphs of Proportions

Try This:

Mary drives the bus.  Her rate is shown in the graph. What is the unit rate? What does it represent?

· Find a point on the graph

click

· Use the point to find the unit rate

click

· What does the unit rate represent?

click

· What coordinate pair represents the unit rate?

click

People

100
90
80
70
60
50
40
30
20
10

0  1  2  3  4  5  6  7  8  9  10
Hours
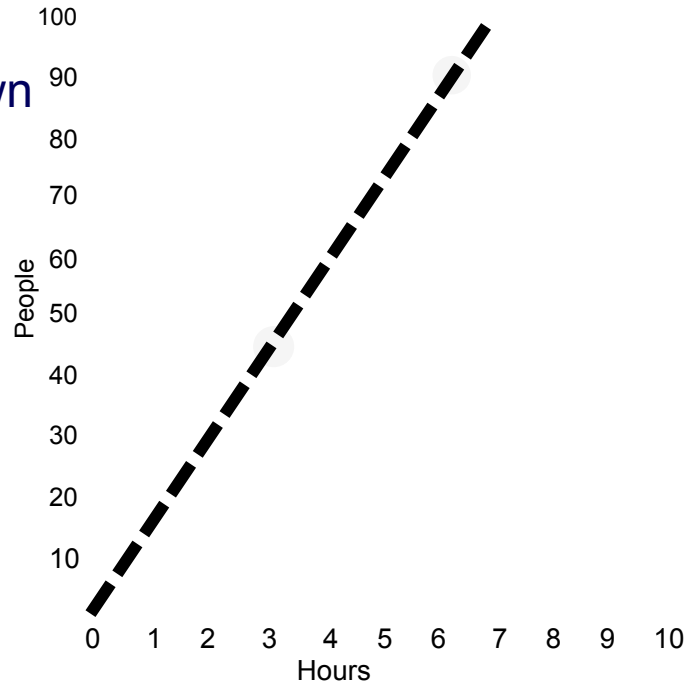
· Does the line run through the unit rate?

click

78 This graph shows the relationship between the pounds of cheese bought at a deli and the total cost, in dollars, for the cheese.

Select _each_ statement about the graph that is true.
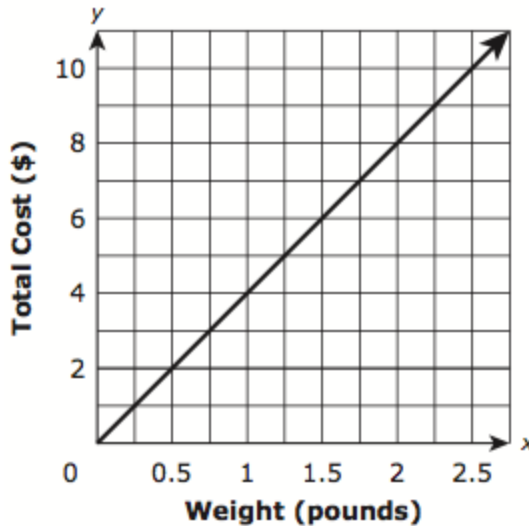


○ The point (0,0) shows the cost is $0.00 for 0 pounds of cheese.

○ The point (0.25, 1) shows the cost is $0.25 for 1 pound of cheese.

○ C  The point (0.5,2) shows that 0.5 pound of cheese costs $2.00.

○ D  The point (1,4) shows the cost is $4.00 for 1 pound of cheese.

○ E  The point (2,8) shows that 8 pounds of cheese cost $2.00.

**From PARCC Sample Test - Non-calculator**

# Problem Solving

# Problem Solving

Chocolates at the candy store cost $6.00 per dozen. How much does one candy cost? Round your answer to the nearest cent.

Solution:

$$\frac{\$}{candy} \qquad \frac{6.00}{12} = \frac{x}{1}$$

(Use equivalent rates to set up a proportions)

$$6.00(1) = 12x$$

$$0.50 = x$$

$$\boxed{\$0.50 \ per \ candy}$$

# Problem Solving

Example 2:
There are 3 books per student. There are 570 students.
How many books are there?

Set up the proportion:

$$\frac{\text{Books}}{\text{Students}}$$

$\dfrac{3}{1} = \underline{\quad\quad}$    Where does the 570 go?

$\dfrac{3}{1} = \dfrac{x}{570}$

$3 \cdot 570 = 1x$

$1{,}710 = x$

*1,710 books*

# Problem Solving

Example 3:

The ratio of boys to girls is 4 to 5.  There are 135 people on a team. How many are girls?

Set up the proportion:

$$\frac{Girls}{People}$$

$$\frac{5}{9} =$$

$$\frac{5}{9} = \frac{x}{135}$$

$$5 \cdot 135 = 9x$$

$$675 = 9x$$

$$x = 75$$

How did we determine this ratio? Where does the 135 go?

75 girls

79  Cereal costs $3.99 for a one pound box. What is the price per ounce? Round your answer to the nearest penny.

80  Which is the better buy?
    Brand A:  $2.19  for 12 ounces
    Brand B:  $2.49 for 16 ounces

○ A    Brand A

○ B    Brand B

81 There are 4 girls for every 10 boys at the party. There are 56 girls at the party. How many boys are there?

82  The farmer has cows and chickens. He owns 5 chickens for every cow. He has a total of 96 animals. How many cows does he own?

83  The auditorium can hold 1 person for every 5 square feet. It is 1210 square feet. How many people can the auditorium hold?

84 The recipe for one serving calls for 4 oz of beef and 2 oz of bread crumbs. 50 people will be attending the dinner. How many ounces of bread crumbs should be purchased?

85 Mary received 4 votes for every vote that Jane received. 1250 people voted. How many votes did Jane receive?

86  To make the desired shade of pink paint, Brandy uses 3 oz. of red paint for each oz. of white paint. She needs one quart of pink paint. How many oz. of red paint will she need? (*1 quart = 32 ounces*)

**Answer**

# Making Sense of
# Your Answers

Sometimes your answer will be a decimal or fraction that may not make sense as an answer.

Double check:
- Reread the problem
- Does your answer make sense?
- Do you need to round your answer?
- If so, which way should you round your answer?

87  Cole earned a total of $11 by selling 8 cups of lemonade.  How many cups of lemonade does Cole need to sell in all to earn $15?  Assume the relationship is directly proportional.

88 Hayley learned a total of 13 appetizer recipes over the course of 3 weeks of culinary school.  How many weeks does she need to complete to have learned 21 appetizers?  Assume the relationship is directly proportional.

89  Kailyn took a total of 2 quizzes over the course of 5 days.  After attending 16 days of school this quarter, how many quizzes will Kailyn have taken in total?  Assume the relationship is directly proportional.

90 Brittany baked 18 cookies with 1 cup of flour. How many cups of flour does Brittany need in order to bake 27 cookies?  Assume the relationship is directly proportional.

91 Shane caught a total of 10 fish over the course of 2 days on a family fishing trip. At the end of what day will Shane have caught his 22 fish? Assume the relationship is directly proportional.

92  In a sample of 50 randomly selected students at a school, 38 students eat breakfast every morning. There are 652 students in the school. Using these results, predict the number of students that eat breakfast.

○ A   76

○ B   123

○ C   247

○ D   496

**Answer**

93  Sal exercised by stretching and jogging 5 days last week.
- He stretched for a total of 25 minutes during the *week*.
- He jogged for an equal number of minutes each of the 5 days.
- He exercised for a total of 240 minutes.

Elena also exercised by stretching and jogging 5 days last week.
- She stretched for 15 minutes each day.
- She jogged for an equal number of minutes each of the 5 days.
- She exercised for a total of 300 minutes.

Determine the number of minutes Sal jogged each day last week and the number of minutes Elena jogged each day last week. Show your work or explain all the steps you used to determine your answers.

**Answer**

# Scale Drawings

# Scale Drawings

Scale drawings are used to represent objects that are either too large or too small for a life size drawing to be useful.

Examples:

A life size drawing of an ant or an atom would be too small to be useful.

A life size drawing of the state of New Jersey or the Solar System would be too large to be useful.

# Scale Drawings

A scale is always provided with a scale drawing.

The **scale** is the ratio:     $\dfrac{drawing}{real\ life\ (actual)}$

When solving a problem involving scale drawings you should:

· Write the scale as a ratio
· Write the second ratio by putting the provided information in the correct location (drawing on top & real life on the bottom)
· Solve the proportion

# Scale Drawings

Example:

This drawing has a scale of "1:10", so anything drawn with the size of "1" would have a size of "10" in the real world, so a measurement of 150mm on the drawing would be 1500mm on the real horse.



Real Horse
1500 mm high
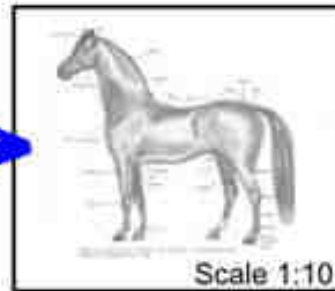
Drawn Horse
150 mm high

Scale 1:10

# Scale Drawings

Example:

The distance between Philadelphia and San Francisco is 2,950 miles. You look on a map and see the scale is 1 inch : 100 miles. What is the distance between the two cities on the map?

$$\frac{drawing}{actual} = \frac{1}{100}$$

Writing the scale as a ratio

$$\frac{1}{100} = \frac{x}{2950}$$

$$100x = 2950$$

$$x = 29.5$$

$29.5 \ inches \ on \ the \ map$

# Scale Drawings

Try This:

On a map, the distance between your town and Washington DC is 3.6 inches.  The scale is 1 inch : 55 miles.  What is the distance between the two cities?

**Answer**

94  On a map with a scale of *1 inch = 100 miles*, the distance between two cities is 7.55 inches.  If a car travels 55 miles per hour, about how long will it take to get from one city to the other.

○A   13 hrs 45 min.

○B   14 hrs  30 min.

○C   12 hrs

○D   12 hrs 45 min.

95 On a map, the scale is $\frac{1}{2}$ $inch = 300$ $miles$. Find the actual distance between two stores that are 5 1/2 inches apart on the map.

○ A    3000 miles

○ B    2,727 miles

○ C    3,300 miles

○ D    1,650 miles

96 The figure is a scale of the east side of a house. In the drawing, the side of each square represents 4 feet. Find the width and height of the door.

○ A    4 ft by 9 ft

○ B    4 ft by 12 ft

○ C    4 ft by 8 ft

○ D    4 ft by 10 ft
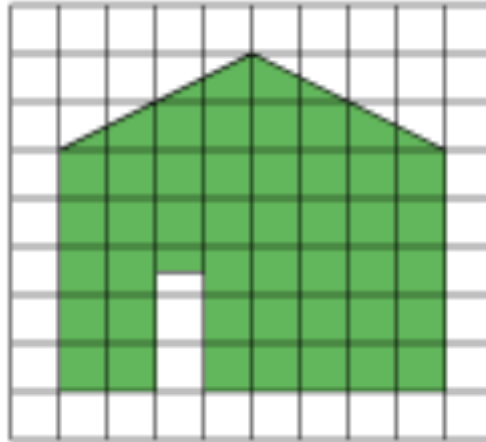
97  The distance between Moorestown, NJ and Duck, NC is 910 miles.  What is the distance on a map with a scale of 1 inch to 110 miles?

98  The distance between Philadelphia and Las Vegas is 8.5 inches on a map with a scale 1.5 in : 500 miles. What is the distance in miles?

99 You are building a room that is 4.6 m long and 3.3 m wide. The scale on the architect's drawing is 1 cm : 2.5 m. What is the length of the room on the drawing?

100  You are building a room that is 4.6 m long and 3.3 m wide. The scale on the architect's drawing is 1 cm : 2.5 m. What is the width of the room on the drawing?

Answer

101 Find the length of a 72 inch wide wall on a scale drawing with a scale 1 inch : 2 feet.

102 You recently purchased a scale model of a car. The scale is 15 cm : 10 m. What is the length of the model car if the real car is 4 m?

103  You recently purchased a scale model of a car.  The scale is 15 cm : 10 m.  The length of the model's steering wheel is 1.25 cm.  What is the actual length of the steering wheel?

104  The scale on a map shows that 5 centimeters = 2 kilometers.

Part A

What number of centimeters on the map represents an actual distance of 5 kilometers?

**Answer**

105  (Continued from previous slide.)

Part B

What is the actual number of kilometers that is represented by 2 centimeters on the map?

**Answer**

# Similar Figures

# Similar Figures

Two objects are **similar** if they are the same shape .

In similar objects:
· corresponding angles are congruent (the same)
· corresponding sides are proportional

# Similar Figures

To check for similarity:

· Check to see that corresponding angles are congruent
· Check to see that corresponding sides are proportional
   (Cross products are equal)

# Similar Figures

Example:

Is the pair of polygons similar?  Explain your answer.
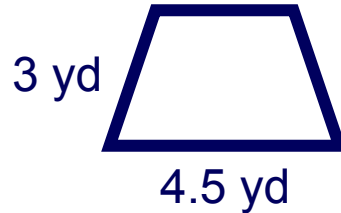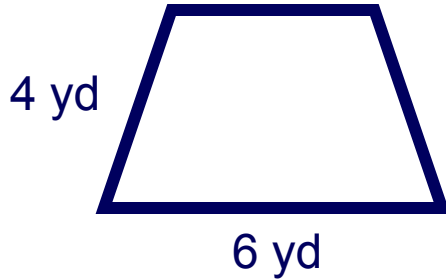


$$\frac{4}{6} = \frac{3}{4.5}$$

$$4(4.5) = 6(3)$$

$$18 = 18$$

*YES*

or

$$\frac{4}{3} = \frac{6}{4.5}$$

$$4(4.5) = 6(3)$$

$$18 = 18$$

*YES*

# Similar Figures

Example:

Is the pair of polygons similar?  Explain your answer.

8 m

5 m

13 m

10 m
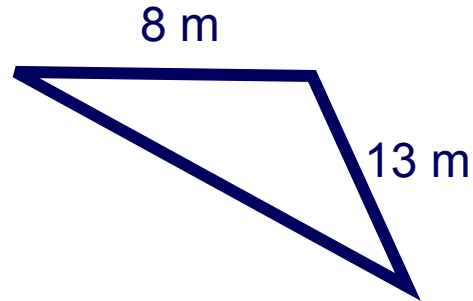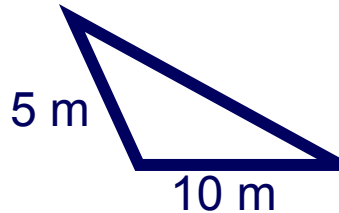
$$\frac{5}{10} = \frac{8}{13}$$

$$5(13) = 10(8)$$

$$65 = 80$$

*NO*

or

$$\frac{5}{10} = \frac{10}{13}$$

$$5(13) = 8(10)$$

$$65 = 80$$

*NO*

106  Are the polygons similar?  You must be able to justify your answer. (Shapes not drawn to scale.)

○ Yes

○ No

15 ft

21 ft

9 ft

12 ft

107 Are the polygons similar?  You must be able to justify your answer. (Shapes not drawn to scale.)

○ Yes

○ No

7.25 cm

7.25 cm

7.25 cm

7.25 cm

7.25 cm
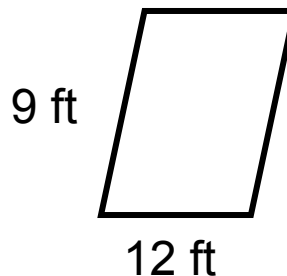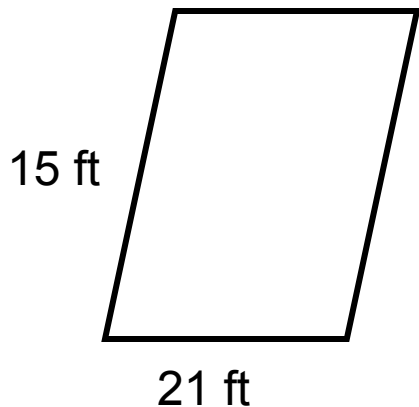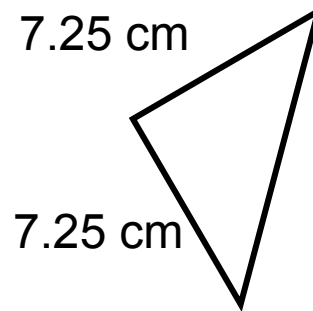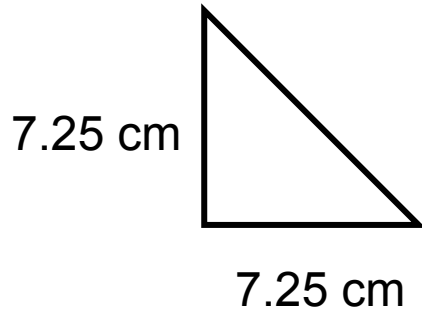
**Answer**

108  Are the polygons similar?  You must be able to justify your answer. (Shapes not drawn to scale.)

○ Yes

○ No

37.5 yd

15 yd

15 yd

6 yd
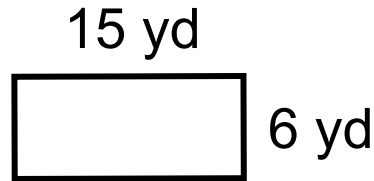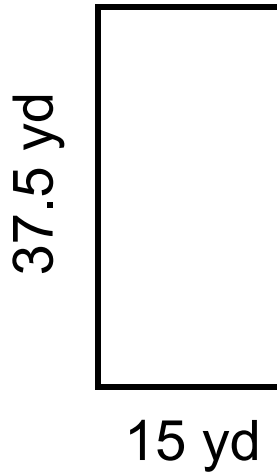
Answer

109  Are the polygons similar?  You must be able to justify your answer. (Shapes not drawn to scale.)

○ Yes

○ No

37.5 yd

15 yd

15 yd

6 yd
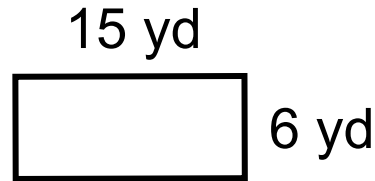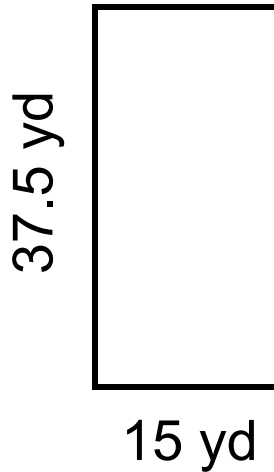
**Answer**

110  A right triangle has legs measuring 4.5 meters and 1.5 meters.

The lengths of the legs of a second triangle are proportional to the lengths of the legs of the first triangle.

Which could be the lengths of the legs of the second triangle? Select *each* correct pair of lengths.

○ 6 m and 2 m

○ B  8 m and 5 m

○ C  7 m and 3.5 m

○ D  10 m and 2.5 m

○ E  11.25 m and 3.75 m

Answer

# Similar Figures

Example:

Find the value of *x* in the pair of similar polygons.
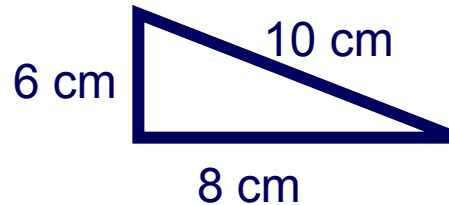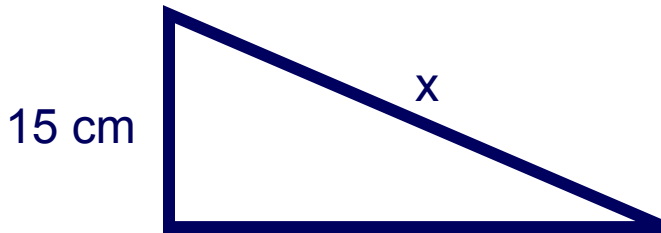


$$\frac{5}{x} = \frac{6}{10}$$

$$15(10) = 6x$$

$$150 = 6x$$

$$25cm = x$$

or

$$\frac{15}{6} = \frac{x}{10}$$

$$15(10) = 6x$$

$$150 = 6x$$

$$25cm = x$$

# Similar Figures

Try This:

Find the value of *y* in the pair of similar polygons.



15 in

y

7.5 in

5 in
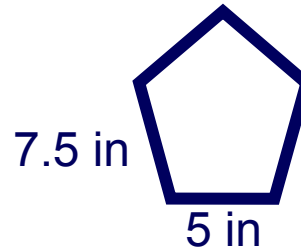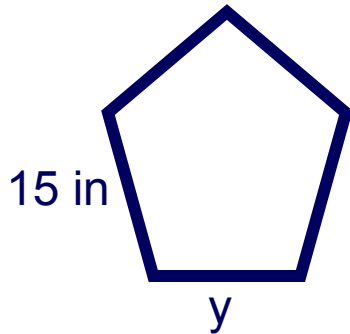
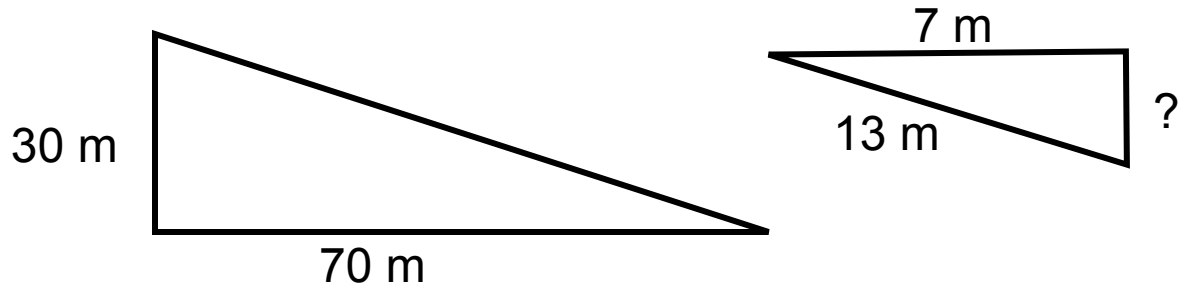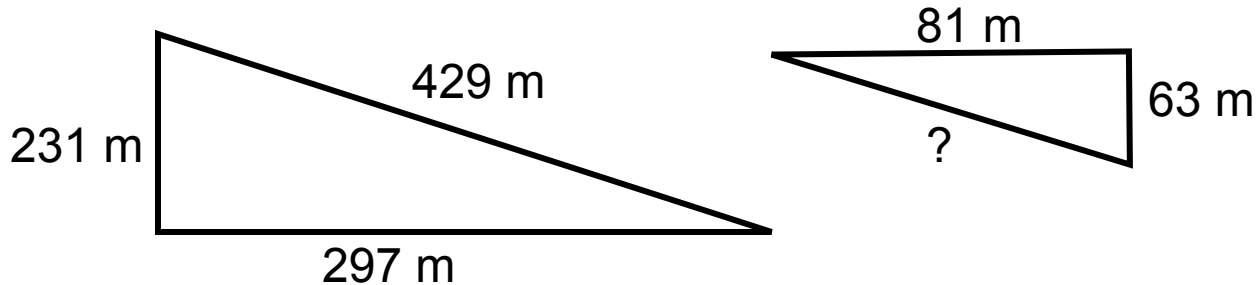111  Find the measure of the missing value in the pair of similar polygons. (Shapes not drawn to scale.)

112 Find the measure of the missing value in the pair of similar polygons. (Shapes not drawn to scale.)

25 ft

25 ft

18 ft

17.5 ft

w

Answer
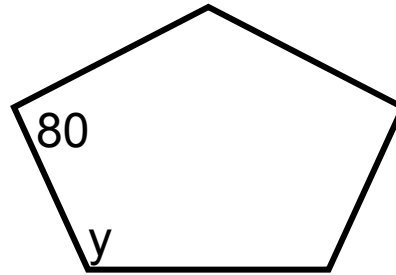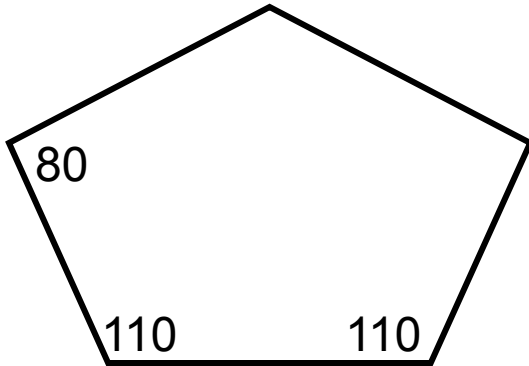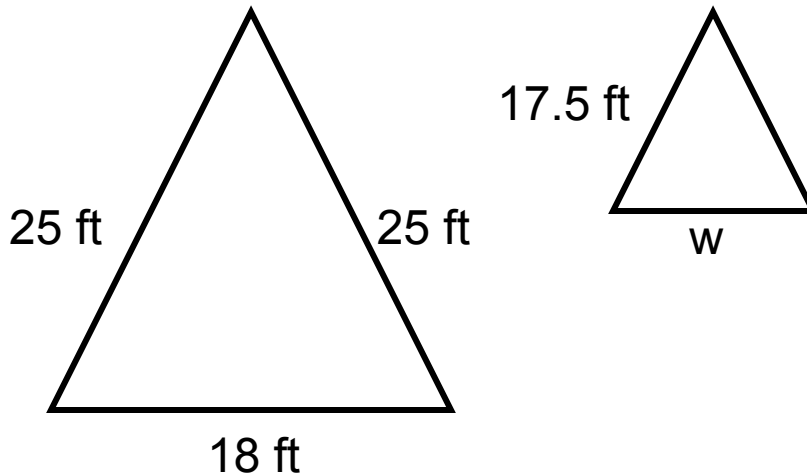
113 Find the measure of the missing value in the pair of similar polygons. (Shapes not drawn to scale.)

x

17 m

4 m

4.25 m
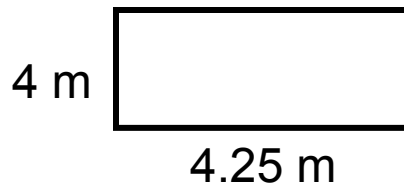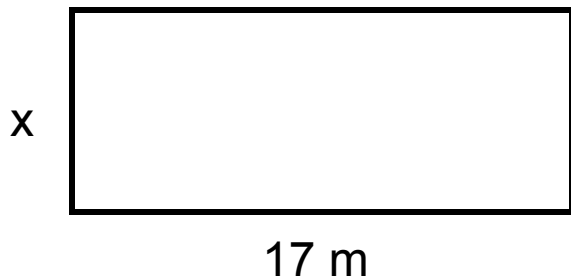
114 Find the measure of the missing value in the pair of similar polygons. (Shapes not drawn to scale.)

6 mm

11 mm

y

38.5 mm
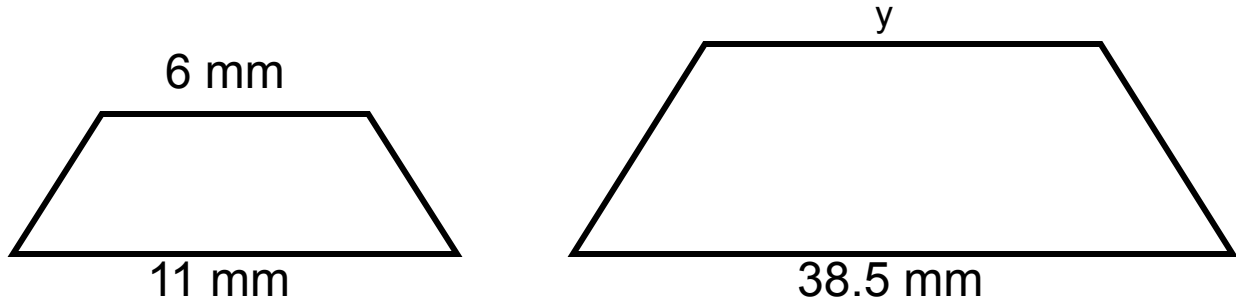
115 Find the measure of the missing value in the pair of similar polygons. (Shapes not drawn to scale.)

30 m

70 m

7 m

13 m

?

116  Find the measure of the missing value in the pair of similar polygons. (Shapes not drawn to scale.)

231 m

429 m

297 m

81 m

63 m

?

117 Find the measure of the missing value in the pair of similar polygons. (Shapes not drawn to scale.)

2 mm

5 mm

27.5 mm

x

Answer
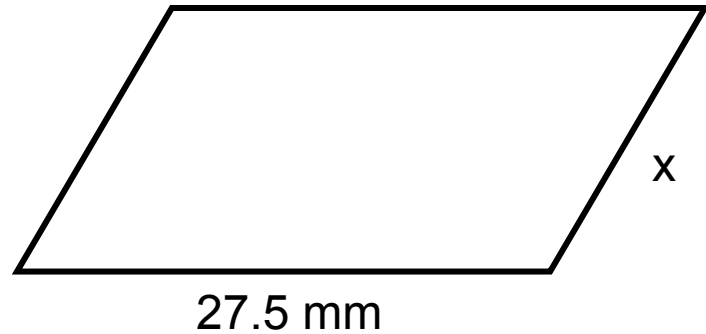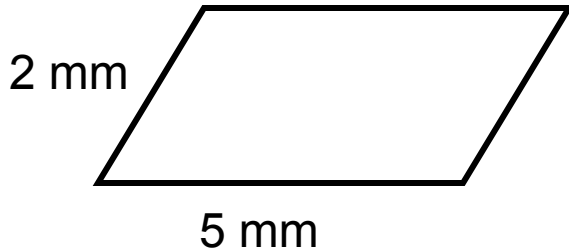
# Glossary

# Constant of Proportionality

A constant ratio (unit rate) in any proportional relationship

| | | |
|---|---|---|
| *Equations:*<br><br>y = kx<br><br>or<br><br>k = $\dfrac{y}{x}$ | y = ⑤x<br><br><br>k = 5 | (3 , 45)<br>  x     y<br>y = kx<br>45 = k3<br>k = 15 |

# Equivalent Ratios

Ratios that have the same value.

$$\frac{3}{6} = \frac{1}{2} = \frac{4}{8}$$

# Population Density

A unit rate of people per square mile.
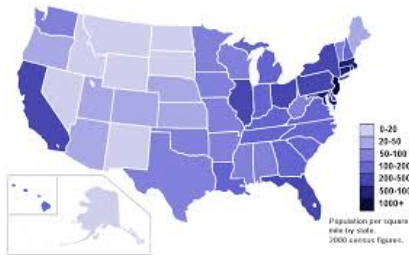
| Population / Area |  | NJ = 8,791,894 people<br>NJ = 7,790 square miles<br><br>$\dfrac{\text{Population}}{\text{Area}} = \dfrac{8,791,894}{7,790}$<br><br>= 1,129 people per square mile |
|---|---|---|

# Proportion

An equation that states that two ratios are equivalent.

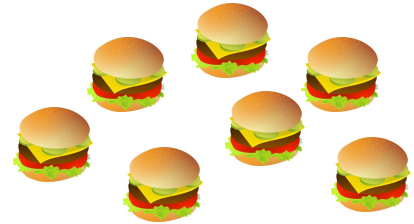| | | |
|---|---|---|
| $\dfrac{2}{3} = \dfrac{14}{21}$ | $\dfrac{1}{2} = \dfrac{20}{40}$ | $\dfrac{5}{8} \overset{x3}{\underset{x3}{=}} \dfrac{15}{x}$ <br> $x = 24$ |

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# Rate

A ratio of two quantities measured in different units.



3 participants/2 teams

5 gallons/3 rooms

7 burgers/2 tomatoes

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# Ratio

A comparison of two numbers by division.

3 different ways:
"the ratio of a to b"

a to b

a : b

$\frac{a}{b}$

There are 48 animals in the field. Twenty are cows and the rest are horses.

*What is the number of cows to the total number of animals?*

20 to 48

20:48

$\frac{20}{48}$
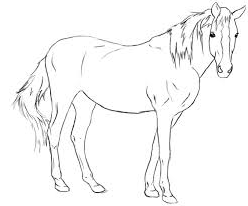
Back to Instruction

# Scale

## The ratio of a drawing to the real life measurement.

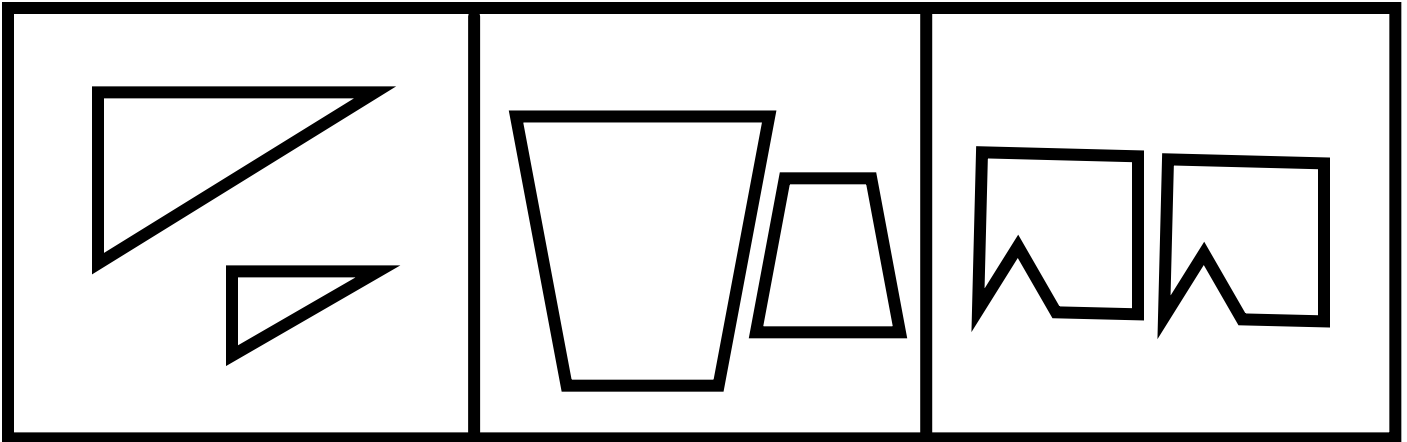$$\frac{\text{drawing}}{\text{real life (actual)}}$$

Scale- 1:10

Real Horse
1500mm high

Drawn Horse
150mm high

# Similar

Two figures that are the same shape.
- corresponding angles are congruent
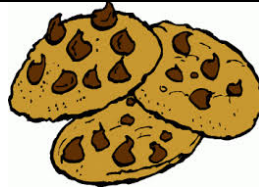- corresponding sides are proportional

# Unit Rate

Rate with a denominator of one.



34 miles/gallon

3 cookies per person

62 words/minute

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# Standards for Mathematical Practices

Throughout this unit, the Standards for Mathematical Practice are used.

MP1: Making sense of problems & persevere in solving them.
MP2: Reason abstractly & quantitatively.
MP3: Construct viable arguments and critique the reasoning of others.
MP4: Model with mathematics.
MP5: Use appropriate tools strategically.
MP6: Attend to precision.
MP7: Look for & make use of structure.
MP8: Look for & express regularity in repeated reasoning.

Additional questions are included on the slides using the "Math Practice" Pull-tabs (e.g. a blank one is shown to the right on this slide) with a reference to the standards used.

If questions already exist on a slide, then the specific MPs that the questions address are listed in the Pull-tab.