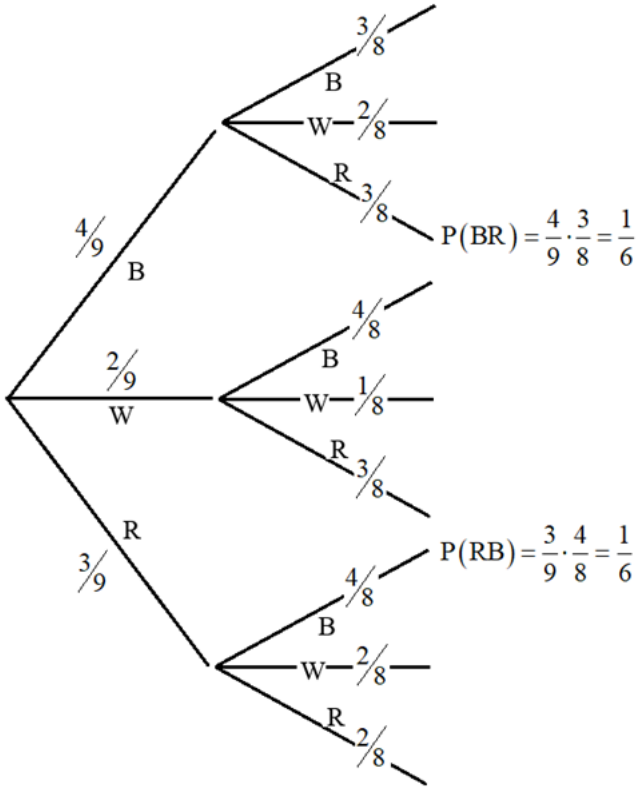
Quiz1 - Probability of Events SOLUTION KEY

[total marks on quiz: 35 marks]

No GDC allowed on questions 1-5; GDC is allowed on the last question #6.

1. A bag contains 4 black balls, 2 white balls and 3 red balls. A ball is chosen at random from the bag and is not replaced. A second ball is chosen. Find the probability of choosing one black ball and one red ball in any order. **[4 marks]**



probability of choosing BR or RB =

$$P(BR)+P(RB)=\frac{1}{6}+\frac{1}{6}=\frac{1}{3}$$

2. Independent events A and B are such that $p(A)=0.2$ and $p(A\cup B)=0.6$. Find $p(B)$. **[4 marks]**

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

since A and B are independent, then $P(A\cap B)=P(A)\cdot P(B)$

thus, $P(A\cup B)=P(A)+P(B)-P(A)\cdot P(B)$

letting $P(B)=x$ and substituting given values:

$$0.6=0.2+x-0.2x \quad\Rightarrow\quad 0.8x=0.4 \quad\Rightarrow\quad x=\frac{0.4}{0.8}=\frac{1}{2} \qquad \text{thus, } p(B)=\frac{1}{2}$$

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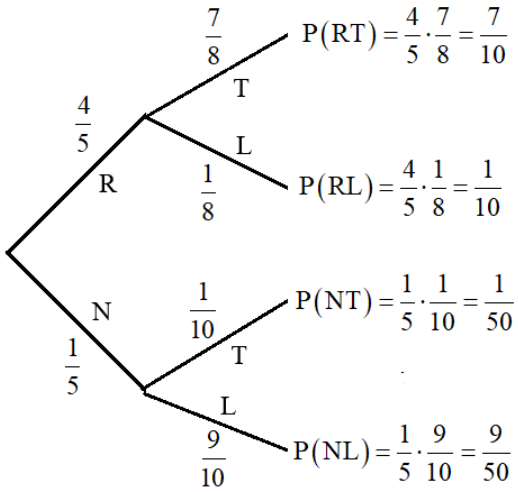
3. An alarm clock is used to wake a student for school. The probability that the alarm rings is $\frac{4}{5}$.

If the alarm rings, there is a probability of $\frac{7}{8}$ that the student arrives at school on time; but, if the alarm does not ring, the probability that the student arrives at school on time is $\frac{1}{10}$.

Find:

(a) the probability that the student arrives at school on time on a given day; **[4 marks]**

(b) the probability that, on a randomly chosen morning on which the student is late for school, the alarm did not ring. **[4 marks]**



R = alarm rings; N = alarm does not ring

T = student is on time; L = student is late

(a) probability student arrives on time =

$$P(RT)+P(NT)=\frac{7}{10}+\frac{1}{50}=\frac{35}{50}+\frac{1}{50}=\frac{36}{50}=\frac{18}{25}=0.72$$

(b) conditional probability:

$$P(N|L)=\frac{P(N\cap L)}{P(L)}=\frac{\frac{9}{50}}{\frac{1}{10}+\frac{9}{50}}=\frac{\frac{9}{50}}{\frac{14}{50}}=\frac{9}{14}\approx 0.643$$

4. Two dice are rolled. The score is the smaller of the two numbers that appear; if the same number appears on both dice, then the score is that number. What is the probability that the score is 3? **[4 marks]**

	1	2	3	4	5	6
1	1,1	2,1	3,1	4,1	5,1	6,1
2	1,2	2,2	3,2	4,2	5,2	6,2
3	1,3	2,3	**3,3**	**4,3**	**5,3**	**6,3**
4	1,4	2,4	**3,4**	4,4	5,4	6,4
5	1,5	2,5	**3,5**	4,5	5,5	6,5
6	1,6	2,6	**3,6**	4,6	5,6	6,6

The table at left shows the complete sample space.

probability score is 3 $=\frac{7}{36}$

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5. Events A and B are given such that $\mathrm{P}(A\cap B)=\frac{1}{5}$, $\mathrm{P}(B\,|\,A)=\frac{1}{2}$, $\mathrm{P}(A\,|\,B)=\frac{3}{10}$. Find:

(a) $\mathrm{P}(B)$; **[3 marks]** (b) $\mathrm{P}(A)$; **[3 marks]** (c) $\mathrm{P}(A\cup B)$ **[2 marks]**

(a) $\mathrm{P}(\mathrm{A}\,|\,\mathrm{B})=\dfrac{\mathrm{P}(\mathrm{A}\cap\mathrm{B})}{\mathrm{P}(\mathrm{B})} \Rightarrow \dfrac{3}{10}=\dfrac{\frac{1}{5}}{\mathrm{P}(\mathrm{B})} \Rightarrow \mathrm{P}(\mathrm{B})=\dfrac{\frac{1}{5}}{\frac{3}{10}}=\dfrac{10}{15}=\dfrac{2}{3}$

(b) $\mathrm{P}(\mathrm{B}\,|\,\mathrm{A})=\dfrac{\mathrm{P}(\mathrm{A}\cap\mathrm{B})}{\mathrm{P}(\mathrm{A})} \Rightarrow \dfrac{1}{2}=\dfrac{\frac{1}{5}}{\mathrm{P}(\mathrm{A})} \Rightarrow \mathrm{P}(\mathrm{A})=\dfrac{\frac{1}{5}}{\frac{1}{2}}=\dfrac{2}{5}$

(c) $\mathrm{P}(\mathrm{A}\cap\mathrm{B})=\mathrm{P}(\mathrm{A})+\mathrm{P}(\mathrm{B})-\mathrm{P}(\mathrm{A}\cap\mathrm{B})=\dfrac{2}{5}+\dfrac{2}{3}-\dfrac{1}{5}=\dfrac{6}{15}+\dfrac{10}{15}-\dfrac{3}{15}=\dfrac{13}{15}$

6. A couple is told that the probability that they will have blue-eyed children is $\frac{1}{4}$. The couple would like to have 6 children.

(a) What is the probability that 3 of the 6 children will be blue-eyed? **[4 marks]**

(b) What is the probability that all 6 children will be blue-eyed? **[3 marks]**

(a) probability that 3 of the 6 are blue-eyed $=\binom{6}{3}\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^3\approx 0.132$

(b) probability that all 6 are blue-eyed $=\left(\frac{1}{4}\right)^6\approx 0.000244$