BASICS

SI Units and Prefixes

SI Units		
Length	meter	m
Mass	kilogram	kg
Time	second	s
Temperature	kelvin	K
Amount of substance	mole	mol
Electrical current	amp	A
Light intensity	candela	cd

Prefix	Symbol	Exponent	Decimal	Word
tera-	T	10^{12}	1,000,000,000,000	trillion
giga-	G	10^{9}	1,000,000,000	billion
mega-	M	10^{6}	1,000,000	million
kilo-	k	10^{3}	1,000	thousand
hecto-	h	10^{2}	100	hundred
deka-	da	10^{1}	10	ten
—	—	10^{0}	1	one
deci-	d	10^{-1}	0.1	tenth
centi-	c	10^{-2}	0.01	hundredth
milli-	m	10^{-3}	0.001	thousandth
micro-	μ	10^{-6}	0.000 001	millionth
nano-	n	10^{-9}	0.000 000 001	billionth
pico-	p	10^{-12}	0.000 000 000 001	trillionth

Big

Small

Unit Conversion

Example: Convert 1 day into units of seconds

1. Find the equal amounts (the relationships) for the units that you're working with
2. Write the starting amount
3. Multiply by the equal amounts as fractions
4. Cross out units that are on both the top and bottom of the list of fractions
5. Multiply the numbers to get the final amount, which will have the units that are remaining

$$\underbrace{\frac{1\ \cancel{\text{day}}}{}}_{\text{Starting Amount}} \times \underbrace{\frac{24\ \cancel{\text{hours}}}{1\ \cancel{\text{day}}} \times \frac{60\ \cancel{\text{minutes}}}{1\ \cancel{\text{hour}}} \times \frac{60\ \text{seconds}}{1\ \cancel{\text{minute}}}}_{\text{Equal Amounts}} = \underbrace{\frac{86{,}400\ \text{seconds}}{}}_{\text{Final Amount}}$$

Scientific Notation

How to write a number in Scientific Notation	3800	0.00024
1. Move the decimal until there is only 1 number to the left of it	3.8.0.0. 3 2 1	0.0.0.0.2.4 1 2 3 4
2. Write down the new number and "x 10"	3.8×10	2.4×10
3. Count how many times you moved the decimal	3.8.0.0. 3 2 1 left + ←	0.0.0.0.2.4 1 2 3 4 → right -
4. Write that number as your exponent - If you moved the decimal left, the exponent is positive - If you moved the decimal right, the exponent is negative	3.8×10^{3}	2.4×10^{-4}

Order of Operations

PEMDAS

Please Excuse My Dear Aunt Sally

1. Parentheses $(1+2)$
2. Exponents 3^2
3. Multiplication $(2)(4)$
4. Division $\frac{4}{2}$
5. Addition $3+2$
6. Subtraction $3-2$

Parentheses $x = \frac{(2-1)(1+3)}{2} + \frac{(5-2)^2}{(8-5)} - 1$

Exponents $x = \frac{(1)(4)}{2} + \frac{3^2}{3} - 1$

Multiplication $x = \frac{(1)(4)}{2} + \frac{9}{3} - 1$

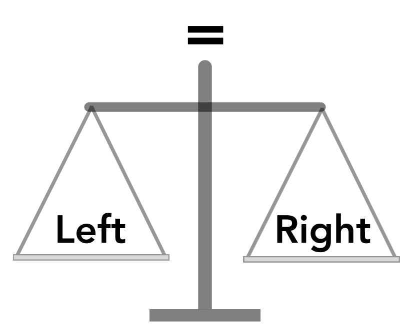
Division $x = \frac{4}{2} + \frac{9}{3} - 1$

Addition, Subtraction $x = 2 + 3 - 1$

$x = 4$

Solving Equations

Do the same thing to both sides of the equation:



$5 + x = 12$

Subtract 5 $\quad 5 + x - 5 = 12 - 5 \quad$ Subtract 5

$x = 7$

$2x = 8$

Divide by 2 $\quad 2x = 8 \quad$ Divide by 2

$x = 4$

$1 = 1$

Add 5 $\quad 6 = 6 \quad$ Add 5

Multiply by 3 $\quad 18 = 18 \quad$ Multiply by 3

Divide by 2 $\quad 9 = 9 \quad$ Divide by 2

Example: Solve for m

Given: $F = ma$, $F = 10$, $a = 2$

A Rearrange the equation, then plug in numbers

$F = ma$

Rearrange equation $\quad \frac{F}{a} = m \quad$ Divide both sides by a

Plug in numbers $\quad \frac{(10)}{(2)} = m$

$5 = m$

B Plug in numbers, then rearrange the equation

$F = ma$

Plug in numbers $\quad (10) = m(2)$

Rearrange equation $\quad 5 = m \quad$ Divide both sides by 2

The Quadratic Formula

If an equation is in this form: $ax^2 + bx + c = 0$

x : any unknown variable
a, b, c : constants

the two solutions (values) of x are:

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

or

$$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

- Some physics equations require you to solve for an unknown variable in a quadratic equation. If you need to solve for the variable by hand (without having a calculator solve it for you), use the quadratic formula.
- Rearrange the equation so it matches the form shown above where each term is added together. If there is no constant in the place of **a** or **b** then the constant would be 1. If a term is being subtracted then change the equation so you're adding a negative term.
- There will be two solutions: one solution when using the "+" and one solution when using the "-" of the "±". Both solutions may not be possible values for the physical quantity being represented, so double check the solutions.

Example:

$$8 = 2 + 5t - t^2$$

$$t^2 - 5t + 6 = 0$$

$$(1)t^2 + (-5)t + (6) = 0$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$at^2 + bt + c = 0$$

$$t = \frac{-(-5) + \sqrt{(-5)^2 - 4(1)(6)}}{2(1)}$$

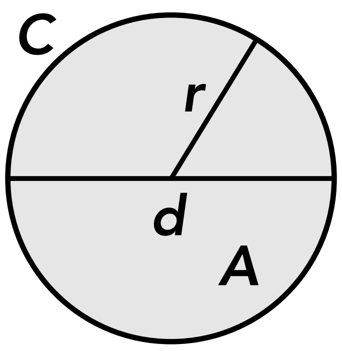
$$t = 3$$

or

$$t = \frac{-(-5) - \sqrt{(-5)^2 - 4(1)(6)}}{2(1)}$$

$$t = 2$$

Circle and Triangle Geometry

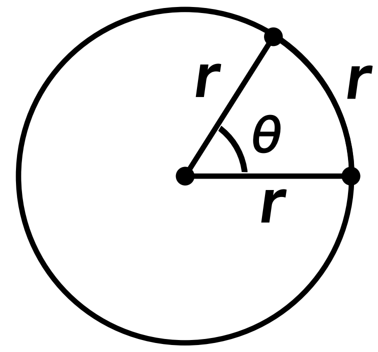


$d = 2r$

$C = 2\pi r = \pi d$

$A = \pi r^2$

r : radius C : circumference
d : diameter A : area

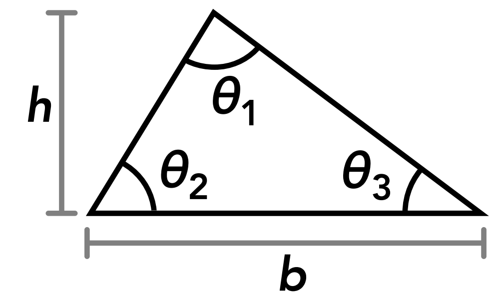


$\theta = 1$ radian

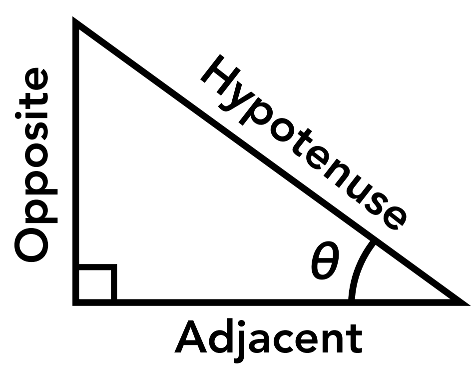
1 revolution $= 360° = 2\pi$ radians

$\theta_1 + \theta_2 + \theta_3 = 180°$

$\text{Area} = \frac{1}{2} b h$



Right Triangle Trigonometry



SOH - CAH - TOA

SOH	CAH	TOA
Sine	Cosine	Tangent
Opposite	Adjacent	Opposite
Hypotenuse	Hypotenuse	Adjacent

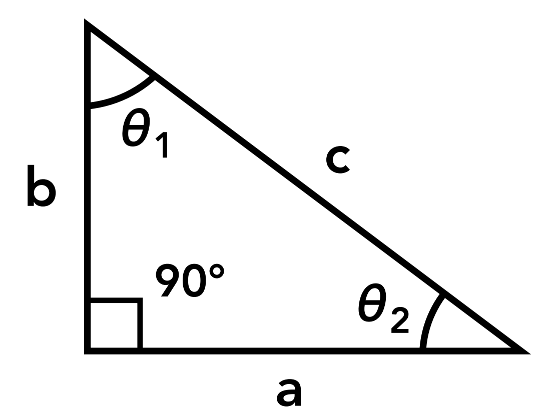
$$\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}} \qquad \theta = \sin^{-1}\left(\frac{\text{Opposite}}{\text{Hypotenuse}}\right) = \text{arcsin}(\,)$$

$$\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}} \qquad \theta = \cos^{-1}\left(\frac{\text{Adjacent}}{\text{Hypotenuse}}\right) = \text{arccos}(\,)$$

$$\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}} \qquad \theta = \tan^{-1}\left(\frac{\text{Opposite}}{\text{Adjacent}}\right) = \text{arctan}(\,)$$

Pythagorean Theorem

$c^2 = a^2 + b^2$



$\theta_1 + \theta_2 = 90°$

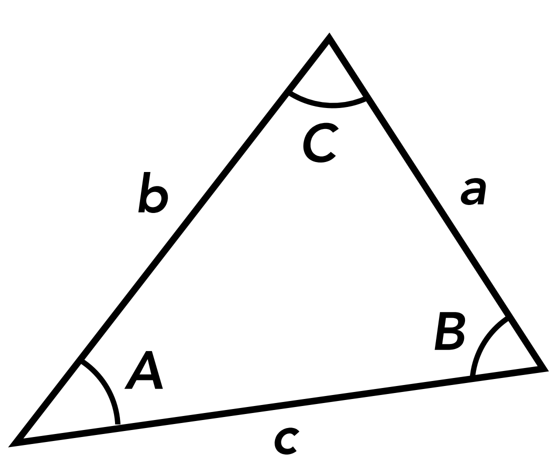
Trigonometric Identities and Laws

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$



Law of sines

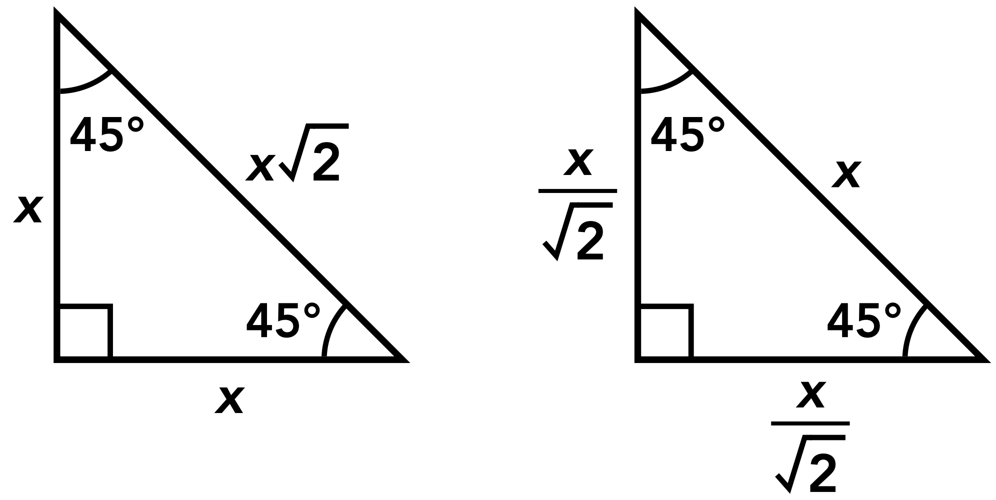
$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

Law of cosines

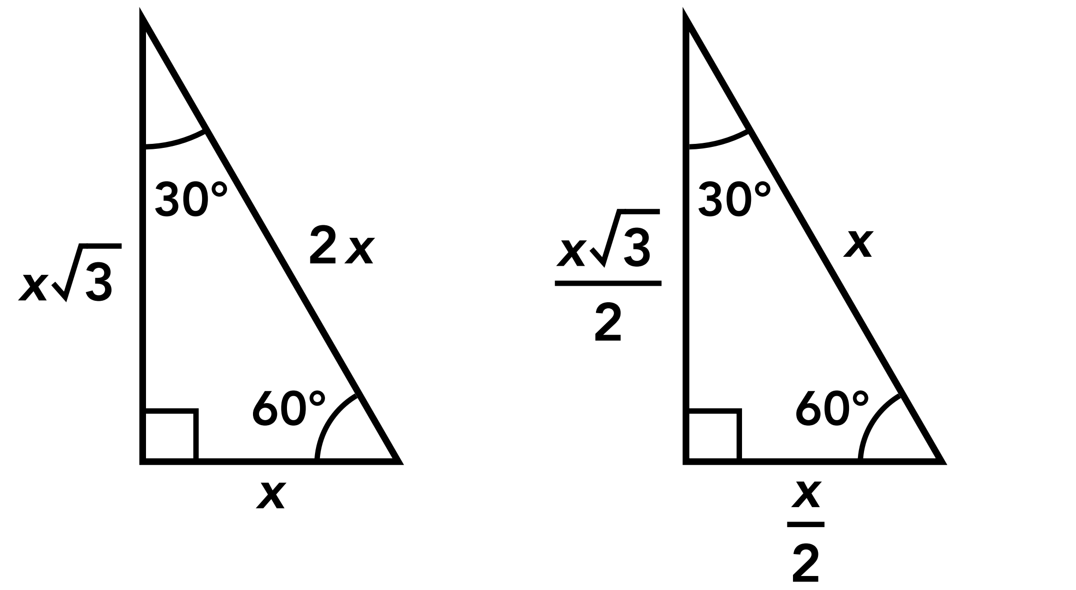
$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

Special Triangles

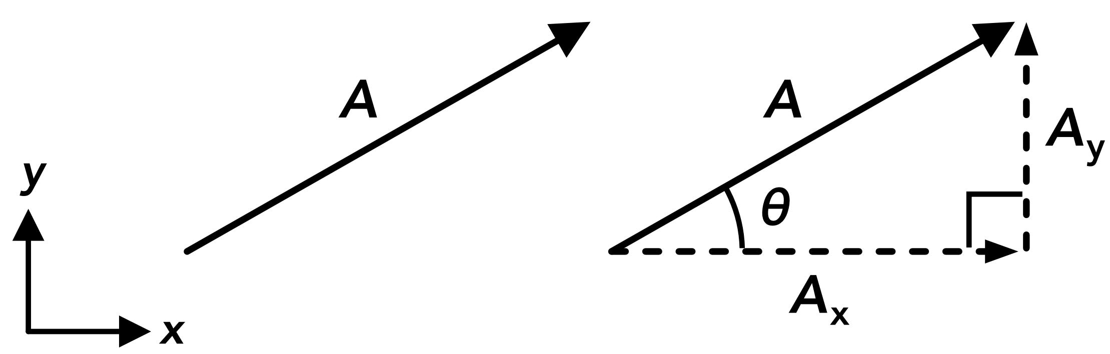
45-45-90 triangle



30-60-90 triangle

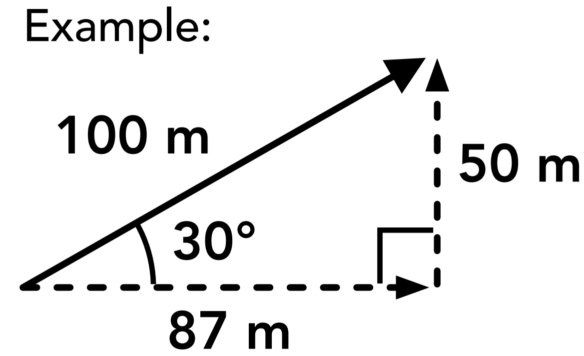


Vectors



A : magnitude
A_x: x component
A_y: y component

Example:



- A **vector** is a quantity that includes a **magnitude** and a **direction**. Some examples of vector quantities are displacement, velocity, acceleration and force.
- Vectors can be represented graphically as arrows and the ***x*** and ***y*** **components** represent the amount of the vector that points in the x and y directions.
- The **magnitude** is the value of the vector (which is always positive) and is represented by the length of the vector arrow.
- The **direction** of a vector is usually described as an angle.
- A vector and its components form a right triangle so we can use right triangle geometry to find the magnitude, angle and components.
- Each vector can be fully described using either the magnitude and direction, or the combination of the x and y components.

Magnitude and direction:

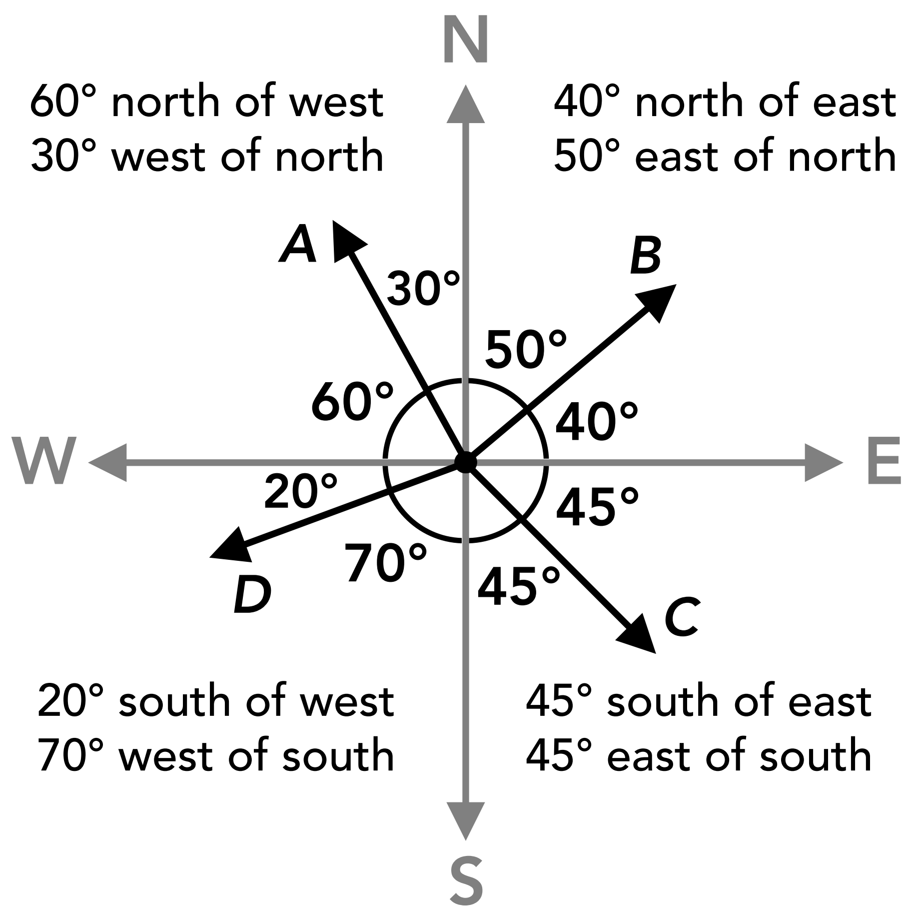
100 m, 30°

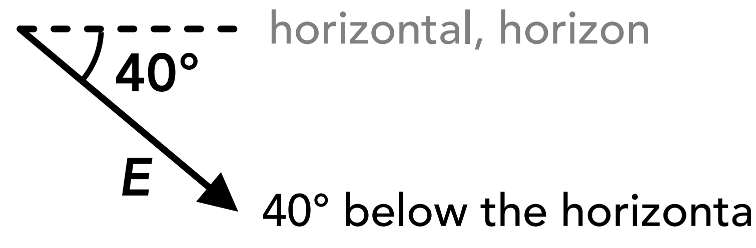
Components:

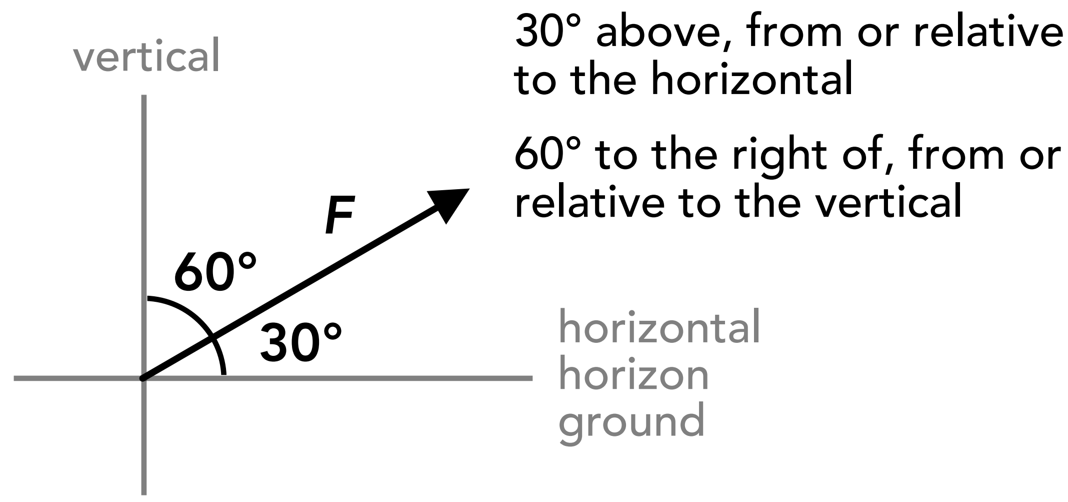
(87 m, 50 m)

$87\text{ m } \hat{i} + 50\text{ m } \hat{j}$

Vector Angles - Using Compass and Other Directions

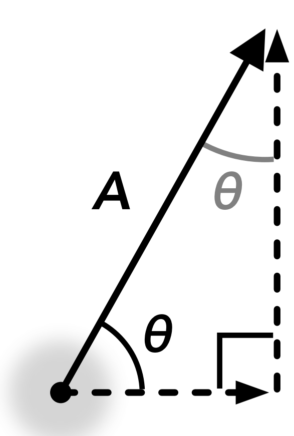




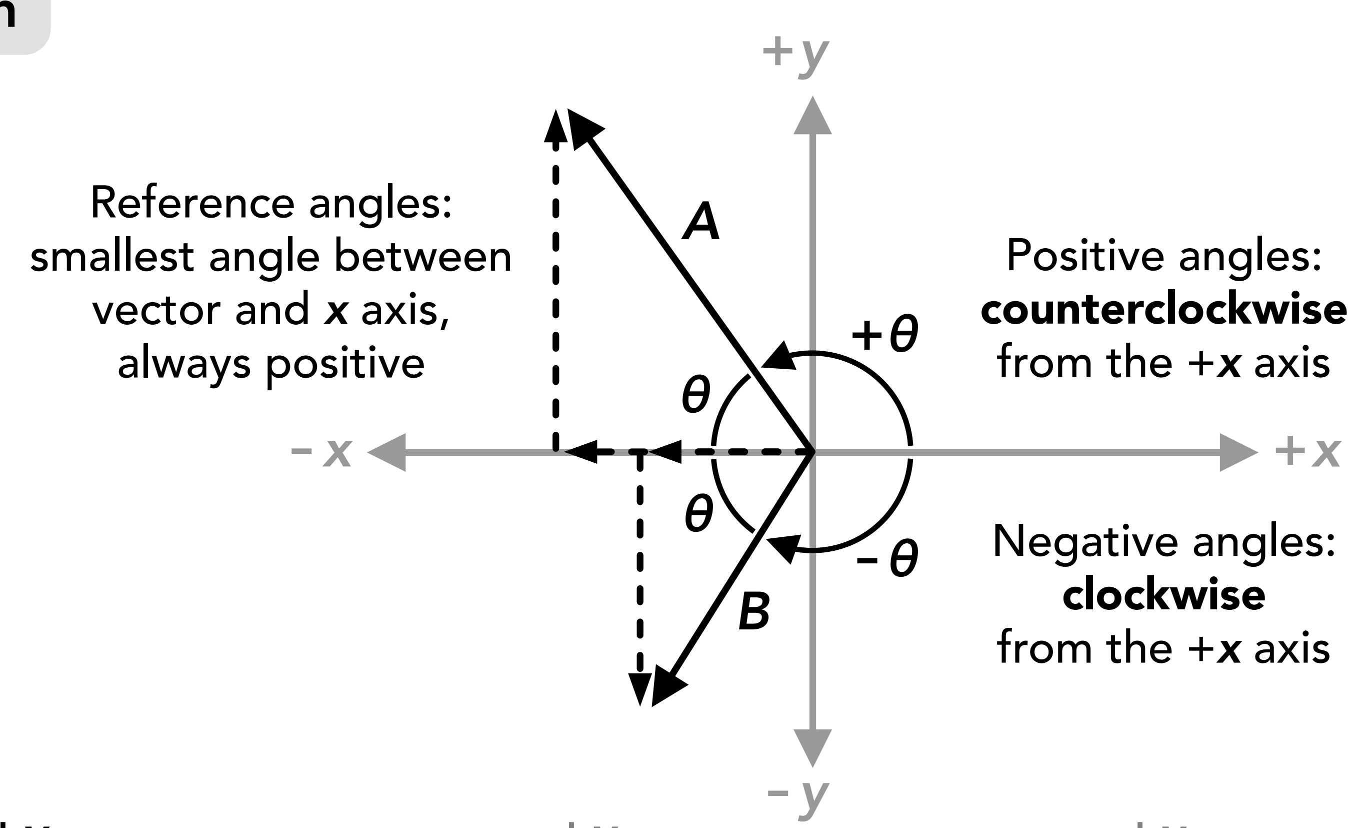


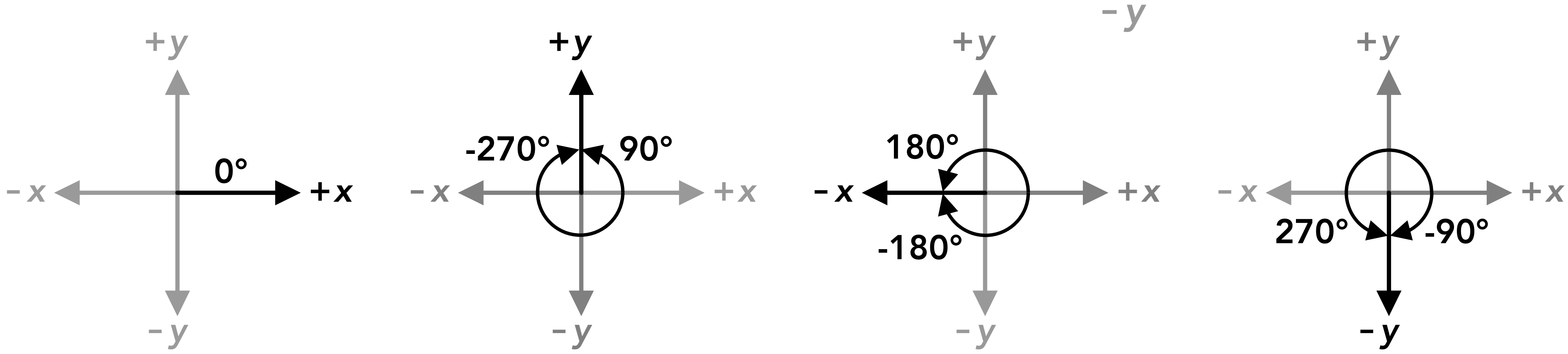
- If an angle is described as "40° north of east" we can imagine a vector that points in the east direction and then rotates 40° so it also points towards the north direction.
- An angle can be described relative to a horizontal line referred to as "the horizontal", "the horizon" or sometimes "the ground" depending on the scenario. An angle can also be described relative to a vertical line or "the vertical".

Vector Angles - Using Convention



A vector's angle is measured at the start of the vector

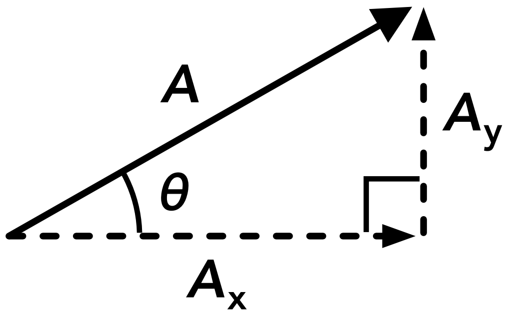




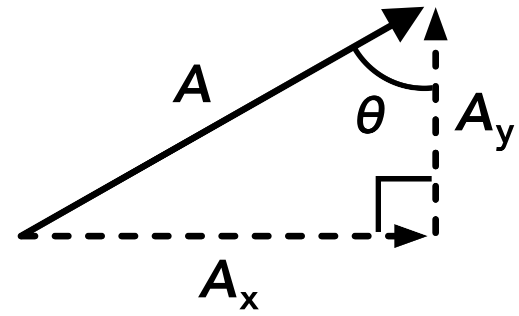
- The conventional way to describe the angle of a vector is counterclockwise from the positive ***x*** axis (0° to 360°).
- If an angle is negative then the angle is clockwise from the positive ***x*** axis (0° to -360°).
- If a vector's angle is described using a single value with no other information (such as "60°") or if the angle is greater than 90°, then the angle is likely the conventional angle.
- This conventional angle (positive or negative, -360° to 360°) can be used with the **sin**( ) and **cos**( ) functions to find the ***x*** and ***y*** components of the vector, and it will result in the correct +/- signs for the component directions.

Finding the Components of a Vector

Using a reference angle:



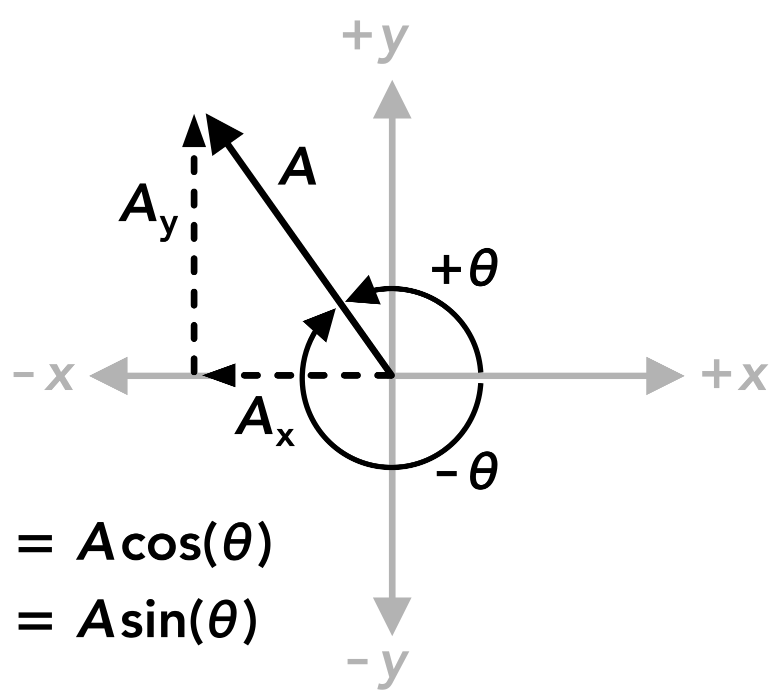
$$A_x = A\cos(\theta)$$
$$A_y = A\sin(\theta)$$



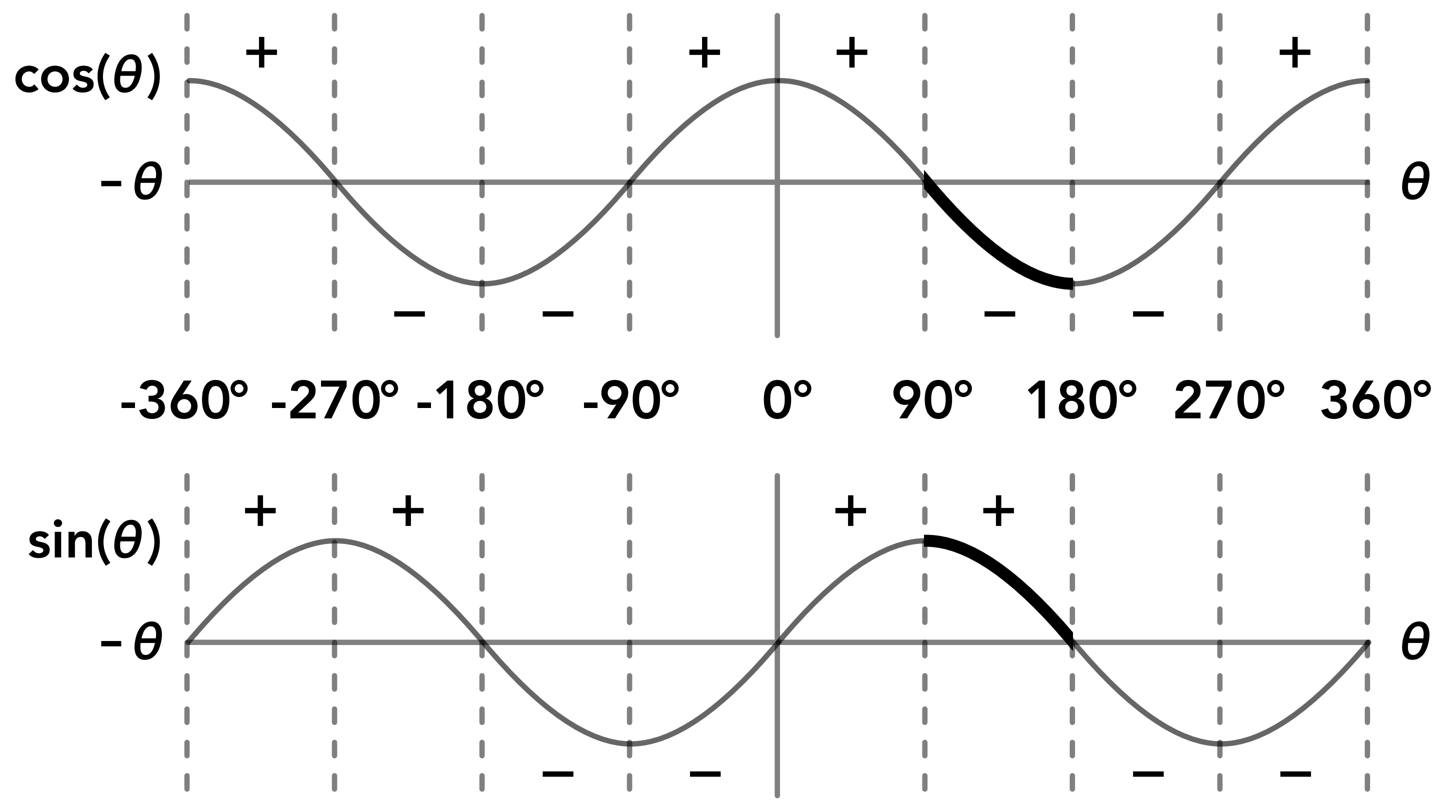
$$A_x = A\sin(\theta)$$
$$A_y = A\cos(\theta)$$

- A vector and its components form a right triangle: the magnitude of a vector is the length of the hypotenuse, and the ***x*** and ***y*** components are the two legs.
- The angle between the vector and the ***x*** component is often used but not always, so don't memorize if the ***x*** or ***y*** components go with **sin**( ) or **cos**( ), just remember how to use the right triangle trig functions.

Using the conventional angle:

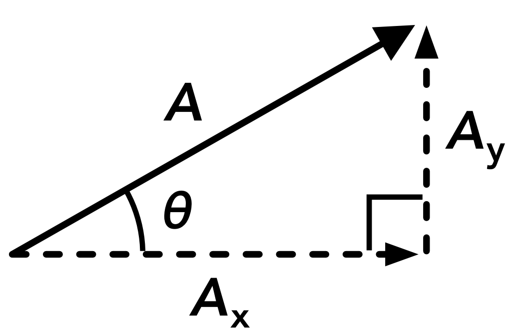


$$A_x = A\cos(\theta)$$
$$A_y = A\sin(\theta)$$

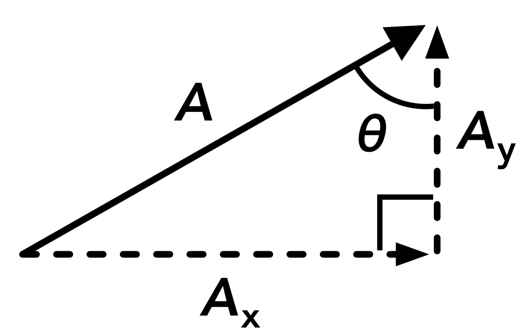


- The conventional angle (-360° to 360°) can be used with the **sin**( ) and **cos**( ) functions to find the components of the vector with the correct +/- signs for the component directions, regardless of the vector's angle.
- The ***x*** component uses **cos**( ) and the ***y*** component uses **sin**( ).

Finding the Magnitude and Angle of a Vector



$$A = \sqrt{A_x^2 + A_y^2}$$
$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

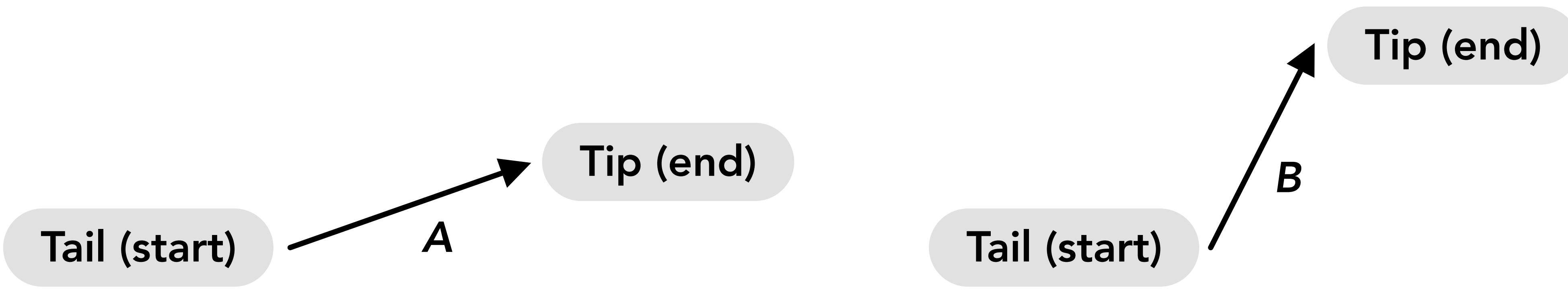


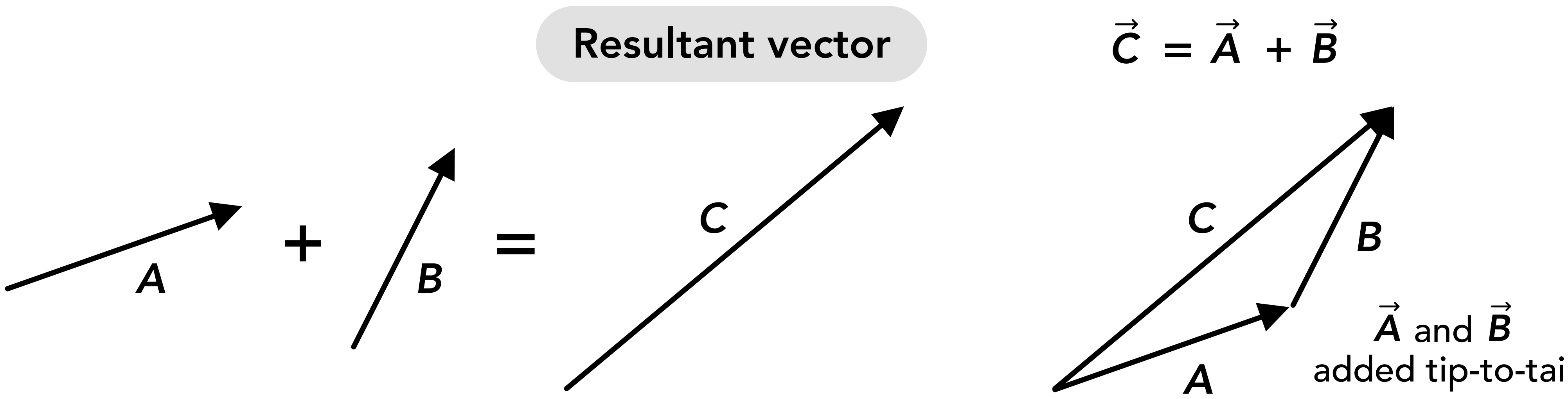
$$A = \sqrt{A_x^2 + A_y^2}$$
$$\theta = \tan^{-1}\left(\frac{A_x}{A_y}\right)$$

Note: Plug positive values into the **tan^{-1}( )** function and the result will be a positive reference angle

- Use the Pythagorean Theorem to find the magnitude of the vector which is the length of the hypotenuse.
- The inverse **tan**( ) relationship can always be used to find the angle, but once we know the components we can also use one of the other inverse trig relationships.

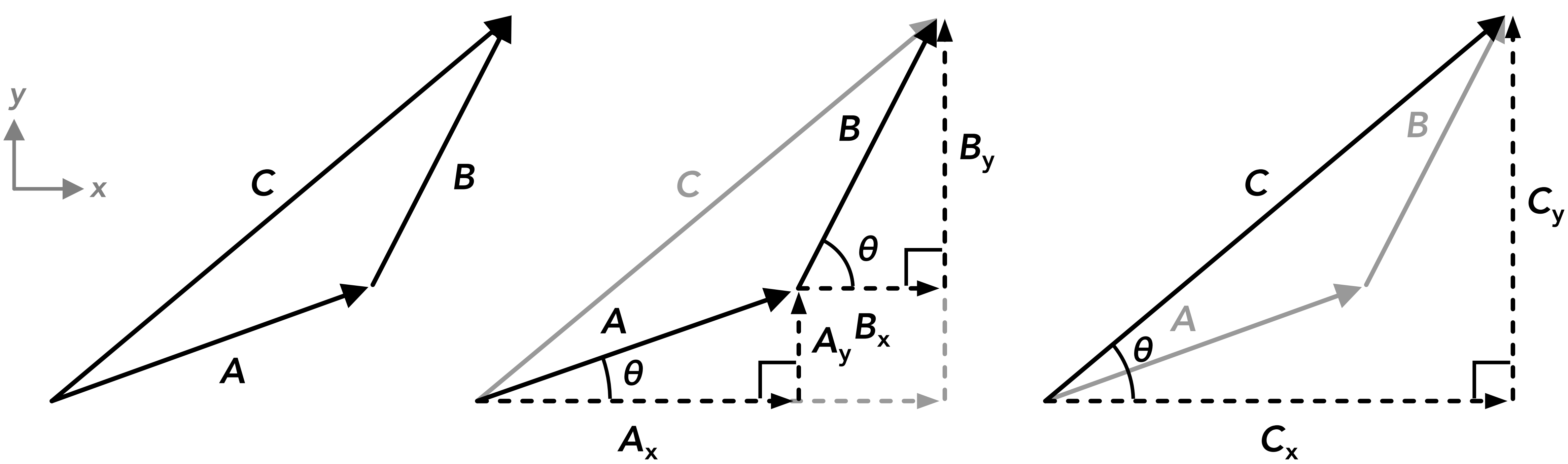
Adding Vectors Graphically Using the Tip-to-Tail Method





- We can add vectors graphically by drawing them out using the **tip-to-tail method**.
- The **tail** is the start of the vector and the **tip** is the end of the vector (the tip of the arrow).
- Each new vector to be added starts at the tip (end) of the previous vector.
- The **resultant** vector is the sum of the other vectors and it points from the tail (start) of the first vector to the tip (end) of the last vector. Any number of vectors can be added together in this way.

Adding Vectors Using Components



- Find the ***x*** and ***y*** components of each individual vector, add the ***x*** components together, then add the ***y*** components together.
- Note that a vector is the sum of its 2 component vectors.

$$\boxed{\vec{C} = \vec{C}_x + \vec{C}_y}$$

$$\vec{C} = \vec{A} + \vec{B}$$

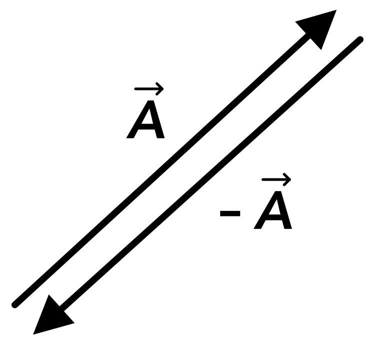
$$\vec{C}_x = \vec{A}_x + \vec{B}_x$$

$$\vec{C}_y = \vec{A}_y + \vec{B}_y$$

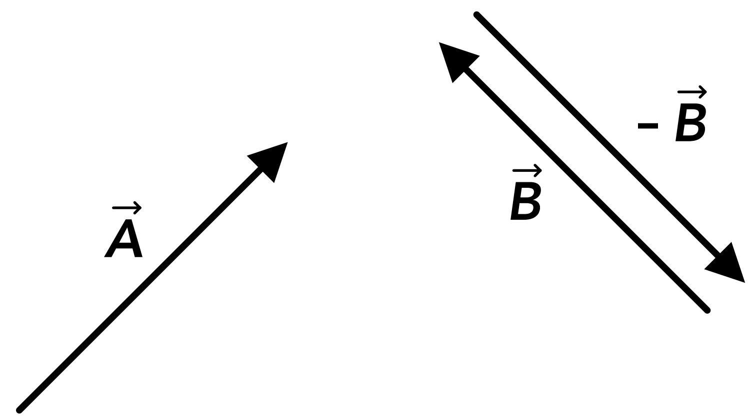
$$C = \sqrt{C_x^2 + C_y^2}$$

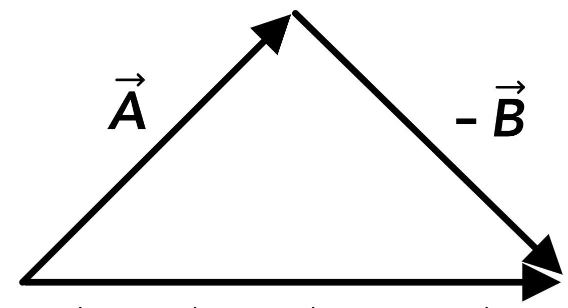
$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right)$$

Negative Vectors and Subtracting Vectors



$\vec{A} + (-\vec{A}) = \vec{0}$

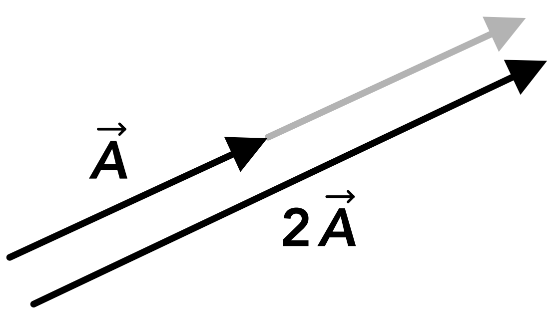


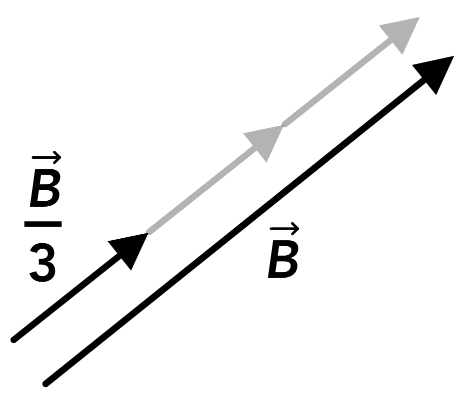


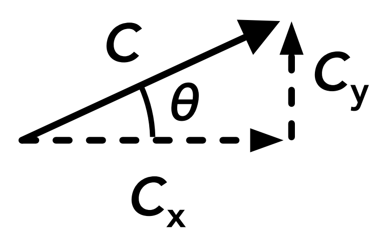
$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

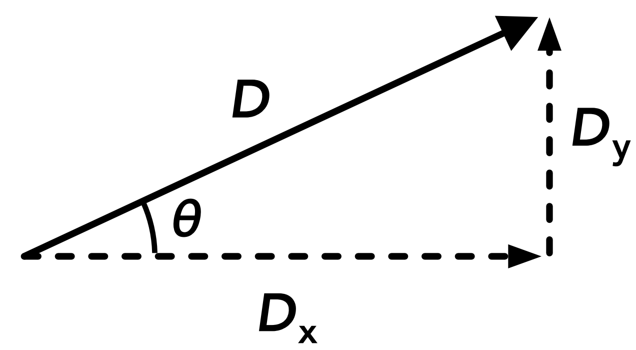
- The negative of a vector has the same magnitude but the opposite direction as the original vector.
- Adding a vector with its negative results in the **0 vector** so they "cancel" each other.
- Subtracting a vector is the same as adding its negative vector.

Multiplying and Dividing Vectors by a Scalar Value









$\vec{D} = 2\vec{C} \quad \vec{D}_x = 2\vec{C}_x \quad \vec{D}_y = 2\vec{C}_y$

- Multiplying or dividing a vector by a scalar (a number) scales the magnitude (length) of the vector but doesn't change its direction (angle).
- The components are each scaled (multiplied or divided) by the same value as the vector.