Tailored Tutors

ALGEBRAIC TECHNIQUES

Algebraic Fractions

- [ ] Simplify rational expressions, including factorising, cancelling, algebraic division

Partial Fractions

- [ ] Decompose rational functions into partial fractions
- [ ] Use partial fractions to integrate, differentiate or expand rational functions

Tailored Tutors

SEQUENCES & SERIES

Binomial Expansion

- [ ] Use the binomial expansion $(a + bx)^n$ for fractional and negative values of n
- [ ] Be aware that the expansion is valid for $|\frac{bx}{a}| < 1$

Notation & Language of Series

- [ ] Work with sequences given by a formula for the nth term
- [ ] Work with sequences generated by a simple relation of the form $x_{n+1} = f(x_n)$
- [ ] Identify increasing sequences, decreasing sequences and periodic sequences
- [ ] Understand and use sigma notation for sums of series

Arithmetic Series & Geometric Series

- [ ] Use the formula for the nth term and the sum to n terms of an arithmetic sequence
- [ ] Use the formula for the nth term and the sum to n terms of a finite geometric sequence
- [ ] Use the formula for the sum to infinity of a convergent geometric series where $|r| < 1$
- [ ] Use sequences and series in modelling, eg amounts paid into saving schemes

Tailored Tutors

NUMERICAL METHODS

Change of Sign Argument

- [ ] Locate roots of $f(x) = 0$ by considering changes of sign of $f(x)$ in an interval of x
- [ ] Understand that change of sign methods can fail if $f(x)$ is not continuous or roots > 1

Simple Iterative Methods

- [ ] Use an iteration in the form $x_{n+1} = f(x_n)$ to find a root to the equation $x = f(x)$
- [ ] Know that the iteration $x_{n+1} = g(x_n)$ converges to a root at $x = a$ if $|g'(a)| < 1$
- [ ] Draw cobweb and staircase diagrams to illustrate simple iterative methods

Newton-Raphson

- [ ] Solve equations using the Newton-Raphson method and other recurrence relations
- [ ] Understand the Newton-Raphson fails if the initial value coincides with a stationary point

Small Angle Approximations

- [ ] Understand and use the small angle approximations for sine: $\sin\theta \approx \theta$
- [ ] Understand and use the small angle approximation for cosine: $\cos\theta \approx 1 - \frac{\theta^2}{2}$
- [ ] Understand and use the small angle approximation for tangent: $\tan\theta \approx \theta$

Tailored Tutors

FUNCTIONS & GRAPHS

Functions

- [ ] Understand and use composite functions, inverse functions and their graphs
- [ ] Understand that a function is a one-one or many-one mapping
- [ ] Understand and find the domain and range of functions
- [ ] Understand that $y = f^{-1}(x)$ is a reflection of the graph $y = f(x)$ in the line $y = x$
- [ ] Use functions in modelling, understanding the limitations and refinements

Modulus Equations & Inequalities

- [ ] Sketch the graphs of $y = |ax + b|$
- [ ] Use the graph to solve modulus equations and inequalities, eg $y = |2x - 1|$
- [ ] Use relations such as $|a| = |b| \Leftrightarrow a^2 = b^2$ and $|x - a| < b \Leftrightarrow a - b < x < a + b$

Graph Transformations

- [ ] Apply multiple transformations to functions of x^2, x^3, x^4, $\frac{1}{x}$, $\frac{1}{x^2}$, $|x|$, $\sin x$, $\cos x$, $\tan x$, e^x, a^x
- [ ] Sketch the graphs of $y = |f(x)|$ and $y = |f(-x)|$ given $y = |f(x)|$

Parametrically Defined Functions

- [ ] Understand and use the parametric equations of curves
- [ ] Convert between Cartesian and parametric forms
- [ ] Use parametric equations in modelling in a variety of contexts

VECTORS & TRIANGLES

Vectors in 3 Dimensions

- [ ] Use vectors in 3 dimensions, in the form of column vectors and as $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ unit vectors
- [ ] Find the magnitude and direction of 3D vectors
- [ ] Use scalar multiplication and vector addition for 3D vectors

Formulae for Sectors

- [ ] Work with radian measure, including arc length $(s = r\theta)$ and area of sector $(\frac{1}{2}r^2\theta)$

Tailored Tutors

TRIGONOMETRY

Reciprocal Trig Functions

- [ ] Know and use exact values of $\sin$, $\cos$ and $\tan$ for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi$ and multiples
- [ ] Understand and use the definitions of secant, cosecant and cotangent
- [ ] Understand the graphs, ranges and domains of the reciprocal trig functions

Pythagorean Identities

- [ ] Understand and use $\sec^2\theta = 1 + \tan^2\theta$ and $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

Addition Formulae

- [ ] Understand and use the formulae $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$
- [ ] Understand the geometrical proofs for these formulae

Double/Half Angle Formulae

- [ ] Understand and use double angle formulae for $\sin$, $\cos$ and $\tan$

Harmonic Form $R\cos(x + a)$

- [ ] Convert the expression $a\cos\theta + b\sin\theta$ into the form $r\cos(\theta \pm \alpha)$ or $r\sin(\theta \pm \alpha)$
- [ ] Solve equations such as $a\cos\theta + b\sin\theta = c$ in a given interval

Inverse Trig Functions

- [ ] Understand and use the definitions of arcsin, arccos and arctan
- [ ] Understand the graphs, ranges and domains of the inverse trig functions

Tailored Tutors

DIFFERENTIATION

Increasing, Decreasing, Concave & Convex Graphs

- [ ] Use the second derivative to determine if a graph is convex or concave over an interval
- [ ] Use the second derivative to find point(s) of inflection of a graph

The Derivatives

- [ ] Differentiate e^{kx}, a^{kx}, $\ln x$, $\sin kx$, $\cos kx$, $\tan kx$ and related multiples
- [ ] Show differentiation from first principles for $\sin x$ and $\cos x$

The Chain Rule

- [ ] Understand and use the chain rule to differentiate functions
- [ ] Use connected rates of changes in models
- [ ] Differentiate parametric functions to find the equations of tangents and/or normals

The Product & Quotient Rules

- [ ] Understand and use the quotient rule and the product rule to differentiate functions
- [ ] Differentiate $\operatorname{cosec} x$, $\cot x$ and $\sec x$
- [ ] Differentiate functions such as $2x^4 \sin x$, $\frac{e^{3x}}{x}$, $\cos^2 x$ and $\tan^2 2x$

Implicit Differentiate and Parametric Differentiation

- [ ] Differentiate functions in the form $x = f(y)$, eg $x = \sin y$, then use $\frac{dy}{dx} = 1 \div (\frac{dx}{dy})$
- [ ] Differentiate simple parametrically defined functions

Tailored Tutors

INTEGRATION

Integrate Fractions, Exponentials and Trig

- [ ] Be able to integrate e^{kx}, $\frac{1}{x}$, $\sin kx$, $\cos kx$ and related multiples

Integration by Substitution

- [ ] Carry out simple cases of integration by substitution
- [ ] Understand that integration by substitution is the inverse of the chain rule
- [ ] Find and use an appropriate substitution for integration by substitution
- [ ] Recognise an integrand of the form $\frac{kf'(x)}{f(x)}$

Parametric Integration

- [ ] Evaluate the area of a region bounded by a parametrically defined curve

Integration by Parts

- [ ] Carry out simple cases of integration by parts
- [ ] Understand that integration by parts is the inverse of the product rule
- [ ] Use more than one application of integration by parts, eg for $x^2 \sin x$
- [ ] Apply integration by parts to the integral $\ln x$ and related functions

Form & Solve Differential Equations

- [ ] Evaluate the solution of simple first order differential equations with separate variables
- [ ] Interpret the solution of a differential equation in context, eg kinematics

PROOF

Proof by Contradiction

- [ ] Use proof by contradiction to prove the irrationality of $\sqrt{2}$
- [ ] Use proof by contradiction to prove the infinity of primes
- [ ] Apply proof by contradiction to unfamiliar proofs

Product Moment Correlation Coefficient

- [ ] Know that the product moment correlation coefficient r satisfies $|r| \leqslant 1$
- [ ] Know that if $r = \pm 1$ all of the data points lie on a straight line
- [ ] Calculate r using a calculator (Edexcel)

Tailored Tutors

PROBABILITY

Independent Events

- [ ] Use set notation to describe events
- [ ] Use $\mathrm{P}(B \mid A) = \mathrm{P}(B)$, $\mathrm{P}(A \mid B) = \mathrm{P}(A)$ when A and B are independent events
- [ ] Use $\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$ when A and B are independent events

Conditional Probability

- [ ] Understand and use conditional probability and the conditional probability formula
- [ ] Use conditional probability in tree diagrams, Venn diagrams and two-way tables
- [ ] Understand and use $\mathrm{P}(A') = 1 - \mathrm{P}(A)$
- [ ] Understand and use $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$
- [ ] Understand and use $\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A)$
- [ ] Model with probability, including critiquing assumptions made and their effect

Tailored Tutors

NORMAL DISTRIBUTION

Key Features of a Normal Distribution

- [ ] Know the shape and symmetry of the normal distribution
- [ ] Know that the points of inflection are at $x = \mu \pm \sigma$
- [ ] Understand and use the notation $X \sim \mathrm{N}(\mu, \sigma^2)$
- [ ] Know approximately two-thirds of the data lies in the range $\mu \pm \sigma$
- [ ] Know approximately $95\,\%$ of the data lies in the range $\mu \pm 2\sigma$
- [ ] Know almost all data lies in the range $\mu \pm 3\sigma$

Using the Normal Distribution

- [ ] Understand and use the normal distribution as a model
- [ ] Find probabilities using the normal distribution by using a calculator
- [ ] Be able to recognise when the binomial or normal model may not be appropriate
- [ ] Be able to link the normal distribution to histograms

Using the Normal to Approximate the Binomial

- [ ] Know when n is large, $p \approx 0.5$, $\mathrm{B}(n, p)$ can be approximated by $\mathrm{N}(np, np[1-p])$
- [ ] Apply a continuity correction to this approximation (Edexcel)

Tailored Tutors

HYPOTHESIS TESTING

Hypothesis Testing for the Mean of a Population

- [ ] Conduct a statistical hypothesis test for the mean of a Normal distribution
- [ ] Know that if $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{x} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$
- [ ] Interpret the results of the hypothesis test in context

Hypothesis Testing for Zero Correlation

- [ ] Be able to interpret a correlation coefficient given a p-value or critical value
- [ ] State hypotheses in terms of ρ where $\rho =$ the population correlation coefficient

Tailored Tutors

CONSTANT ACCELERATION

Projectiles

- [ ] Model motion under gravity in a vertical plane using vectors
- [ ] Derive formulae for time of flight, range, and greatest height

VARIABLE ACCELERATION

2D Variable Acceleration

- [ ] Differentiation and integration of vectors with respect to time

Tailored Tutors

FORCE ACCELERATION PROBLEMS

Friction

- [ ] Understand and use the coefficient of friction, μ
- [ ] Understand that, for a body in motion, $F = \mu R$
- [ ] Understand that, for a body at equilibrium, $F \leqslant \mu R$
- [ ] Solve problems involving a body on a rough surface

Resolving Forces

- [ ] Understand and use the term resultant as applied to 2+ forces acting at a point
- [ ] Understand and use Newton's second law when forces need to be resolved
- [ ] Understand and use Newton's third law when forces need to be resolved

Motion on an Inclined Plane

- [ ] Use the addition of forces and resultant forces to model motion on an inclined plane

MOMENTS

Moments

- [ ] Calculate the moment of a force about an axis through a point in the plane of the body
- [ ] Understand that when a rigid body is in equilibrium the resultant moment is zero
- [ ] Apply moments to simple static problems, eg ladders, uniform/non-uniform rods, laminas