STUDY GUIDE

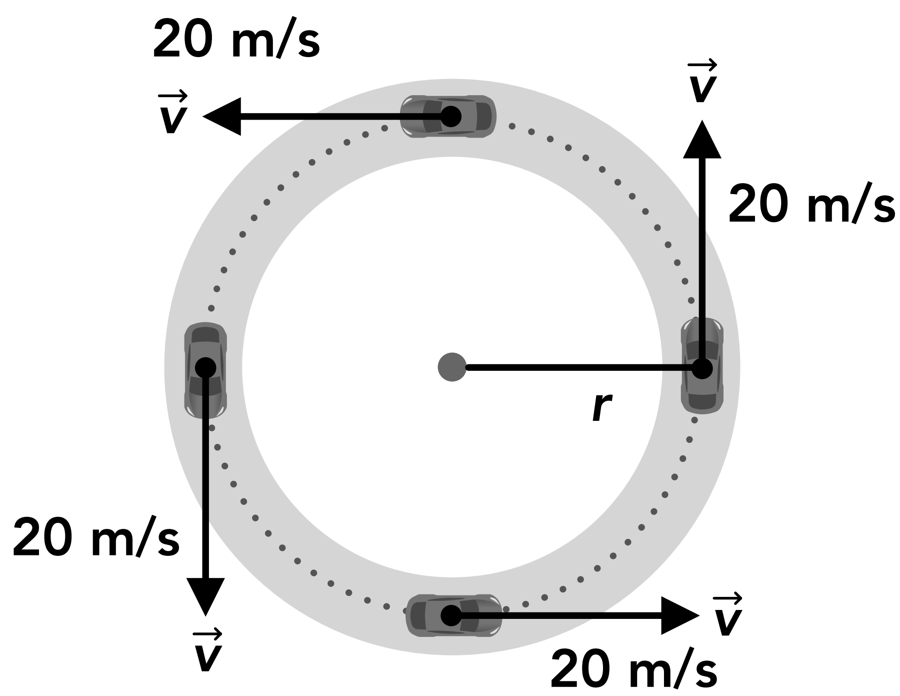
CENTRIPETAL ACCELERATION & FORCE

Uniform Circular Motion

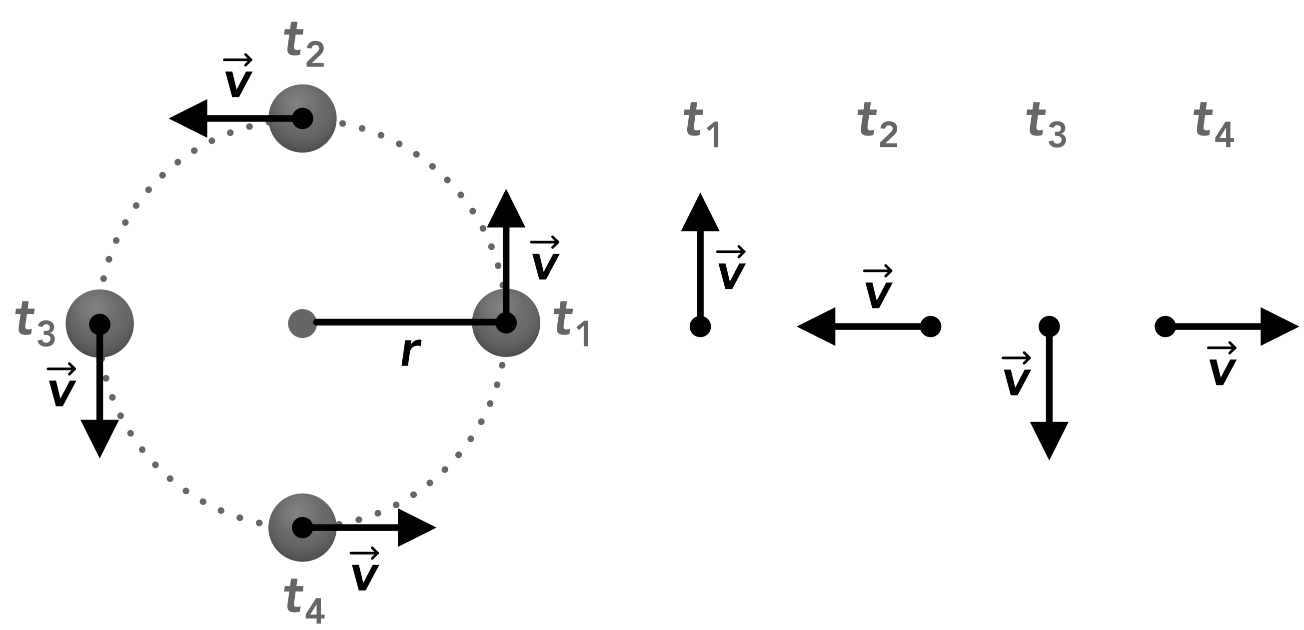
- **Uniform circular motion** is when an object travels in a circular path with a constant speed.
- The direction of the velocity is constantly changing, but the magnitude (speed) stays the same.
- We call the acceleration of the object the **centripetal acceleration** which is covered in another section.
- An object can be in uniform circular motion without completing a full circle.

Variables		SI Unit
v	velocity	$\frac{m}{s}$
r	radius	m
T	period	s
f	frequency	$Hz = \frac{cycles}{s}$
ω	angular velocity	$\frac{rad}{s}$

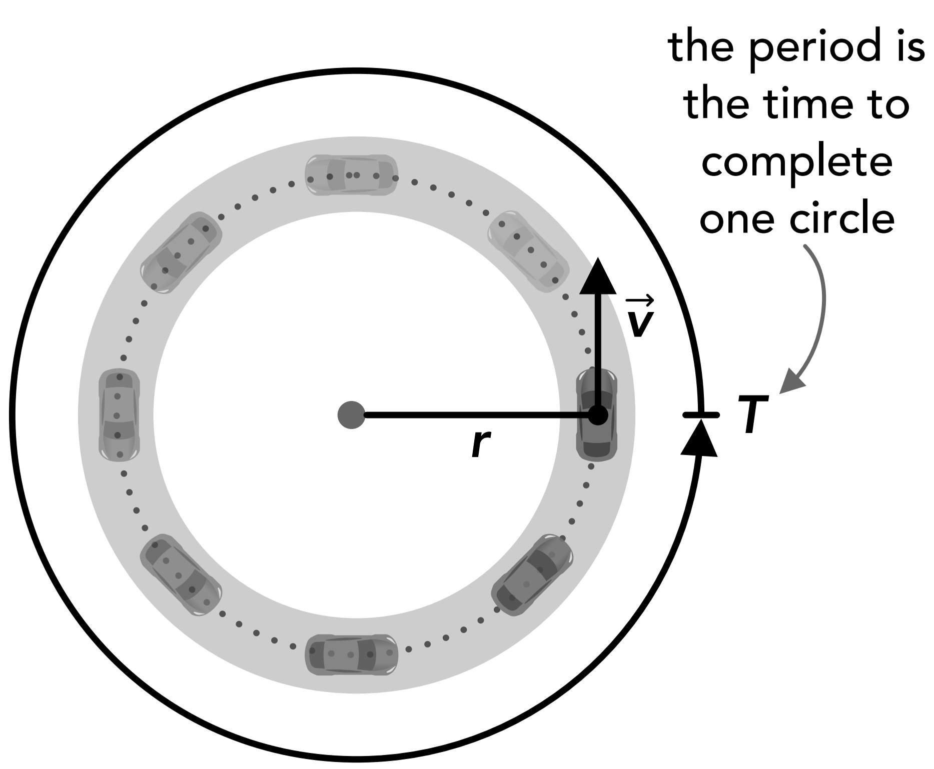
A car in uniform circular motion



The direction of the the velcocity is constantly changing but the magnitude (speed) stays the same



- Sometimes an object in uniform circular motion will repeat several revolutions over and over. In those cases we can describe the motion using period and frequency.
- **Period** (T) is the amount of time it takes to complete one circle or revolution. The unit of period is seconds (s).
- **Frequency** (f) is the inverse of the period (**1/**T) and is the number of circles traveled per second. The unit of frequency is Hertz (Hz) which is cycles/second or 1/s (the numerator has no unit, it's just "something" per second like circles/second, revolutions/second, etc).



Frequency

$$f = \frac{1}{T}$$

f : frequency (Hz, cycles/s)
T : period (s)

an object travels one circumference in one period, so the velocity is related to the period and frequency

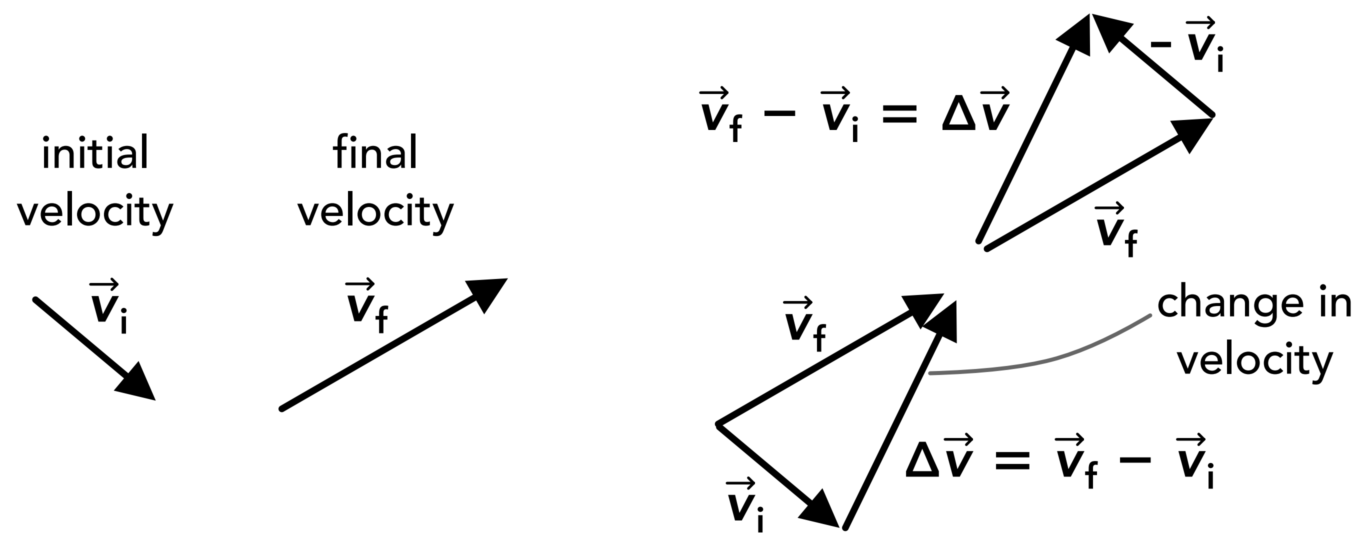
$$v = \frac{2\pi r}{T} \qquad v = 2\pi r f$$

Centripetal Acceleration

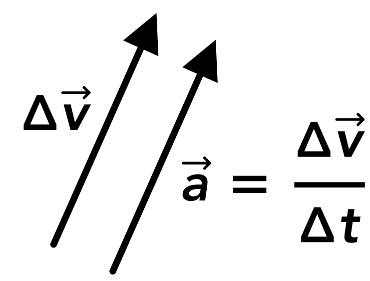
- To understand centripetal acceleration and its related motion, it will help to review velocity and acceleration vectors.
- Acceleration is the change in velocity divided by a period of time.
- Acceleration and velocity are both vector quantities which have a magnitude (value) and a direction, so the acceleration can change the **magnitude** of the velocity (the speed) or the **direction** of the velocity.
- Remember that vectors can be added or subtracted using the tip-to-tail method.

Variables		SI Unit
a_c	centripetal acceleration	$\frac{m}{s^2}$
a	acceleration	$\frac{m}{s^2}$
v	velocity	$\frac{m}{s}$
r	radius	m
t	time	s

The vector representing the change in velocity is the final velocity vector minus the initial velocity vector, which can be found using the tip-to-tail method in one of two ways

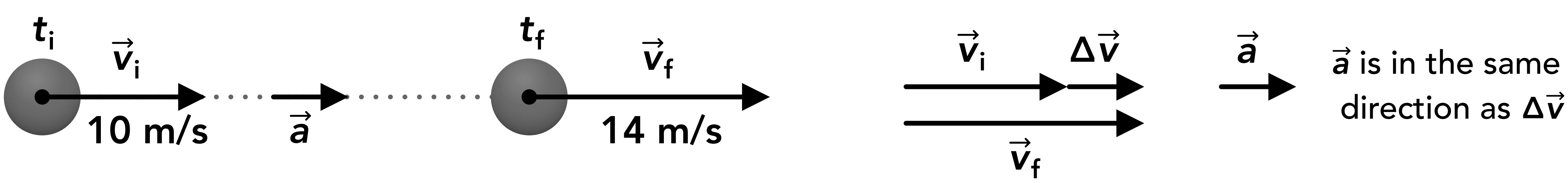


The acceleration vector points in the same direction as the change in velocity vector

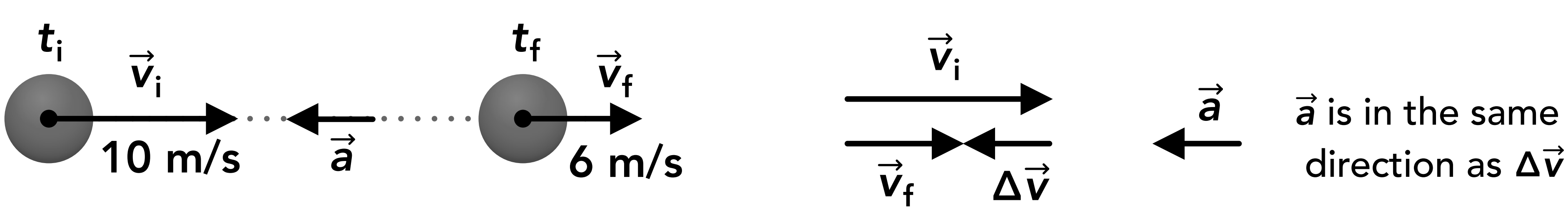


- Let's look at two examples below of an object moving in a straight line. The velocity vectors at two timepoints are shown. What is the direction of the acceleration vector that would cause that change in the velocity vector?

If $\vec{a}$ is parallel to $\vec{v}$ and points in the same direction, the magnitude (speed) of $\vec{v}$ increases and the direction of $\vec{v}$ doesn't change

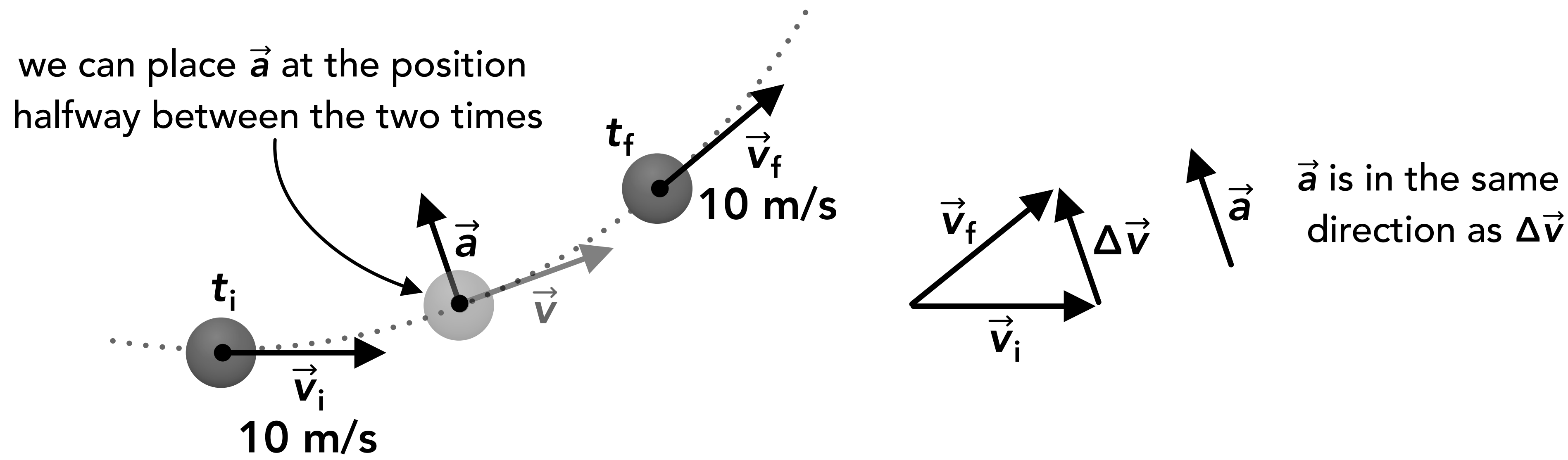


If $\vec{a}$ is parallel to $\vec{v}$ and points in the opposite direction, the magnitude (speed) of $\vec{v}$ decreases and the direction of $\vec{v}$ doesn't change

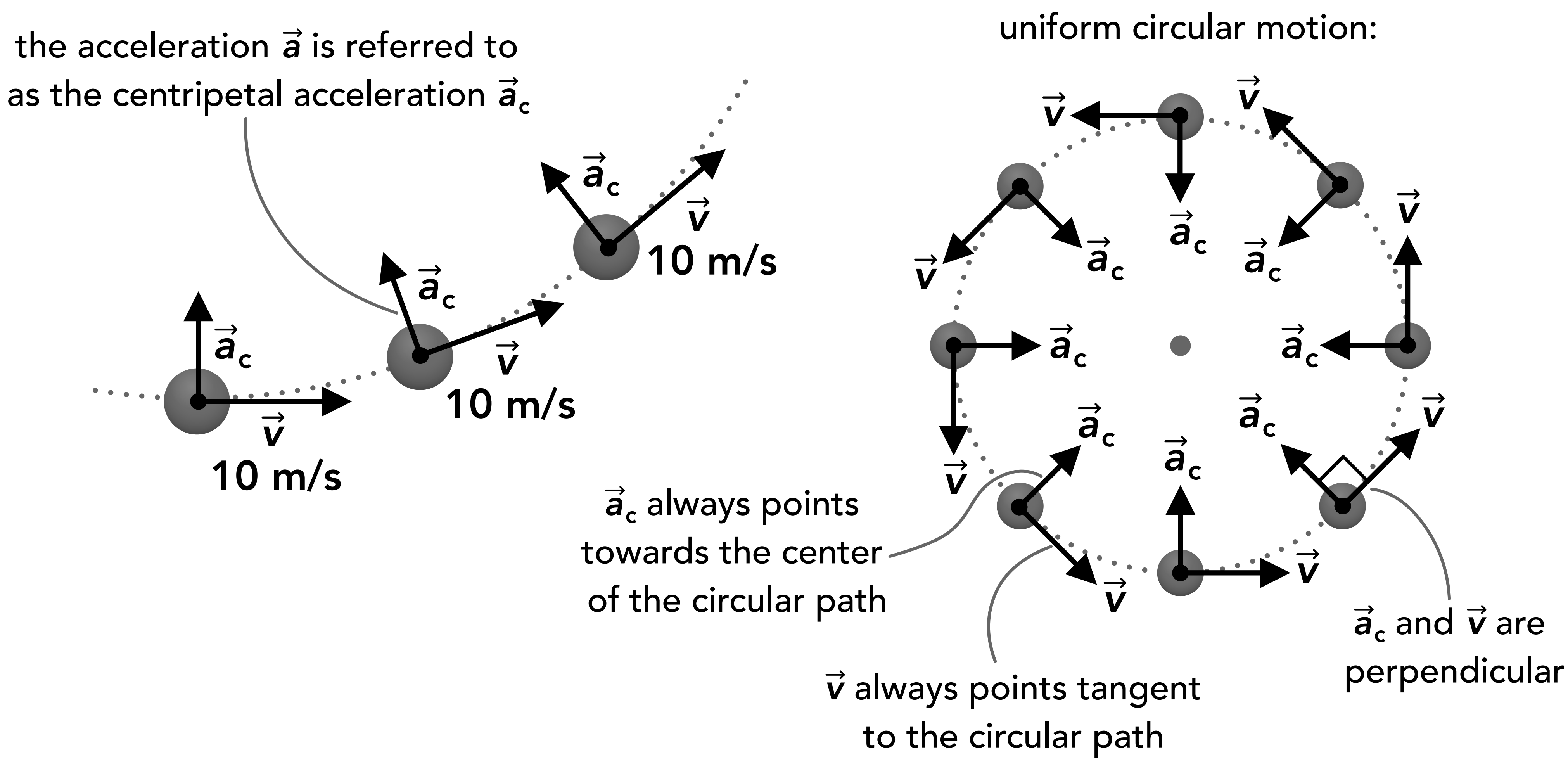


- Now let's look at an object following a curved circular path where the magnitude of the velocity vector (speed) doesn't change. Remember, acceleration can change the direction of the velocity vector, not just the magnitude. What is the direction of the acceleration vector that would cause this change in the velocity vector?

If $\vec{a}$ is not parallel to $\vec{v}$, it causes $\vec{v}$ to change direction and follow a curved path



If $\vec{a}$ is always perpendicular to $\vec{v}$, the direction of $\vec{v}$ changes and the object follows a circular path. The magnitude of $\vec{v}$ doesn't change because no component of $\vec{a}$ points in the same direction as $\vec{v}$



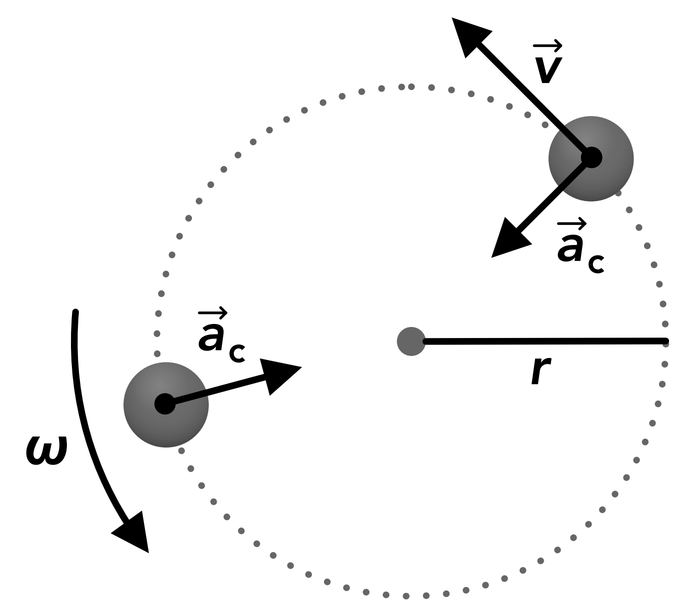
- If the acceleration vector is continuously perpendicular to the velocity vector, the velocity will change direction and end up following a circular path without changing speed. This is uniform circular motion.

- We call the acceleration which results in circular motion the **centripetal acceleration**. "Centripetal" means **acting towards the center** which is the direction of the centripetal acceleration.
- Conceptually, it's important to note that there is nothing special about a centripetal acceleration and this is not some new "type" of acceleration. If an object is moving along a circular path there must be an acceleration that is perpendicular to its velocity and points towards the center of the circle.
- In a physical scenario, the cause of a centripetal acceleration is a **centripetal force** which is any net force that acts towards the center of the circular path. This could be the tension force of a rope, the normal force of a circular track, or any other type of force. Centripetal force is covered in another section.
- The magnitude of the centripetal acceleration is given by the equation below. The greater the speed of the object and the smaller the radius of the circle, the greater the centripetal acceleration that is required to keep the object moving in a circle at that speed.

Centripetal acceleration

$$\vec{a}_c = \frac{v^2}{r} \text{ (towards center of circle)}$$

v : tangential speed (m/s)
r : radius of circular path (m)



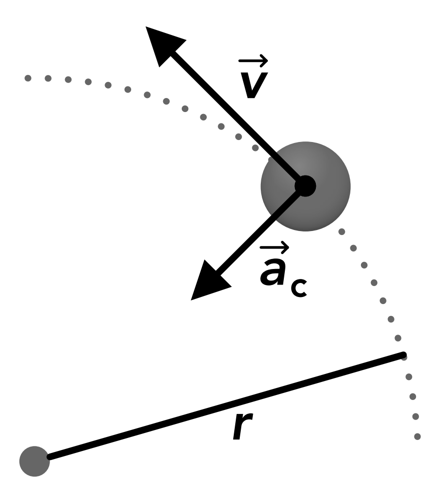
Centripetal acceleration
(other variables substituted for speed)

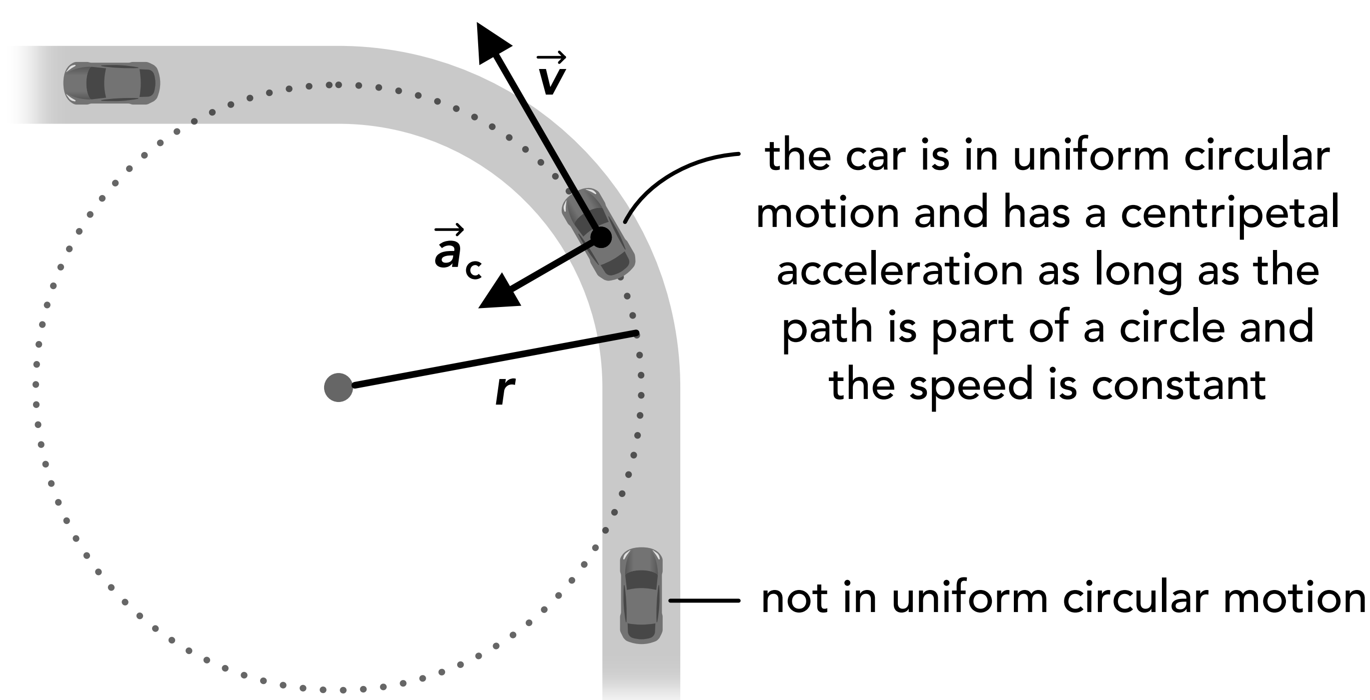
$$a_c = \frac{v^2}{r} = \omega^2 r = (2\pi f)^2 r = \left(\frac{2\pi}{T}\right)^2 r$$

ω : angular speed (rad/s)
f : frequency (Hz = rev/s)
T : period (s)

$$f = \frac{1}{T}$$

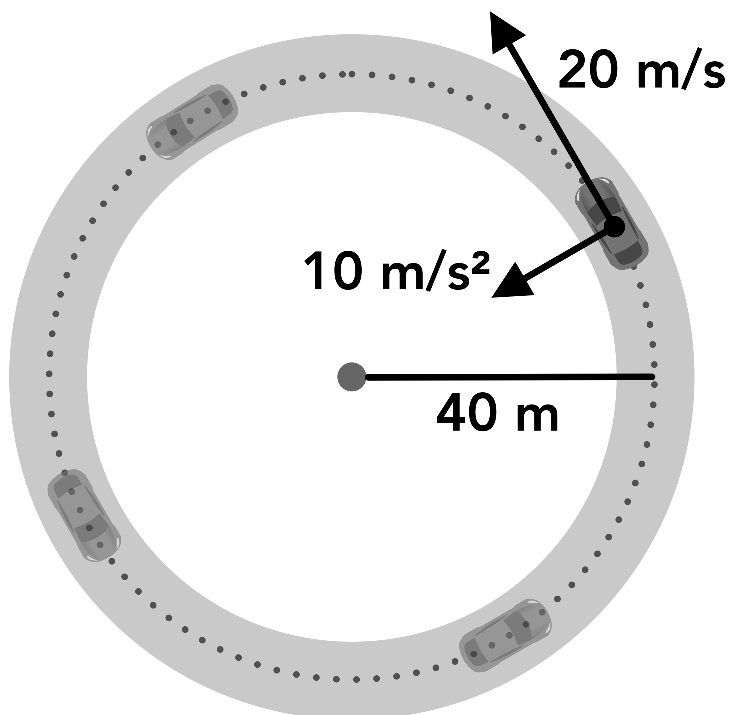
- Even if an object only travels along a segment of a circular path (instead of a full circle) we still consider the object to be in uniform circular motion and the acceleration is still centripetal acceleration.





Calculating centripetal acceleration using different variables for speed

A car is driving around a circle at a constant speed



$$a_c = \frac{v^2}{r} = \frac{(20\text{ m/s})^2}{40\text{ m}} = 10\text{ m/s}^2$$

$$\omega = \frac{v}{r} = \frac{20\text{ m/s}}{40\text{ m}} = 0.5\text{ rad/s}$$

$$a_c = \omega^2 r = 10\text{ m/s}^2$$

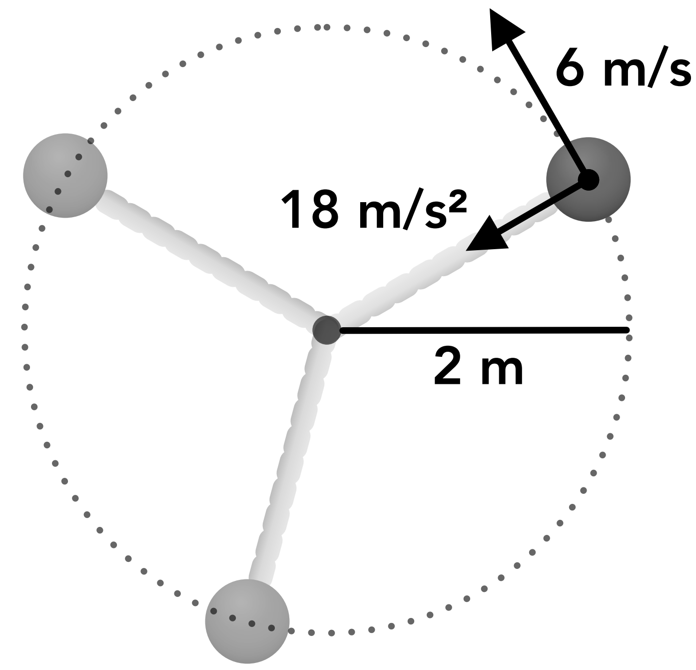
$$T = \frac{2\pi r}{v} = \frac{2\pi(40\text{ m})}{20\text{ m/s}} = 12.57\text{ s}$$

$$a_c = \left(\frac{2\pi}{T}\right)^2 r = 10\text{ m/s}^2$$

$$f = \frac{1}{T} = \frac{1}{12.57\text{ s}} = 0.0796\text{ Hz}$$

$$a_c = (2\pi f)^2 r = 10\text{ m/s}^2$$

An object is tied to a rope and swings around in a circle



$$a_c = \frac{v^2}{r} = \frac{(6\text{ m/s})^2}{2\text{ m}} = 18\text{ m/s}^2$$

$$\omega = \frac{v}{r} = \frac{6\text{ m/s}}{2\text{ m}} = 3\text{ rad/s}$$

$$a_c = \omega^2 r = 18\text{ m/s}^2$$

$$T = \frac{C}{v} = \frac{2\pi(2\text{ m})}{6\text{ m/s}} = 2.1\text{ s}$$

$$a_c = \left(\frac{2\pi}{T}\right)^2 r = 18\text{ m/s}^2$$

$$f = \frac{1}{T} = \frac{1}{2.1\text{ s}} = 0.48\text{ Hz}$$

$$a_c = (2\pi f)^2 r = 18\text{ m/s}^2$$

Centripetal Force

- Remember that if an object is traveling in circular motion then there must be a **centripetal acceleration** (an acceleration vector that points towards the center of the circular path). According to Newton's 2nd law of motion ($\vec{F}_{net} = m\vec{a}$) there must be a net force acting on the object in the direction of that acceleration.
- A **centripetal force** is what we call that **net force** acting in the radial direction (towards the center of the circle) which is causing a centripetal acceleration, which results in the circular motion.

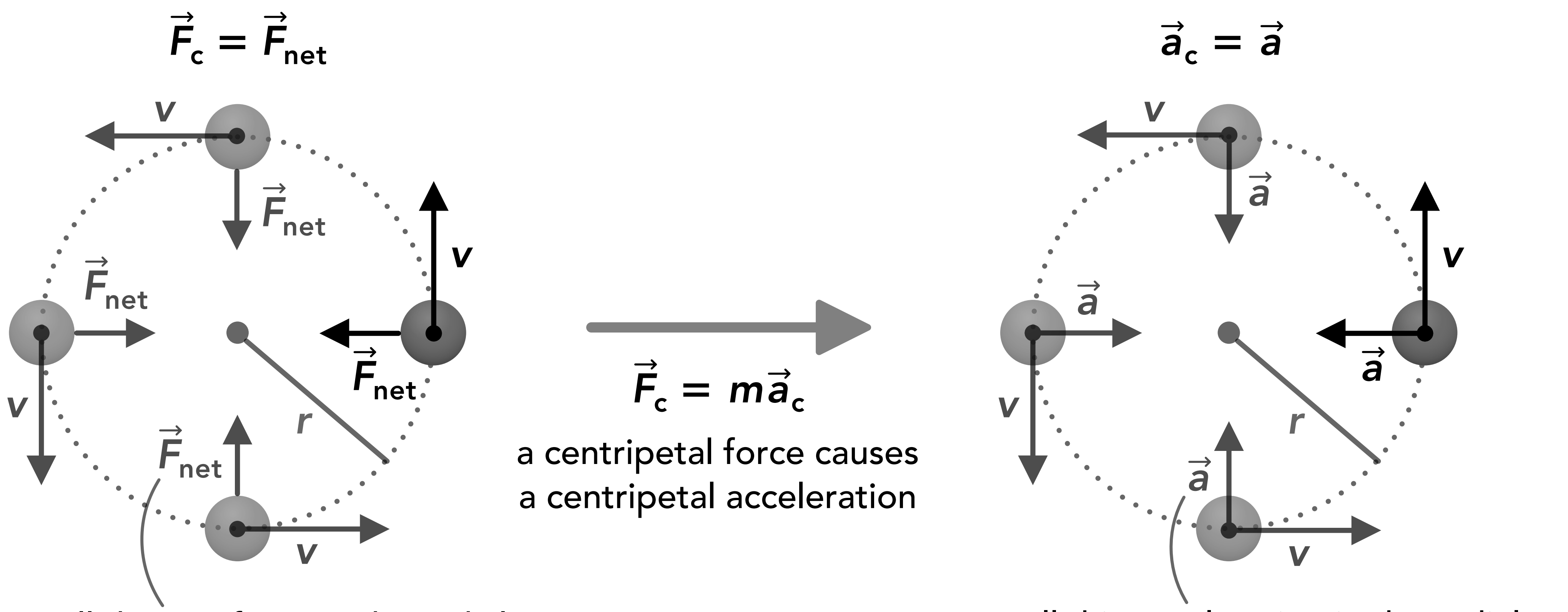
Variables		SI Unit
F_c	centripetal force	N
a_c	centripetal acceleration	$\frac{m}{s^2}$
v	velocity	$\frac{m}{s}$
r	radius	m

Centripetal force

$$\vec{F}_c = m\frac{v^2}{r} \quad \text{(towards center of circle)}$$

Centripetal acceleration

$$\vec{a}_c = \frac{v^2}{r} \quad \text{(towards center of circle)}$$

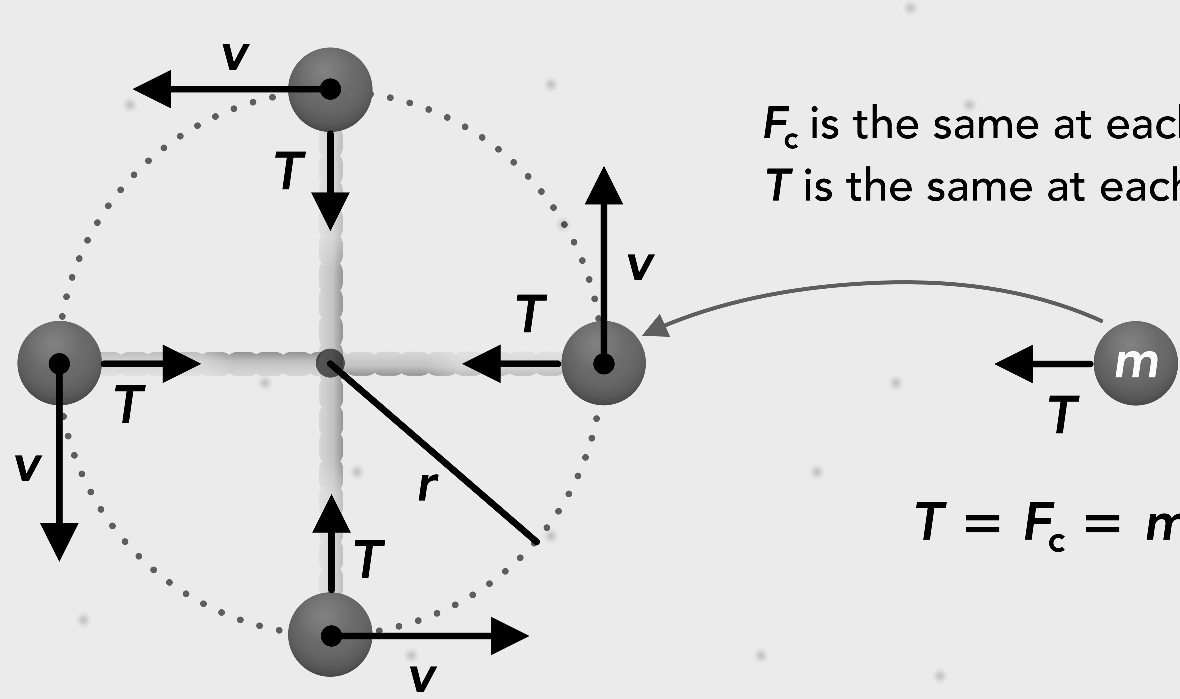


we call this net force in the radial direction the "centripetal force" $\vec{F}_c$

we call this acceleration in the radial direction the "centripetal acceleration" $\vec{a}_c$

- Centripetal force refers to the **net force** acting in the radial direction (towards the center of the circle) which is causing the object to move in circular motion.

A ball attached to a rope swings in uniform circular motion in space (assuming no gravity). The tension force on the ball from the rope is acting as the centripetal force, keeping the ball in circular motion.

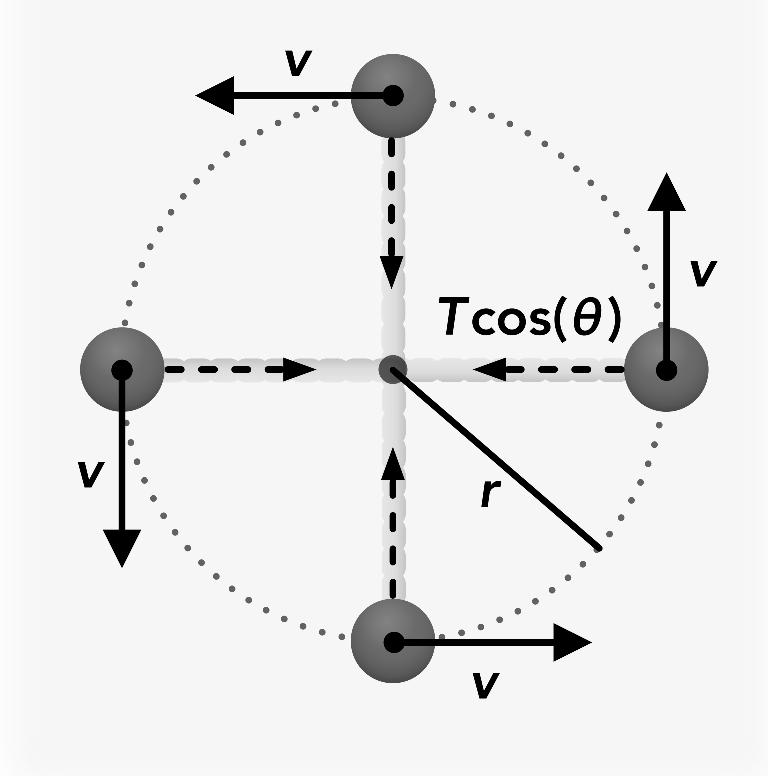


F_c is the same at each point
T is the same at each point

$$T = F_c = m\frac{v^2}{r}$$

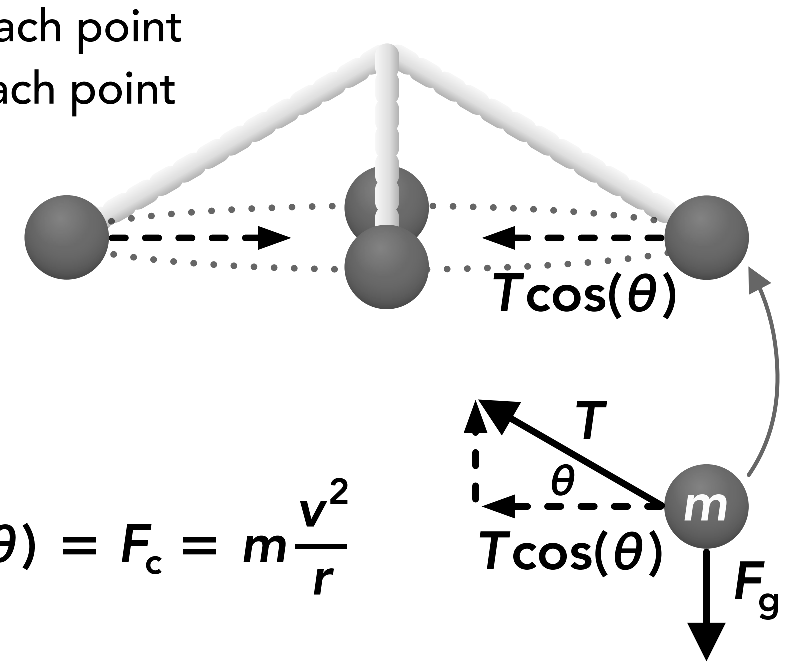
A ball is attached to a rope and swings in uniform circular motion. The circle is horizontal, parallel to the ground, but gravity causes the ball to pull the rope down at an angle. The horizontal component of the tension force, which always points towards the center of the circle, is acting as the centripetal force (not the entire tension force).

top view



side view

F_c is the same at each point
T is the same at each point

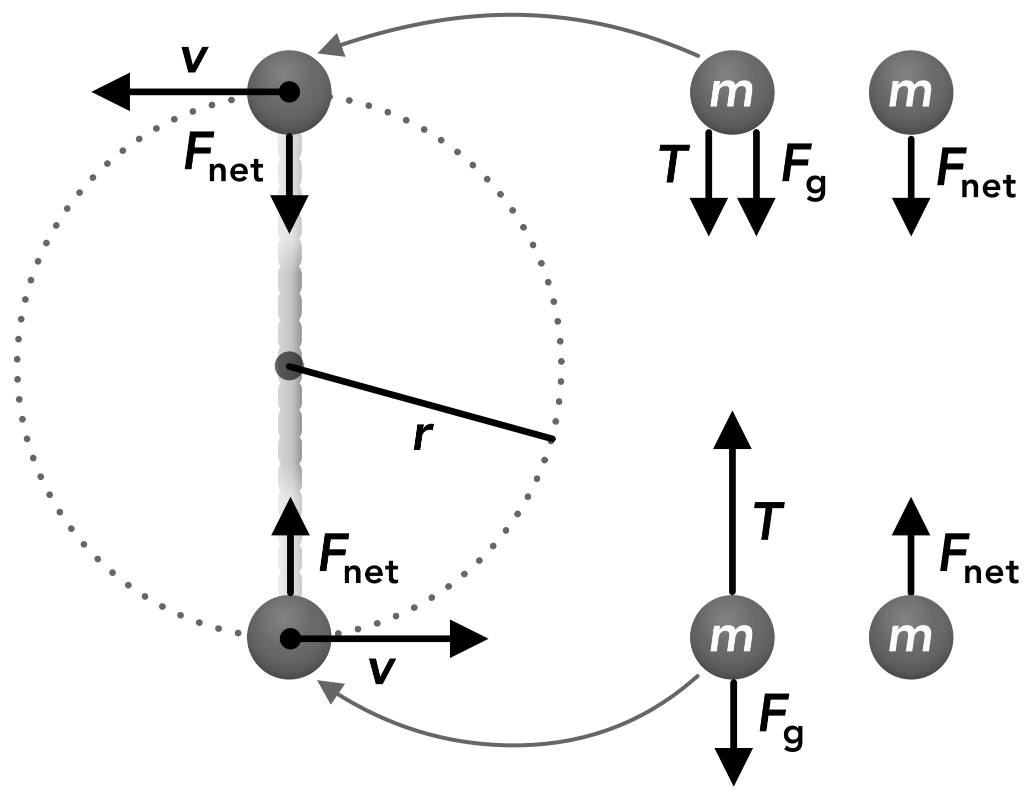


$$T\cos(\theta) = F_c = m\frac{v^2}{r}$$

A ball is attached to a rope and swings in a vertical circle. At each point there is a tension force and a gravitational force acting on the ball. Because the ball is in circular motion, the net force acting on the ball in the radial direction at any time is equal to the centripetal force.

F_c is the same at each point

F_g is the same at each point
T changes around the circle



$$F_{net} = F_c$$
$$T + F_g = m\frac{v^2}{r}$$

$$F_{net} = F_c$$
$$T - F_g = m\frac{v^2}{r}$$

- A common confusion when working with circular motion is the concept of "centrifugal force".
- **Centrifugal force** is a "fictitious force" which is a force that does not actually exist. When the circular motion of an object is viewed in the rotating reference frame (in which the object appears to be stationary) it may appear that a force is pushing or pulling the object **away from the center of the circle**, which we call a centrifugal force. This imaginary force only arises because of the rotating reference frame.
- The only real force acting on the object is an **inwards centripetal force**, not an outwards centrifugal force.
- According to Newton's 1st law of motion an object will maintain its velocity (continue moving in a straight line at a constant speed) unless acted on by a net force. In circular motion we call that net force the centripetal force. If that centripetal force suddenly disappeared the object would travel in a straight line tangent to the circle.