Topic: Sketching direction fields

Question: Sketch the direction field.

$$
y^{\prime}=y+x
$$

## Answer choices:



## Solution: B

Before you try to sketch the direction field, you want to make sure your equation is solved for $y^{\prime}$.

$$
y^{\prime}=y+x
$$

We'll need to make several tables, and our strategy will be to keep $x$ constant in each table. So our first table will be for $x=-2$. We'll explore $y$-values on the interval $[-2,2]$, and then pairing those $x$ and $y$ values together, we'll solve for values of $y^{\prime}$.

The table for $x=-2$ is

$$
\begin{array}{llllll}
y^{\prime} & -4 & -3 & -2 & -1 & 0 \\
x & -2 & -2 & -2 & -2 & -2 \\
y & -2 & -1 & 0 & 1 & 2
\end{array}
$$

The table for $x=-1$ is

$$
\begin{array}{cccccc}
y^{\prime} & -3 & -2 & -1 & 0 & 1 \\
x & -1 & -1 & -1 & -1 & -1 \\
y & -2 & -1 & 0 & 1 & 2
\end{array}
$$

The table for $x=0$ is

$$
\begin{array}{llllll}
y^{\prime} & -2 & -1 & 0 & 1 & 2 \\
x & 0 & 0 & 0 & 0 & 0 \\
y & -2 & -1 & 0 & 1 & 2
\end{array}
$$

The table for $x=1$ is

$$
\begin{array}{llllll}
y^{\prime} & -1 & 0 & 1 & 2 & 3 \\
x & 1 & 1 & 1 & 1 & 1 \\
y & -2 & -1 & 0 & 1 & 2
\end{array}
$$

The table for $x=2$ is

| $y^{\prime}$ | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $x$ | 2 | 2 | 2 | 2 | 2 |
| $y$ | -2 | -1 | 0 | 1 | 2 |

The values of $y^{\prime}$ that we found represent the slope of the function at the corresponding point $(x, y)$. For example, in this last table, we see the point $(2,-2)$, and the corresponding value of $y^{\prime}=0$. This means that the slope of the function at $(2,-2)$ is 0 , so we'd draw a small, short horizontal line right at $(2,-2)$. Plotting all of the other point-slope pairs, the direction field starts to look something like this:


If we add more points, maybe ones that are half-way between those that we already found, a more complete direction field should look something like this:
$\longrightarrow$

Topic: Sketching direction fields

Question: Sketch the direction field and the solution curve at the given point.

$$
y^{\prime}=y-2 x
$$

$$
\text { at }(1,1)
$$

## Answer choices:



## Solution: C

Before you try to sketch the direction field, you want to make sure your equation is solved for $y^{\prime}$.

$$
y^{\prime}=y-2 x
$$

We'll need to make several tables, and our strategy will be to keep $x$ constant in each table. So our first table will be for $x=-2$. We'll explore $y$-values on the interval $[-2,2]$, and then pairing those $x$ and $y$ values together, we'll solve for values of $y^{\prime}$.

The table for $x=-2$ is

$$
\begin{array}{llllll}
y^{\prime} & 2 & 3 & 4 & 5 & 6 \\
x & -2 & -2 & -2 & -2 & -2 \\
y & -2 & -1 & 0 & 1 & 2
\end{array}
$$

The table for $x=-1$ is

$$
\begin{array}{cccccc}
y^{\prime} & 0 & 1 & 2 & 3 & 4 \\
x & -1 & -1 & -1 & -1 & -1 \\
y & -2 & -1 & 0 & 1 & 2
\end{array}
$$

The table for $x=0$ is

$$
\begin{array}{llllll}
y^{\prime} & -2 & -1 & 0 & 1 & 2 \\
x & 0 & 0 & 0 & 0 & 0 \\
y & -2 & -1 & 0 & 1 & 2
\end{array}
$$

The table for $x=1$ is

| $y^{\prime}$ | -4 | -3 | -2 | -1 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $x$ | 1 | 1 | 1 | 1 | 1 |

$$
\begin{array}{llllll}
y & -2 & -1 & 0 & 1 & 2
\end{array}
$$

The table for $x=2$ is

| $y^{\prime}$ | -6 | -5 | -4 | -3 | -2 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $x$ | 2 | 2 | 2 | 2 | 2 |
| $y$ | -2 | -1 | 0 | 1 | 2 |

The values of $y^{\prime}$ that we found represent the slope of the function at the corresponding point $(x, y)$. For example, in this last table, we see the point $(2,-2)$, and the corresponding value of $y^{\prime}=6$. This means that the slope of the function at $(2,-2)$ is 6 , so we'd draw a small, short line with slope 6 right at $(2,-2)$. Plotting all of the other point-slope pairs, the direction field starts to look something like this:


If we add more points, maybe ones that are half-way between those that we already found, a more complete direction field should look something like this:


Sketching the solution curve through $(1,1)$ gives


