Test – Polynomial Functions, Equations & Inequalities

Syllabus content on test: ▪ polynomial functions ▪ quadratic formula and discriminant ▪ factor and factor theorems ▪ polynomial division ▪ sum and product of roots of a polynomial equation ▪ rational functions ▪ solving equations involving radicals, absolute value and in quadratic form ▪ solving inequalities

total marks on test: **60**

Part I: **No** calculator – questions 1-7 **[39 marks]**

1. The polynomial x^3+3x^2+ax+b has the same remainder when divided by $x-2$ as when divided by $x+1$. Find the value of a. [5 marks]

2. The function g is defined as $g(x)=\dfrac{4x+8}{2-x}$, $x\in\mathbb{R}$, $x\neq 2$.

Sketch the graph of $y=g(x)$, clearly indicating and stating the equations of any asymptotes and the coordinates of any axes intercepts. [5 marks]

3. Given that $2-i$ is a root of the equation $x^3-6x^2+13x-10=0$ find the other two roots. [5 marks]

4. $x-2$ and $x+2$ are factors of x^3+ax^2+bx+c, and it leaves a remainder of 10 when divided by $x-3$. Find the values of a, b and c. [6 marks]

5. Given $f(x)=x^2+x(k-1)+k^2$, find the range of values of k so that $f(x)>0$ for all real values of x. [5 marks]

6. If α and β are the roots of the equation $2x^2+6x-5=0$, find a quadratic equation with integer coefficients whose roots are:

(a) 2α, 2β (b) $\dfrac{1}{\alpha+1}, \dfrac{1}{\beta+1}$ [8 marks – 4 marks each]

7. The polynomial $h(x)=(px+q)^3$ leaves a remainder of -8 when divided by $(x+1)$, and a remainder of -27 when divided by $(x+3)$. Find the value of p and the value of q. [5 marks]

Part II: calculator allowed – questions 8-12 **[21 marks]**

8. Find the values of x for which $\dfrac{x^2+3}{x-1}>7$ [4 marks]

9. Find a polynomial of lowest degree with integer coefficients that has zeros $x=\dfrac{3}{2}$ and $x=1-2\mathrm{i}$. [4 marks]

10. Solve the inequality $x+3>\sqrt{7x+9}$ [4 marks]

11. Find the value(s) of m such that the following equation has two distinct real solutions.

$mx^2-2x-m-2=0$ [4 marks]

12. Find the roots of the quadratic equation $x^2-6x+k=0$ given that one root is three times the other root and that k is a constant. [5 marks]

Bonus question: The polynomial $P(x)=x^3+mx^2+nx-8$ is divisible by $(x+1+i)$. Find the value of m and the value of n. [+3 marks]