AS#10 LINEAR MODELLING



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APPRENTICE

The cost of electricity, E_{i} , in pounds and the number of kilowatt hours, h_{i} are shown in the table.

Kilowatt hours, <i>h</i>	0	15	40	60	80	110
Cost of electricity, E	45	46.8	49.8	52.2	54.6	58.2

- a. Draw a graph of the data.
- b. Explain how you know a linear model would be appropriate.
- c. Deduce an equation in the form E = ah + b
- d. Interpret the meaning of the coefficients a and b.
- e. Use the model to find the cost of electricity for 3 days..

EXPERT



The scatter graph shows the height h and foot length f of 8 students. A line of best fit is drawn on the scatter graph.

- a. Explain why the data can be approximated to a linear model.
- b. Use points A and B on the scatter graph to write a linear equation in the form h = af + b.
- c. Calculate the expected height of a person with a foot length of 26.5cm.

MASTER

The average August temperature in Exeter is 20°C or 68°F. The average January temperature in the same place is 9°C or 48.2°F.

- a. Write an equation linking Fahrenheit F and Celsius C in the form F = aC + b
- b. Interpret the values of a and b.
- c. The highest recorded un the UK was 101.3°F. Calculate this temperature in Celsius.

AS#11 QUADRATIC MODELLING



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APPRENTICE



The diagram shows a section of a suspension bridge carrying a road over water.

The height of the cables above water level in metres can be modelled by the function $h(x) = 0.00012x^2 + 200$, where x is the displacement in metres from the centre of the bridge.

- a. Interpret the meaning of the constant term 200 in the model.
- b. Use the model to find the two values of x at which the height is 346m.
- c. Given that the towers at each end are 346m tall, use your answer to part (b) to calculate the length of the bridge to the nearest metre.

EXPERT

A car manufacturer uses a model to predict the fuel consumption, y miles per gallon (mpg), for a specific model of a car travelling at a speed of x mph.

$$y = -0.01x^2 + 0.975x + 16, x > 0$$

- a. Use the model to find two speeds at which the car has a fuel consumption of 32.5mpg.
- b. Rewrite y in the form $A B(x C)^2$, where A, B and C are constants to be found.
- c. Using your answer to part (b), find the speed at which the car has the greatest fuel efficiency.
- d. Use the model to calculate the fuel consumption of a car travelling at 120mph. Comment on the validity of using this model for very high speeds.

MASTER

A fertiliser company uses a quadratic model to determine how the amount of fertiliser used, f kilograms per hectare, affects the grain yield g, measured in tonnes per hectare.

The maximum grain yield possible is $\frac{39}{4}$ tonnes per hectare, which is achieved by using 250 kg of fertiliser. Without any fertiliser, the yield of grain is 6 tonnes per hectare.

- a. Find an equation giving g in terms of f
- b. One farmer currently uses 20 kilograms of fertiliser per hectare. How much more fertiliser would he need to use to increase his grain yield by 1 tonne per hectare?



APPRENTICE

- a. Factorise $9x 4x^3$ completely
- b. Sketch the curve *C* with equation $y = 9x 4x^3$

Show on your sketch the coordinates at which the curve meets the x-axis.

EXPERT

f(x) = (x - 2)(2x + 3)(5 - x)The graph of y = f(x) is translated by $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ to obtain the graph of y = g(x).

State the equation of y = g(x) and sketch its graph, showing the coordinate intercepts.

MASTER



The sketch shows part of the curve $f(x) = (2x - 5)^2(x + 3), x \in \mathbb{R}$

a. Given that the curve with equation y = f(x) + k passes through the origin, find the value of the constant k,

b. The curve with equation y = f(x + c) passes through the origin. Find the possible values of the constant c.

AS#13 GRAPH TRANSFORMATIONS



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APPRENTICE

The diagram above shows a sketch of the curve with equation y = f(x). The curve has a maximum point A at (-2,3) and a minimum point B at (3, -5).

On separate diagrams sketch the curve with equation



b. y = 2f(x)

a. y = f(x + 3)

On each diagram show clearly the coordinates of the maximum and minimum points.

c. The graph of y = f(x) + a has a minimum at (3,0), where a is a constant. Write down the value of a.

EXPERT

The graph of the function y = f(x) is shown.

It has a horizontal asymptote y = 4 and a minimum point at P(0, -2)

Sketch the graph of 2 - f(3x) showing the equation of the asymptote and the new coordinates of the point *P*



MASTER

A function f(x) is defined by $f(x) = x^2 - 2x$.

The minimum point of the graph of y = kf(x) is the same coordinate as the minimum point of the graph of y = f(x) - c.

Find the relationship between k and c



APPRENTICE

- a. The graph of $f(x) = 2 (x + 3)^2$ is translated left 2 then stretched scale factor 3 parallel to the y axis. What is the equation of this transformed graph?
- b. The graph of f(x) = -2(x-3)(x+5) is translated right 1 then reflected over the x axis.

i. What is the equation of this transformed graph?

ii. What is the equation of the transformed graph if we reflect over the x axis first, then translate right 1?

EXPERT

- a. Factorise completely $x^3 6x^2 + 9x$
- b. Sketch the curve with equation $f(x) = x^3 6x^2 + 9x$ showing the coordinates of the points at which the curve meets the x-axis.
- c. State and fully simplify the equation of f(-2x)
- d. Using your answer to part (b), sketch, on a separate diagram, the curve with equation

$$y = (x - 2)^3 - 6(x - 2)^2 + 9(x - 2)$$

showing the coordinates of the points at which the curve meets the x-axis

MASTER

- a. The graph of $g(x) = 4^x$ is translated right 1 then stretched scale factor 2 parallel to the y axis. Write the equation of the transformed graph in the form 4^k where k is a function of x
- b. Describe fully a single transformation that transforms the curve $y = 2\sqrt{x}$ to $y = 3\sqrt{5x}$.



APPRENTICE

Sketch the following graphs on separate axes:

a.
$$y = (x - 4)^3 + 1$$

b. $y = 4 - (x + 1)^3$

c. $y = 4(x - 1)^4 + 1$ b. $y = 2 - 5(x + 1)^4$

EXPERT

Describe the sequence of transformations which maps

a.
$$y = (x + 1)^4$$
 onto $y = -(x - 2)^4$

b. $y = -2x^3 + 1$ onto $y = -4x^3 + 1$

MASTER

Find the equation of the graph $y = x^3$ after the following transformations:

a. Reflection over the y axis followed by translation $\begin{pmatrix} -4\\ 1 \end{pmatrix}$

b. Translation $\begin{pmatrix} 0\\ -2 \end{pmatrix}$ followed by vertical stretch scale factor 6



APPRENTICE

Sketch the following graphs on separate axes:

a.
$$y = \frac{2}{x-4}$$
 b. $y = 4 - \frac{1}{x+1}$

c.
$$y = \frac{4}{x^2} - 1$$
 d. $y = 2 - \frac{1}{(x+3)^2}$

EXPERT

- a. Describe the transformations which maps the graph of $y = \frac{1}{5x}$ onto the graph of $y = \frac{1+10x}{5x}$
- b. Describe the transformations which maps the graph of $y = \frac{1}{x+1}$ onto the graph of $y = \frac{1}{x-4} + 3$

MASTER

a. Find the equation of the graph $y = \frac{1}{x}$ after the following transformations: Reflection over the y axis followed by translation $\begin{pmatrix} -4\\1 \end{pmatrix}$ b. Find the equation of the graph $y = \frac{1}{x^2}$ after the following transformations: Translation $\begin{pmatrix} 0\\-2 \end{pmatrix}$ followed by vertical stretch scale factor 6



APPRENTICE

State the equations of the asymptotes of the graph of $y = \frac{x-2}{2x+1}$

EXPERT

Write $\frac{x-2}{x+1}$ in the form $A + \frac{B}{x+1}$ where A and B are constants to be found

Sketch $y = \frac{x-2}{x+1}$ showing the axis intercepts and equations of the asymptotes

MASTER

Sketch $y = \frac{2x-1}{x+5}$ showing the axis intercepts and equations of the asymptotes

AS#18 SOLVING INEQUALITIES



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APPRENTICE

A curve has equation $y = 2x^2 + x - 10$.

Determine the set of values of x for which the graph of the curve lies above the x-axis.

EXPERT

Find the set of values of x for which

- a. 3(2x+1) > 5 2x
- b. $2x^2 7x + 3 > 0$
- c. both 3(2x + 1) > 5 2x and $2x^2 7x + 3 > 0$.

MASTER

The width of a rectangular sports pitch is x metres, x > 0. The length of the pitch is 20m more than its width. Given that the perimeter of the pitch must be less than 300m,

a. Form a linear inequality in x

Given that the area of the pitch must be greater than $4800 \mathrm{m}^2$,

- b. Form a quadratic inequality in x.
- c. By solving your inequalities, find the set of possible values of x.

AS#19 SKETCHING INEQUALITIES



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APPRENTICE

The sketch shows the graphs of the lines x = 1, y = x + 1 and y = 7 - x

- a. Find the coordinates of the points of intersection of these lines
- b. Write down a set of inequalities that represent the shaded region R shown in the sketch



EXPERT

On coordinate axes, sketch the region satisfied by the following inequalities:

 $x \ge -1$ y + x < 4 $2x + y \le 5$ y > -2

MASTER

a. On coordinate axes, shade the region satisfying the inequalities

 $x \ge -1$ y + x < 4 $2x + y \le 5$ y > -2

b. Which of the vertices of the shaded region lie inside the region identified by the inequalities?

c. Find the area of the shaded region