KEY

1.4: Solving Absolute Value Equations

The ***absolute value*** of a number is its *distance from zero* on a number line. Since distance is always non-negative, absolute values are always non-negative.

Symbol: $|x|$

Another way of understanding it is that the absolute value bars are like a "positivity machine." Any number that enters the positivity machine will come out *positive*. Zero will come out as zero.

Ex #1: Please evaluate the following if $x = -2$.

a. $|4x+3|-3\frac{1}{2}$

$|4(-2)+3|-3\frac{1}{2}$

$|-8+3|-3\frac{1}{2}$

$|-5|-3\frac{1}{2}$ → $5-3\frac{1}{2}$

$1\frac{1}{2}$

b. $-2|3-x|+8$

$-2|3-(-2)|+8$

$-2|5|+8$

$-2(5)+8$

$-10+8$

-2

Solving Absolute Value Equations – "BIFURCATE" – meaning, dividing into two branches

$|x-5|=3$

positive branch: $x-5=3$ → $x=8$

negative branch: $x-5=-3$ → $x=2$

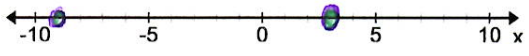
(then solve both branches)

Ex #2: Please solve each equation. Then graph your solution(s) on a number line.

a) $|x+3|=6$

$x+3=6$ → $x=3$

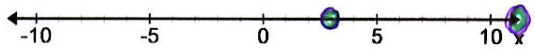
$x+3=-6$ → $x=-9$



b) $|x-7|=4$

$x-7=4$ → $x=11$

$x-7=-4$ → $x=3$



No solution?

We know that an absolute value is always equal to a positive number.

Thus, whenever an absolute value equation equals a *negative number*, there is ***no solution***.

Here are some examples of an equation having "no solution" for the variable, 'a'.

$|a| = -8$ (there is no number that a can be that would make the equation true)

So there is no solution for a

$-2|3a| = 8$ (divide both sides by –2, to see that abs. value = neg.)

$|3a| = -4$

abs value ≠ neg, so no solution exists for a.

Ex #3: **Extraneous Solutions** – When an absolute value expression is set equal to an expression containing a variable, ***extraneous solutions*** may be encountered.

(Hint: first combine like terms. Then isolate the absolute value. Then bifurcate, and solve each.)

$2|x+1| - x = 3x - 4$

$+x \quad +x$

$\frac{2|x+1|}{2} = \frac{4x-4}{2}$

$|x+1| = 2x-2$

$x+1 = 2x-2$

$-x+2 \quad -x+2$

$3 = x$ (plug in to check)

$2|3+1| - 3 = 3(3) - 4$

$2|4| - 3 = 9 - 4$

$8 - 3 = 5$

$5 = 5$ ✓ so $x = 3$

$x+1 = -(2x-2)$

$x+1 = -2x+2$

$+2x \quad -1 \quad +2x \quad -1$

$3x = 1$

$x = \frac{1}{3}$ (plug in to check)

$2\left|\frac{1}{3}+1\right| - \frac{1}{3} = 3\left(\frac{1}{3}\right) - 4$

$2\left|\frac{4}{3}\right| - \frac{1}{3} = 1 - 4$

$\frac{8}{3} - \frac{1}{3} = -3$

$\frac{7}{3} \neq -3$

so $x = \frac{1}{3}$ is extraneous

So $x = 3$ is the solution.